ELEC 4700
Assignment 2

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1 No Resistive Region

1.1 Free Y Dimension

The solution to Laplace's Equation by finite difference for a rectangular region set to 1 Volt at x = 0, and grounded at x = L, with free y dimensions can be found in Figure 1.

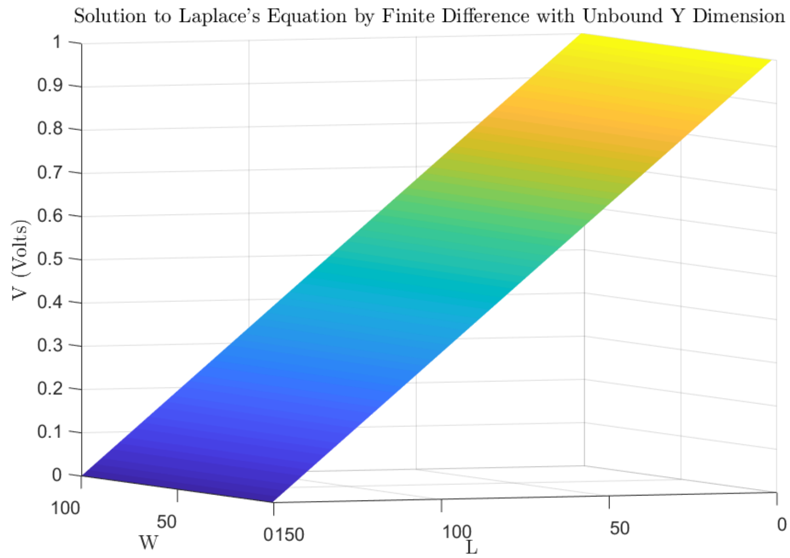

Figure 1: Surface plot of solution to Laplace's Equation by Finite Difference. Rectangular region is set 1V when L = 0, and grounded at L=150. The bounds of W are free.

1.2 Grounded Y Dimension

Grounding the Y dimension of the previously simulated rectangular region results in the surface plot found in Figure 2.

In the case of the G matrix solution, a smaller mesh resulted in a saddle pattern that appears much less smooth than that of the one seen in Figure 2. The most affected portion of the surface plot is the large slopes, at the points closest to L = 0 and the maximum L value. In contrast to the surface plot in 2, that appears to transition smoothly from the highest voltage to the middle point of the saddle, the larger mesh doesn't begin the saddle transition until far below the maximum voltage. This is akin to missing high frequency transients in electrical signals through inadequate sampling rates, although a spatial case of improper sampling as opposed to temporal. A tighter mesh pattern results in a more accurate solution, but also requires more time to compute for a given method. This introduces the trade off and optimization requirements of time spent computing versus solution accuracy that must be considered for the application of the solution.

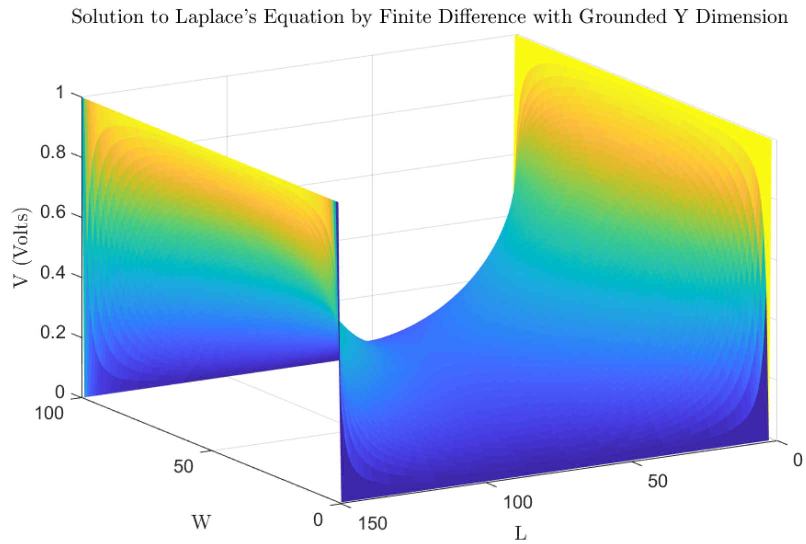

Figure 2: Surface plot of solution to Laplace's Equation by Finite Difference. Rectangular region is set 1V when L = 0 and L = 150. The bounds of W are both set to 0 V.

