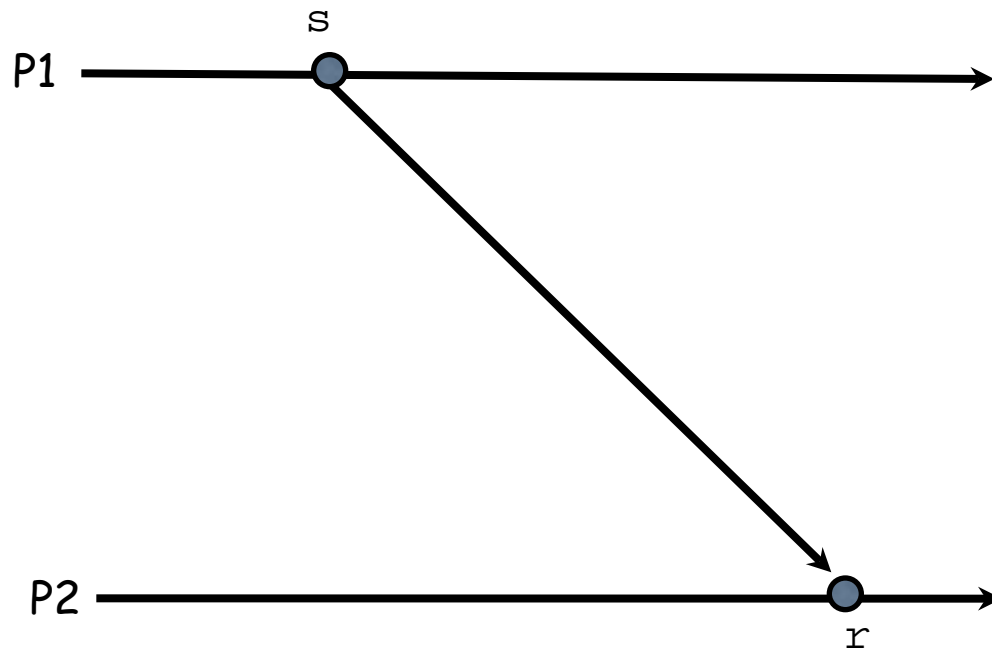


# Lecture 15

□ Administration



# "happens before" Relation

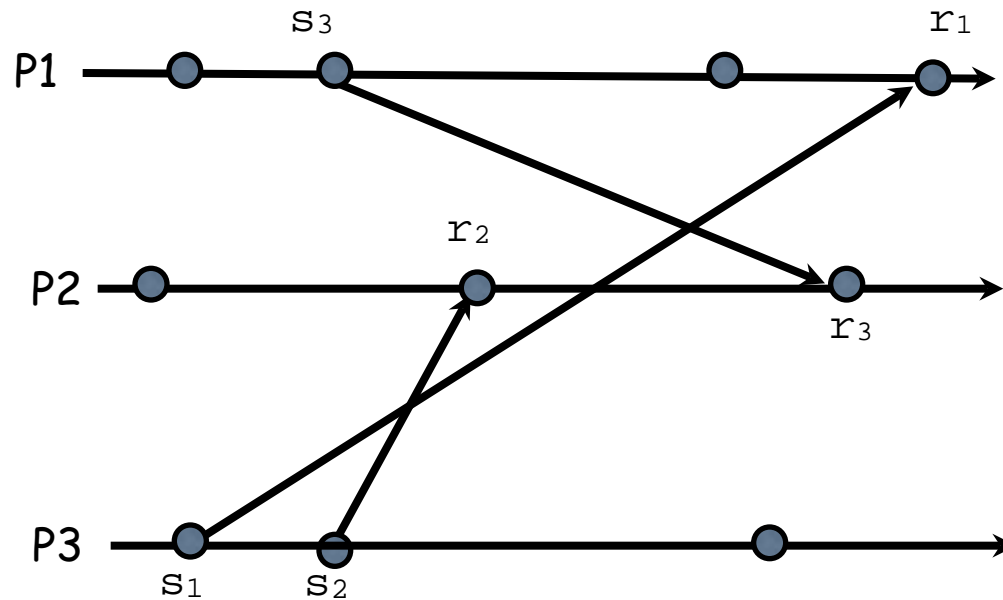
$$e_i^x \rightarrow e_j^y \Leftrightarrow \begin{cases} e_i^x \xrightarrow{i} e_j^y (i == j) \wedge (x < y) \\ e_i^x \xrightarrow{msg} e_j^y \\ \exists e_k^z \in H \text{ s.t. } e_i^x \rightarrow e_k^z \wedge e_k^z \rightarrow e_j^y \end{cases}$$

