Den Haan And Drechsel (2018)

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When models are written in state-space form, we have two equations and a lot of nice methods to estimate them. These are the transition equation and the measurement equation:

$$\begin{aligned} s_t &= T(\theta)s_{t-1} + R(\theta)\epsilon_t \\ y_t &= Z(\theta)s_t + D(\theta) + v_t. \end{aligned}$$

In those equations, θ is a vector of the parameters of the DSGE, ϵ_t is a vector of shocks, and v_t is the measurement error. However, with DSGEs, we don't start with those equations. We have to linearize the model (or not, if you want to use particle filter methods) and then we have to solve that system of linearized equations. Usually, we're solving systems of the form (as in Sims 2002):

$$\Gamma_1 s_t = \Gamma_0 s_{t-1} + C + \Psi z_t + \Pi \eta_t.$$

That equation relates present values to past values, shocks, and expectation errors. This is kind of like the formulation that Den Haan and Drechsel use in 3.2.1, except they don't stack their equations to get it in that form. How we get them in the form of the transition equation isn't particularly important. There are many different ways to do it (e.g. Sims 2002, Anderson 1997, King and Watson 1997, etc.). Just take note that the transition equation looks like the form in section 3.2.3 of Den Haan and Drechsel.

What Den Haan and Drechsel do is they take the measurement equation above and augment the vector of the shocks with what they call Agnostic Structural Disturbances (ASDs). As an aside, I'm not sure how I feel about the use of the word "structural" in this case because, when I hear it, I hear "orthogonal to other shocks." That may not be the case for ASDs. In effect, they're reduced form shocks and aren't related to the state by any particular combination of structural parameters. What that means is when ϵ_t is augmented with ASDs v_t, the equation can be written in the form

$$s_t = T(\theta)s_{t-1} + \begin{bmatrix} R(\theta) & R' \end{bmatrix} \begin{bmatrix} \epsilon_t \\ v_t \end{bmatrix}.$$

In that equation, R' isn't a function of the structural parameters. The entries of R' are just left free to be estimated along with the structural parameters θ, which can make the model a bit bigger. So, when Den Haan and Drechsel say that these ASDs are a somewhat more general form of the wedges in the Business Cycle Accounting of Chari, Kehoe, and McGrattan (2007), this is what they mean. They're both reduced form disturbances, but while CKM are guided by theory in the placement of their wedges, Den Haan and Drechsel put fewer restrictions on how they can enter in each equation. Then, at the same time, you can interpret these ASDs by observing which state variables they affect and how.

So, how does that help with misspecification of shocks? Well, if you have "structural" shocks that don't enter into the model correctly (their coefficients in $R(\theta)$ are wrong), then a model where they're replaced by an ASD will dominate them (by likelihood ratio test or whatever) because the reduced form coefficients yield more freedom for the initial impact of that shock. As a result, this method takes care only of misspecification of shocks.

I would also like to mention some things about DSGE-VARs, since they mention them in the paper as a competing method of weeding out misspecification. As covered in Del Negro, Schorfheide, Smets, and Wouters (2007), the way misspecification is dealth with in the DSGE-VAR formulation is that we take the VAR approximation of the DSGE model, with the implied coefficient matrix for deep parameter vector θ being $\Phi(\theta)$ and variance covariance matrix being $\Sigma_u(\theta)$. This is the prior distribution for the underlying data generating process, which we suppose is a VAR. For some hyperparameter λ defining the weight of the prior against the data ($\lambda = 0$ is an unrestricted VAR and $\lambda = \infty$ is the DSGE), the posterior for the parameters of the model, are the coefficient matrix $\Phi = \Phi(\theta) + \Phi^{\Delta}$ and variance covariance matrix $\Sigma_u = \Sigma_u(\theta) + \Sigma_u^{\Delta}$. So, Σ_u^{Δ} and Φ^{Δ} give us the misspecification in the DSGE, but a more compact measure would be λ since it tells us how hard we shrink to the subset of the parameter space implied by the VAR approximation of the DSGE. Low λ implies big misspecification and vice versa.