THE THREE EQUATION NEW KEYNESIAN MODEL IN DYNARE

April 2020

OUTLINE

- Recap the three equation model
- Take a brief detour to talk about filtering
- Going through a .mod file
- What is Dynare doing?
- Some results and implications for filtering
- A larger-scale example

MODEL EQUATIONS

- New Keynesian IS (NKIS) Curve

$$x_t = \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t[\pi_{t+1}]\right) + (\rho_z - 1)\frac{1+\eta}{\eta+\sigma} z_t$$

- New Keynesian Phillips Curve (NKPC)

$$\pi_t = \frac{(\sigma+\eta)(1-\omega\beta)(1-\omega)}{\omega} x_t + \beta \mathbb{E}_t[\pi_{t+1}] + v_t$$

- Taylor Rule

$$i_t = \phi_\pi \pi_t + \phi_x x_t + u_t$$

- Shock Processes (Why do we need three?)

$$\begin{aligned} z_t &= \rho_z z_{t-1} + \sigma_z \epsilon_t^z \\ v_t &= \rho_v v_{t-1} + \sigma_v \epsilon_t^v \\ u_t &= \rho_u u_{t-1} + \sigma_u \epsilon_t^u \end{aligned}$$

DATA AND DETRENDING

- Take inflation, FFR, and rGDP data from FRED
- Demean inflation and FFR and detrend rGDP
- Detrending is something we should think about briefly
- Two methods I'll talk briefly about
    1. Hodrick-Prescott Filter
    2. Simple Regression Filter
- Want to think about what the filter does to different frequencies (i.e. its gain). Which frequencies does it silence or turn up?

HODRICK-PRESCOTT FILTER

- Involves solving the problem

$$\min_{\tau_t} \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(y_t - \tau_t) - (\tau_t - y_{t-1}) \right]^2 .$$

- λ controls the degree of smoothing (1600 is common) and we have a nice, closed-form solution for the problem
- Can also write in transition/observation form where $\sigma_c^2 / \sigma_v^2 = \lambda$

$$\begin{aligned} \tau_t &= 2\tau_{t-1} - \tau_{t-2} + v_t \\ y_t &= \tau_t + c_t \end{aligned}$$

- Gain of the filter has the form

$$g(\omega) = \frac{4\lambda(1 - \cos(\omega))^2}{1 + 4\lambda(1 - \cos(\omega))^2}$$

REGRESSION FILTER

- Conceptually simple. Instead of assuming that our trend is just low frequency movements, this approach tries to model the trend and calls prediction error the cyclical component
- Run regressions of the form

$$\begin{aligned} y_{t+h} &= \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + \epsilon_{t+h} \\ y_t &= \beta_0 + \beta_1 y_{t-h} + \beta_2 y_{t-h-1} + \beta_3 y_{t-h-2} + \beta_4 y_{t-h-3} + \epsilon_t \end{aligned}$$

- Hamilton suggests $h = 8$ and $p = 4$.
- Then the trend and cyclical components are

$$\begin{aligned} \tau_t &= \beta(L) y_t = (\beta_1 L^h + \beta_2 L^{h+1} + \beta_3 L^{h+2} + \beta_4 L^{h+3}) y_t \\ c_t &= (1 - \beta(L)) y_t = C(L) y_t. \end{aligned}$$