2 Current Flow Through Bottle Neck

Using a similar G matrix method as the previous section, the current travelling through a bottle neck cause by regions with low conductivity can also be determined. A visual representation of the simulated bottleneck can be found in Figure 3.

The voltage throughout a rectangular region containing a bottle neck created by high resistivity material can be found in Figure 4.

The electric field throughout the rectangular region with the bottle neck can be found given that the electric field within a material is equal to the potential inside the material. The electric field in the region can be seen in Figure 5.

A closer view of the electric field vectors can be found in Figure 6.

The current density can be found using Ohm's Law, as laid out in Eq. (1).

$$\vec{J} = \sigma \vec{E} \tag{1}$$

The current through the bottleneck between the two contacts can be determined by integrating the current density over a cross section of the rectangular region. Using the trapezoidal method of integration across the center of the bottleneck on the y component of the current density, the current through the bottleneck for a variety of situations was calculated. The first simulation carried out varied the tightness of the meshing used when simulating the rectangular region. The ratio of number of Y components to number of X components was kept constant for all configurations at 2/3. The ratio of the area of the bottleneck was also kept constant, where the area of the bottleneck for each mesh size was the number of elements in the X dimension divided by 6, all squared. This simulation resulted in the current pattern found in Figure 8.

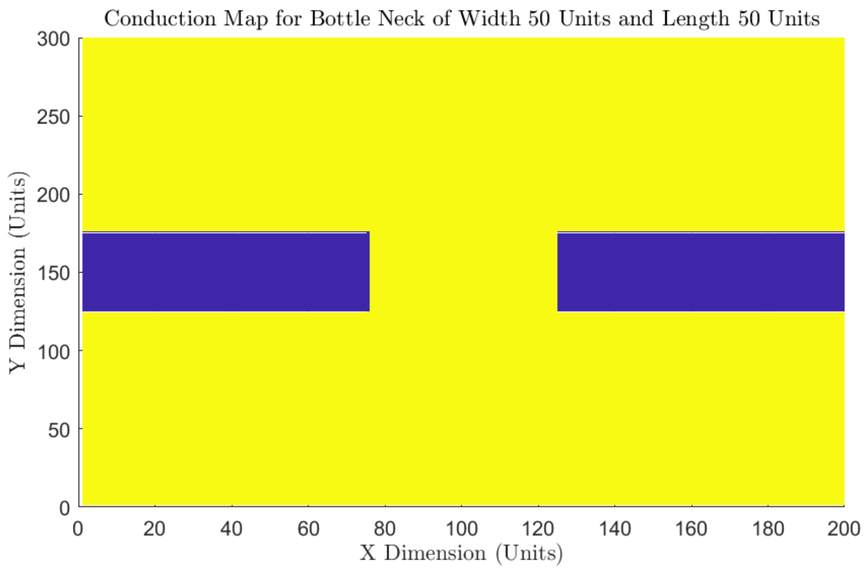

Figure 3: Simulated map of conductivity of a sample of material. The purple regions denote areas of low conductivity, in this case 10^{-5}. Yellows denotes areas of higher conductivity, 1 in this case. The X and Y axes are in terms of generic units, and denote the number of points in the solution mesh.

