SIGN RESTRICTIONS IN SVARs

Spring 2020

BACK TO BASICS

- Why SVARs?
- We only have a reduced form VAR(p):

$$Y_t = \sum_{j=0}^{p} B_j Y_{t-j} + U_t.$$

- Problem: U_ts are correlated, but we think they contain bits and pieces of uncorrelated shocks or treatments. How do we recover them?
- We need contemporaneous relationships (i.e. some variable affects another variable within the period)
- In the notation of the SVAR:

$$A_0 Y_t = \sum_{j=0}^{p} A_j^{+} Y_{t-j} + E_t.$$

- A_0 is a matrix representation of the relationships of contemporaneous variables

HOW DO WE GET A_0?

1. Zero restrictions: We say that some variables don't affect others with a period (this includes long-run restrictions)
2. Heteroskedasticity: Using variation in the variances of shocks in different periods to recover A_0, assuming A_0 doesn't change over time
3. External instruments: Use a noisy measure of a desired shock as a kind of instrument for that shock
4. Sign restrictions: We restrict A_0 to include only sets where the responses to a shock match our specified sign over a horizon (e.g. we say that interest rate shocks can't increase the price level for n quarters)

POINTS VS. SETS

- In the first few schemes, if we have an appropriate number of restrictions, there is only ONE A_0 that satisfies those restrictions given B and Σ (reduced form covariance matrix). That is, there is one POINT that satisfies the restrictions
- For a given set of sign restrictions, there can be MANY A_0s for which those restrictions are satisfied given a set of parameters
- Why? We can rotate A_0 a little, but not so much that the restictions are violated. Then, this rotated A_0 is also another potential candidate for the structural matrix.
- So, there is a SET of A_0s that satisfy the restrictions
- This is the difference between point and set identification
- A problem: as a result, we might get responses that are actually responses to a linear combination of separate structural shocks

WHY SIGN RESTRICTIONS?

- In some sense, they don't make you assume so much. Even a few restrictions on the sign of responses can narrow things down significantly
- Easy to understand (I hope) and pretty easy to implement, especially compared to some other approaches
- The approach is flexible. With a few modifications to the procedure, you can implement them alongside zero restrictions, narrative schemes, etc.

HOW CAN WE IMPLEMENT IT?

1. Draw the reduced form parameters B and Σ from the posterior
2. Compute $A^* = \Sigma^{1/2}$, which is the lower triangular matrix from the Cholesky decomposition
3. Obtain a matrix X with the same dimensions as A^*, but each element is drawn from $\mathcal{N}(0,1)$
4. Compute the Q matrix of the QR decomposition of X
5. Let $A_0 = QA^*$ and compute the impulse responses under this structural matrix
6. If the computed impulse responses satisfy the restrictions, store the draw. Otherwise, reject
7. Repeat until you've achieved the desired number of accepted draws.

WHY DOES THIS WORK?

- Since $(\Sigma^{1/2}Q)(\Sigma^{1/2}Q)' = \Sigma$, we can draw $\Sigma^{1/2}$ and then think about drawing Q, the rotation matrix
- In this above implementation, we assume that each rotation is equally likely, except for the ones that don't satisfy the restrictions
- It turns out that drawing a random matrix X and computing Q from the QR decomposition is a way to draw it from a uniform distribution!
- Then, rejecting draws that don't satisfy the restrictions is akin to putting another prior on the rotation matrices, where the rejected rotations have probability zero.

A SIMPLE EXAMPLE

- Four monthly variables log$(INDPRO)$, log(CPI), EBP, and FFR from 1973 to 2016.
- We make the following restrictions

Variable	Shock			
	log$(INDPRO)$	log(CPI)	EBP	FFR
log$(INDPRO)$				
log(CPI)				-
EBP				+
FFR		+		+

A SIMPLE EXAMPLE

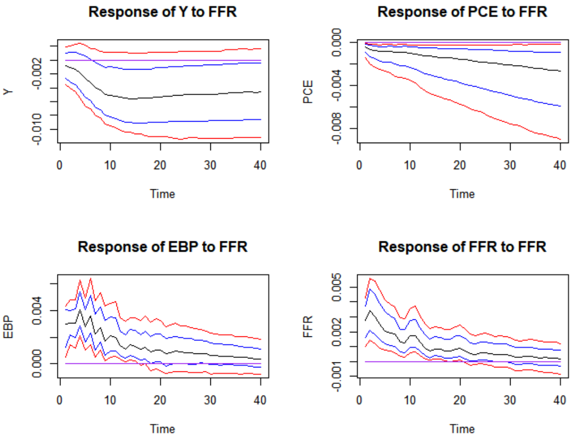

Figure 1: Monetary Policy Shock Responses

WHY NOT SIGN RESTRICTIONS?

- Even though we draw rotation matrices from a uniform distribution, the implications for the distributions of the responses can look really informative! That's a standard problem of uniform distributions, in general
- Our priors for the distribution of the responses won't die out even if we have infinite data. Only the boundaries will become sharper
- The restrictions on the rotation matrices might not be informative enough! As mentioned before, there will be estimated structural shocks that are linear combinations of the true structural shocks. This can lead to terrible misidentification in some cases