- Further, we can calculate the gain as

$$g = \left(C(e^{i\omega}) C(e^{-i\omega})\right)^{\frac{1}{2}}$$

SPECTRAL COMPARISON

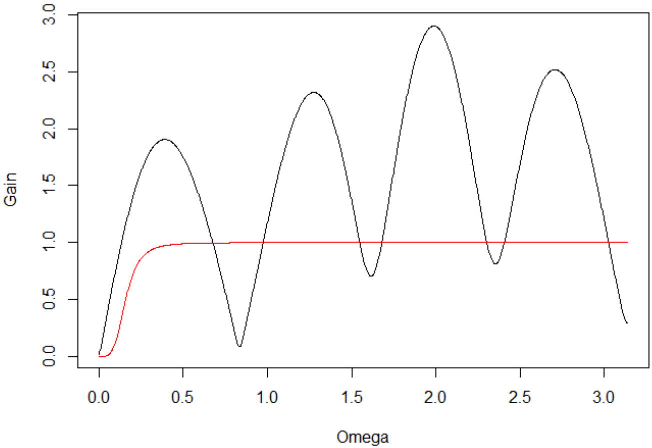

Figure 1: Regression v. HP Gain

TREND COMPARISON

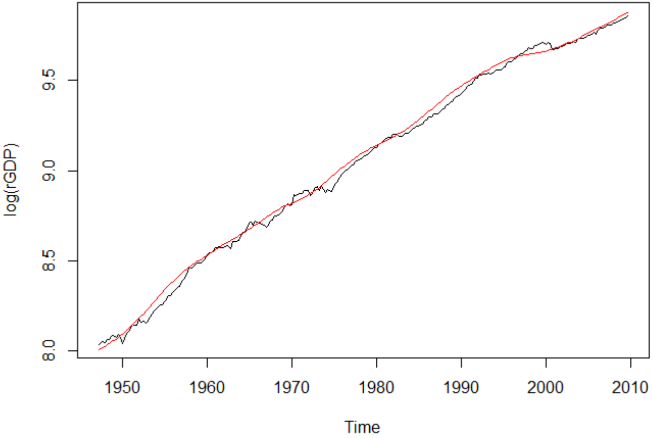

Figure 2: Regression v. HP Trends

THE ANATOMY OF A .MOD FILE

- For this, we'll take a look at the .mod file for the three equation New Keynesian model

UNDER THE HOOD (SOLVING THE MODEL)

- We want a simple transition equation for the state variables in terms of past states and exogeneous variables. The form we want is

$$s_t = Qs_{t-1} + R\epsilon_t$$

- Dynare first takes the equations and stacks them systematically to get a matrix equation that can be solved to get the transition equation of the form above
- That starting matrix equation is sometimes called a canonical form
- Blanchard-Kahn form is one (ss are state variables, ps are jump variables, and Zs are exogenous processes):

$$\begin{bmatrix} s_{t+1} \\ \mathbb{E}_t[p_{t+1}] \end{bmatrix} = A \begin{bmatrix} s_t \\ p_t \end{bmatrix} + \gamma Z_t$$

UNDER THE HOOD (SOLVING THE MODEL)

- What Dynare actually uses is Sims' (2002) canonical form and method (code and paper can be found on his website). The canonical form is

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + C + \Psi z_t + \Pi \eta_t$$

- In the above, η is expectational error. Each x_t can be written as $\mathbb{E}_{t-1}[x_t] + \eta_t$. In this way, we can handle expectations
- Notice that we don't have any parametric form for η_t. We must use the structure of the model to pin it down
- In the most abstract terms, this method involves decomposing the Γ_0 and Γ_1 matrices (which might not be full rank or square) and solving for η_t
- This gives us conditions for the stability and uniqueness of the equilibrium

UNDER THE HOOD (MAXIMIZING THE POSTERIOR)

- Before you start your MCMC, you want to start at a point of high probability and you also want a decent proposal density
- Maximizing the posterior can get you both
- But first, have to compute the posterior likelihood for a given set of parameters
- If we add an observation equation to our transition equation, our problem is in state space form and we can use the Kalman filter to evaluate the likelihood $P(Y|\theta)$. That is

$$
\begin{aligned}
s_t &= Q(\theta)s_{t-1} + R(\theta)\epsilon_t \\
y_t &= P(\theta)s_t + u_t
\end{aligned}
$$

- Multiply that likelihood by the prior and you have the posterior likelihood
- Give that function to an optimizer that will return the Hessian and the maximum of the posterior likelihood

UNDER THE HOOD (ESTIMATING THE MODEL)

- Now you can use an MCMC to sample from the posterior
- Your initial parameter draw $\theta_0 = \theta^{MAX}$ and your proposal density $\theta_t \sim \mathcal{N}(\theta_{t-1}, -cH^{-1})$, where c is a scale factor and H is the Hessian at the posterior maximum
- Then, the RWMH algorithm is
  - Initialize θ_0 and LLH_0
  - For t in 1:#DRAWS
    - Draw $\theta^p \sim \mathcal{N}(\theta_{t-1}, -cH^{-1})$
    - Compute the posterior likelihood $LLH^p = P(Y|\theta^p)\pi(\theta^p)$
    - If $(LLH^p/LLH_{t-1}) > 1$, $\theta_t = \theta_p$
    - Else, draw $u \sim \mathcal{U}(0,1)$
    - If $(LLH^p/LLH_{t-1}) > u$, $\theta_t = \theta_p$
    - Else $\theta_t = \theta_{t-1}$
- Now we have our draws and we can compute posterior statistics (means, credible intervals, etc.)