Properties of H:

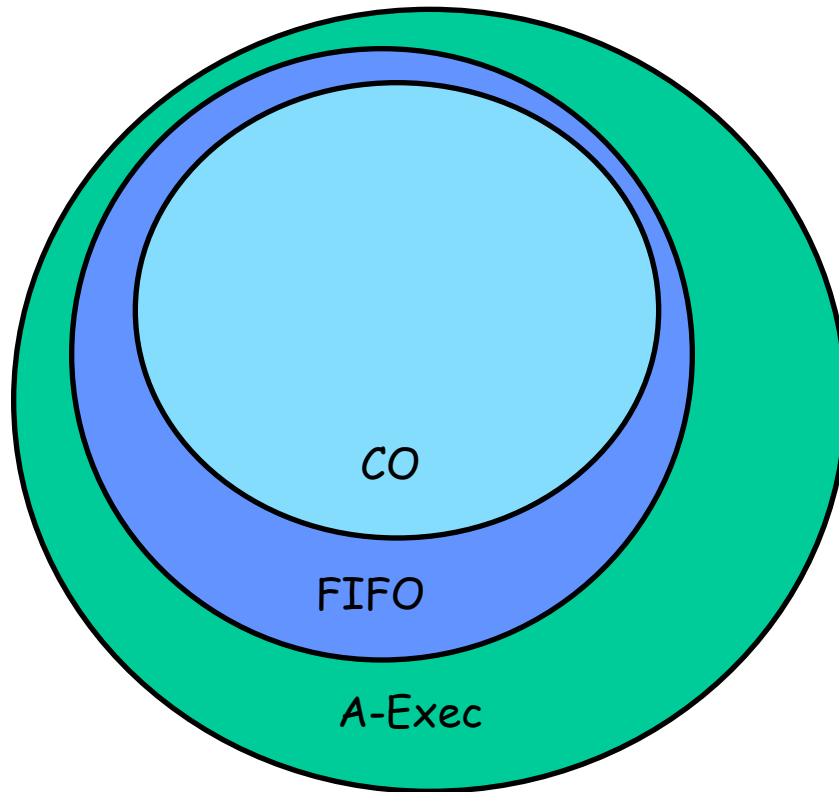
1. NOT  $a < a$  (irreflexive)
2. If  $a < b$  Then NOT  $b < a$  (asymmetric)
3. Transitive