UNDER THE HOOD (COMPUTING IRFs)

- The IRF of a state variable is (over a horizon h)

$$Q(\theta)^h R(\theta)$$

- To compute pointwise (i.e. for each horizon) credible intervals, we compute IRFs for each draw of θ and compute the credible intervals (percentiles, basically) of the value of the IRF at eash horizon

ON THE MENU

- The estimation command has multiple different options available to you for computing the mode, defining the jump distribution, using the Kalman Filter, initializing the states, etc.
- Pick the ones that make sense for your model, but the defaults are usually okay
- One to be aware of: in `mode_compute`, I use option 6 for several reasons
    - Sometimes quasi-Newton methods (which approximate the hessian instead of calculating it) will produce a hessian that's not positive definite
    - You don't have to worry about tuning the scale parameter here
- Drawback is that it's not really an optimizer

POSTERIORS FOR HP DATA

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\sigma - 1$	gamm	0.100	0.0500	0.216	0.0075	0.2047	0.2279
η	norm	1.000	0.2500	2.453	0.0607	2.3547	2.5474
ω	beta	0.500	0.1000	0.059	0.0026	0.0541	0.0627
β	beta	0.990	0.0025	0.996	0.0003	0.9954	0.9964
ϕ_π	gamm	1.700	0.1000	1.918	0.0100	1.9045	1.9360
ϕ_x	gamm	0.125	0.1000	0.203	0.0050	0.1960	0.2117
ρ_z	beta	0.800	0.0500	0.982	0.0009	0.9806	0.9832
ρ_v	beta	0.800	0.0500	0.636	0.0149	0.6155	0.6583
ρ_u	beta	0.800	0.0500	0.791	0.0093	0.7752	0.8019
ϵ^z	invg	1.000	2.0000	0.215	0.0118	0.1961	0.2357
ϵ^v	invg	1.000	2.0000	0.596	0.0154	0.5740	0.6222
ϵ^u	invg	1.000	2.0000	0.130	0.0006	0.1290	0.1304

MARGINAL POSTERIOR DISTRIBUTIONS WITH HP DATA

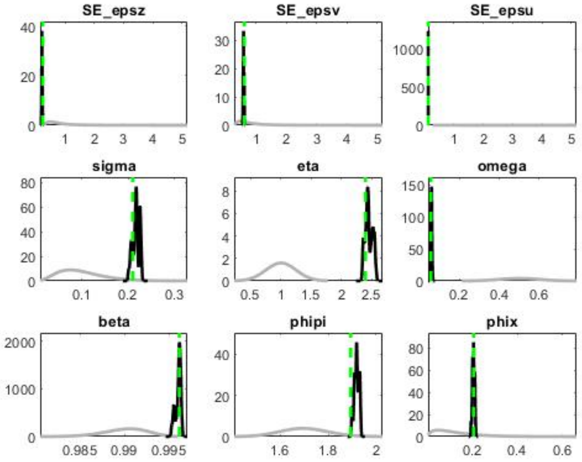

Figure 3: Marginals 1

MARGINAL POSTERIOR DISTRIBUTIONS WITH HP DATA

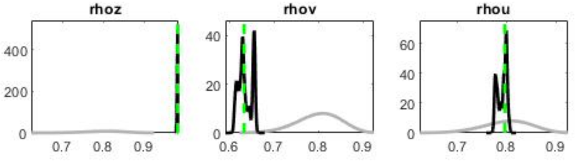

Figure 4: Marginals 2

IRFs WITH HP DATA (MP SHOCK)

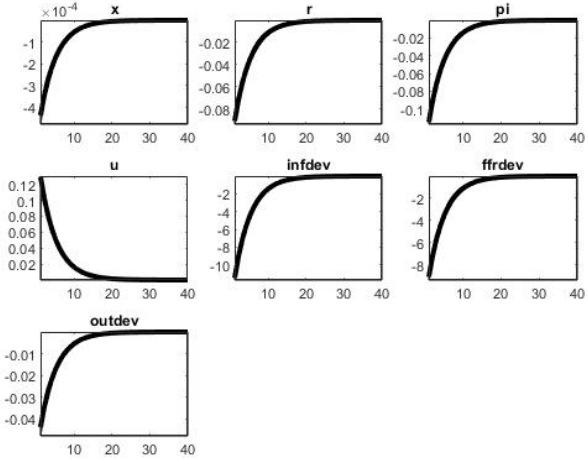

Figure 5: MP Shock IRFs

IRFs WITH HP DATA (CP SHOCK)

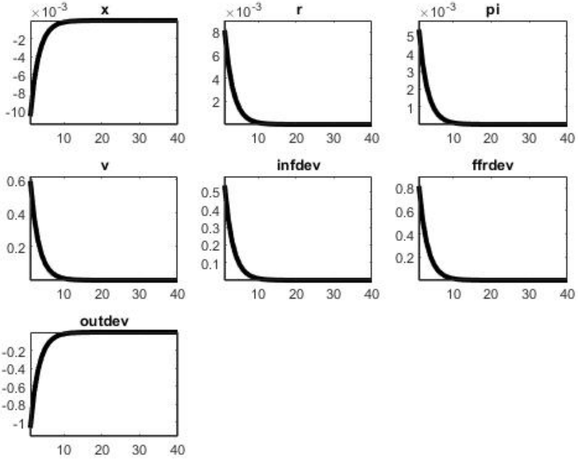

Figure 6: CP Shock IRFs

IRFs WITH HP DATA (PROD. SHOCK)

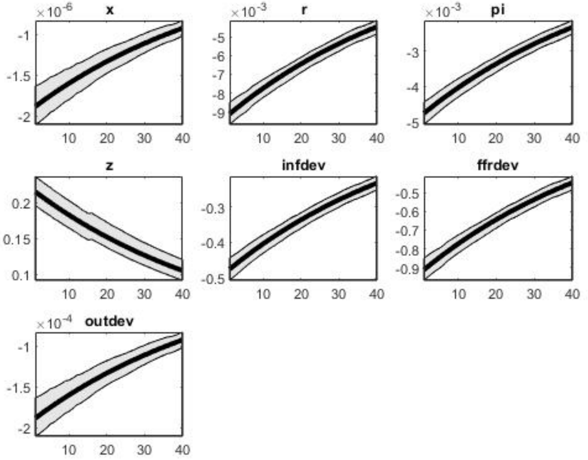

Figure 7: Prod. Shock IRFs

SHOCKS WITH HP DATA

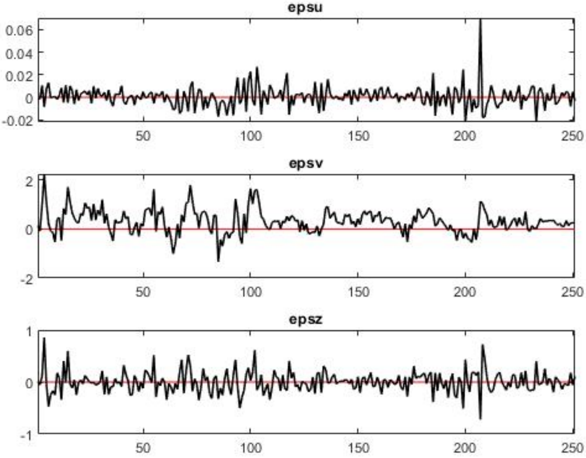

Figure 8: Estimated Shocks

SMOOTHED ESTIMATES WITH HP DATA

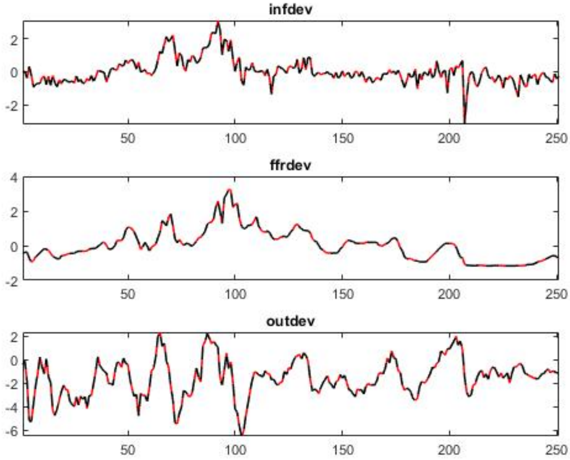

Figure 9: Smoothed Data

POSTERIORS FOR RF DATA

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\sigma - 1$	gamm	0.100	0.0500	0.005	0.0024	0.0024	0.0100
η	norm	1.000	0.2500	1.406	0.0377	1.3531	1.4582
ω	beta	0.500	0.1000	0.475	0.0047	0.4690	0.4833
β	beta	0.990	0.0025	0.991	0.0001	0.9907	0.9910
ϕ_π	gamm	1.700	0.1000	2.127	0.0184	2.1024	2.1577
ϕ_x	gamm	0.125	0.1000	0.104	0.0223	0.0718	0.1367