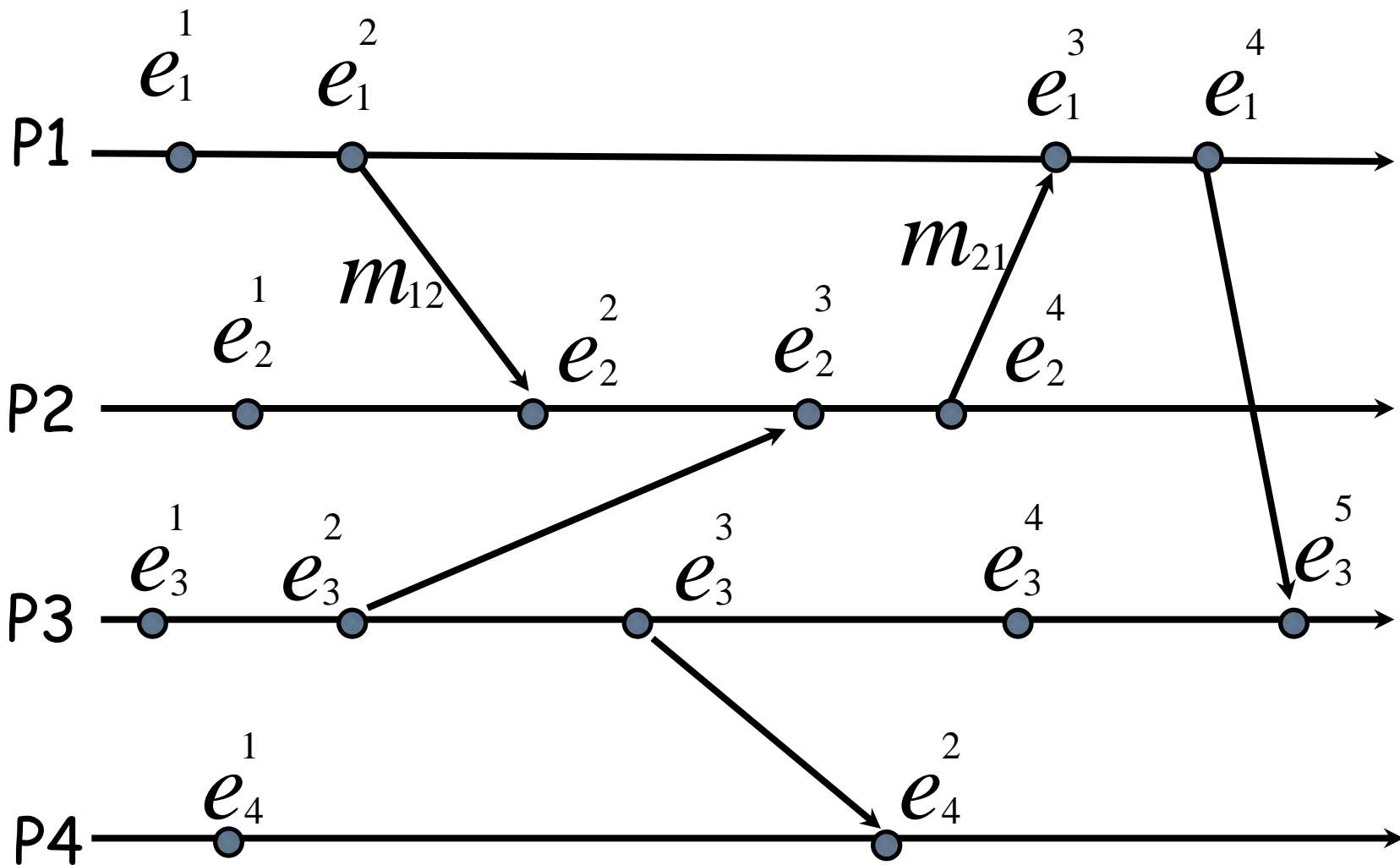
Defines a strict partial order on events  
(causal ordering)

# Asynchronous Communication



# Ordered Communication Hierarchy





# Communication

- Not a POSET

$$e_i^x \rightarrow e_j^y \wedge e_j^y \rightarrow e_i^x$$

- ASYNC (a POSET)

- FIFO

$$e_i^x \rightarrow e_j^y \wedge e_i^{x'} \rightarrow e_j^{y'}; e_i^x \rightarrow e_i^{x'} \Rightarrow e_j^y \rightarrow e_j^{y'}$$

- Causal Order

$$\text{send}(m_{ij}) \rightarrow \text{send}(m_{kj}) \Rightarrow \text{recv}(m_{ij}) \rightarrow \text{recv}(m_{kj})$$

- SYNC

# State of Channel

All messages that have been sent but not yet received.

$$S_{ij}^{x,y} = \{m_{ij} : \text{send}(m_{ij}) \leq \text{recv}(m_{i,j}) > LS_j^y\}$$

$LS_j^y$  The state of process j after the occurrence of event  $e_j^y$



# Global State

$$GS = \left\{ \bigcup_i LS_i^{x_i}, \bigcup_{j,k} S_{jk}^{y_j, z_k} \right\}$$

Consistent or Inconsistent

# Terms

- ❑ Concurrent
- ❑ Cut some Global State
- ❑ Consistent, transitless (no outstanding messages), strongly consistent (consistent and transitless)

# Logical clocks

Assign sequence numbers to messages

- m All cooperating processes can agree on order of events
- m vs. **physical clocks**: time of day

Assume no central time source

- m Each system maintains its own local clock
- m No total ordering of events
  - No concept of *happened-when*