ρ_z	beta	0.800	0.0500	0.799	0.0003	0.7986	0.7993
ρ_v	beta	0.800	0.0500	0.827	0.0140	0.8047	0.8549
ρ_u	beta	0.800	0.0500	0.844	0.0075	0.8344	0.8564
ϵ^z	invg	1.000	2.0000	0.130	0.0006	0.1290	0.1304
ϵ^v	invg	1.000	2.0000	1.347	0.0283	1.2825	1.3826
ϵ^u	invg	1.000	2.0000	0.130	0.0008	0.1290	0.1305

MARGINAL POSTERIOR DISTRIBUTIONS WITH RF DATA

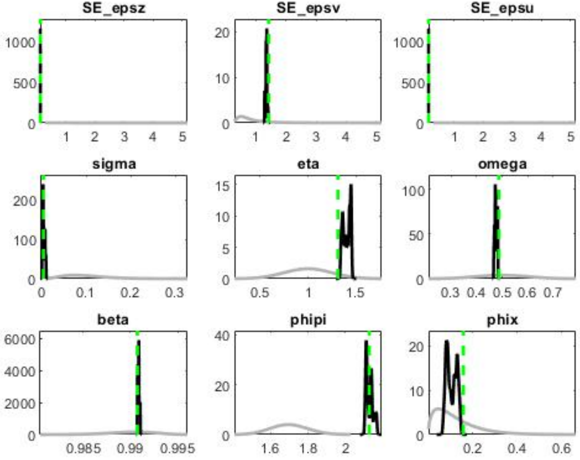

Figure 10: Marginals 1

MARGINAL POSTERIOR DISTRIBUTIONS WITH RF DATA

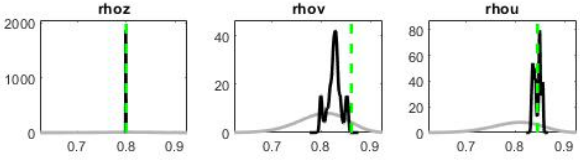

Figure 11: Marginals 2

IRFs WITH RF DATA (MP SHOCK)

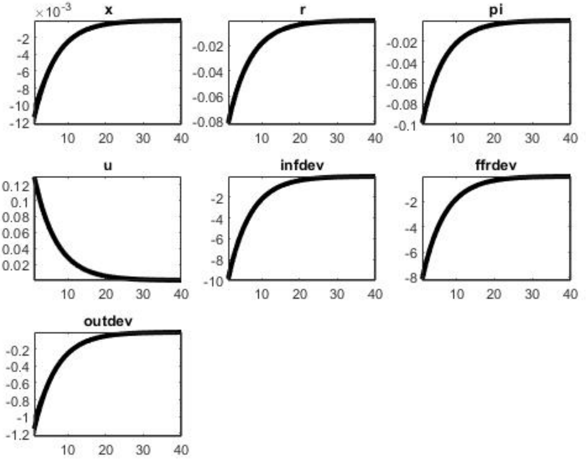

Figure 12: MP Shock IRFs

IRFs WITH RF DATA (CP SHOCK)

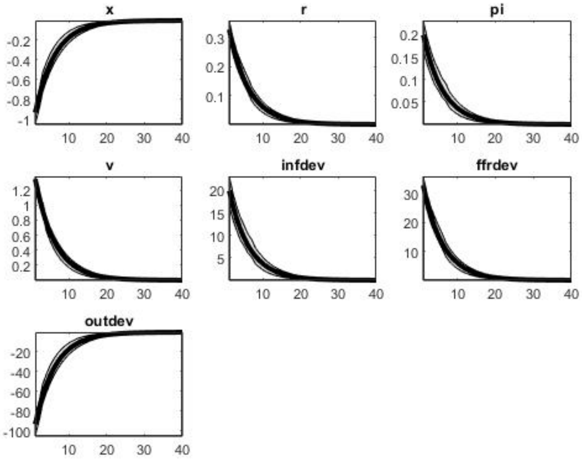

Figure 13: CP Shock IRFs

IRFs WITH RF DATA (PROD. SHOCK)

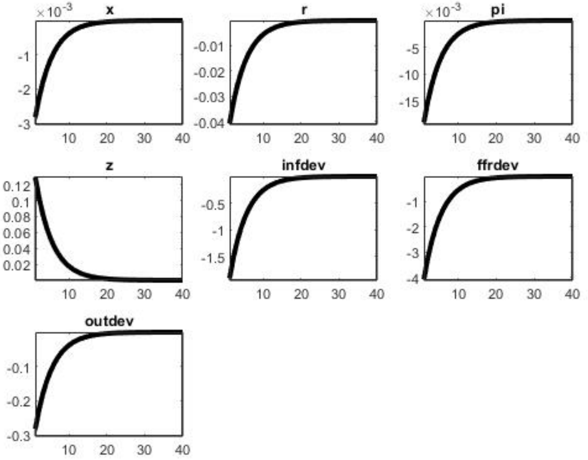

Figure 14: Prod. Shock IRFs

SHOCKS WITH RF DATA

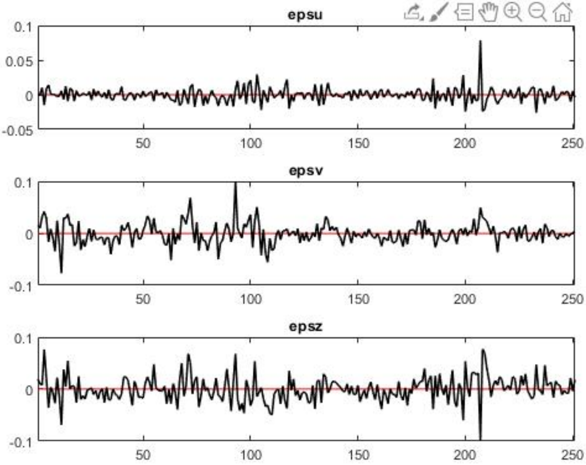

Figure 15: Estimated Shocks

SMOOTHED ESTIMATES WITH RF DATA

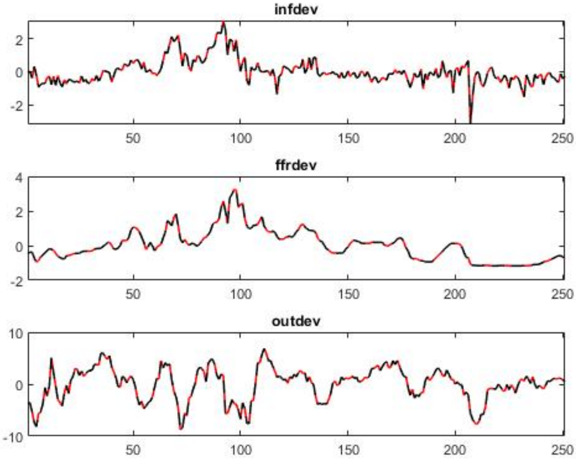

Figure 16: Smoothed Data

THINGS TO NOTE

- In this model the shock processes completely determine the shape of the responses
- We also note that differences in filtering choices have big impacts on the sizes of shocks and estimates of their persistence
- Other parameters of note are also affected (Risk aversion, labor elasticity, price rigidity, and Taylor Rule coefficients)
- Policy and theory conclusions are sensitive to method of detrending and also potentially sensitive to smoothness parameters

WHAT CAN DYNARE DO FOR YOU?

- Even larger DSGEs with many equations (Open economy models, Two-Agent NK models, etc.)
- DSGEs with sticky or lagged expectations à la Mankiw and Reis, which are otherwise unwieldy
- DSGE-VARs, which use the DSGE as a prior for a VAR
- Toolboxes for heterogeneous agent models (Winberry et al.)
- SVARs with a number of identification schemes
- Support for Julia interface (hopefully coming soon!)

DSGE-VARs IN DYNARE (AN EXAMPLE)

- You've heard about BVARs already. This is very similar!
- The DSGE-VAR is basically a BVAR, but you use the DSGE as a prior rather than the Minnesota Prior
- Main complication is figuring out how to convert the DSGE into a prior for the parameters of the VAR
- Once that's taken care of, you estimate as before. You can even use the same Gibbs sampler for the VAR parameter draws (DSGE parameter draws necessitate RWMH)!
- There is a hyperparameter for the weight placed on the DSGE prior and we can use it for model comparison
- Now, let's take a look at one of the .mod files for it

REFERENCES

- "Why you should never use the Hodrick-Prescott Filter" Hamilton (WP 2017)
- "Solving Linear Rational Expectation Models" Sims (Comp. Econ. 2002)
- "The Solution of Linear Difference Models Under Rational Expectation" Blanchard and Kahn (ECMA 1980)