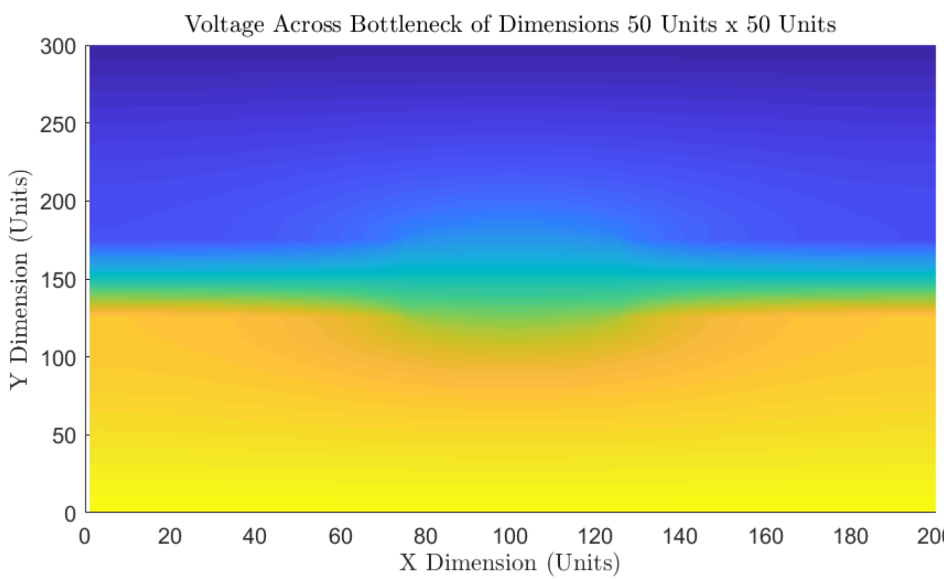

Figure 4: Simulated map of voltage across a rectangular region containing a low resistivity bottle neck. The high resistivity regions are as outlined in Figure 3.

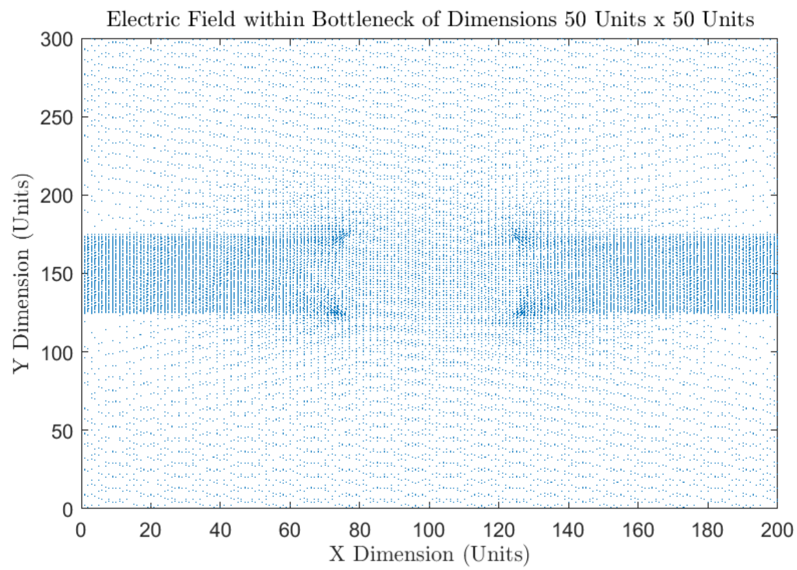

Figure 5: Electric Field inside the bottle neck region. Arrows not visible due to tight meshing.

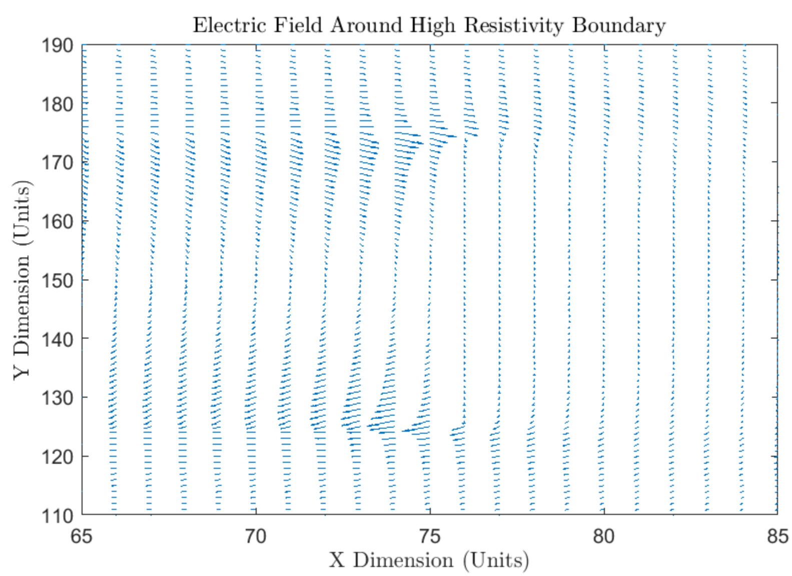

Figure 6: Closer view of the boundary between high and low resistivity regions to better visualize electric field vectors.

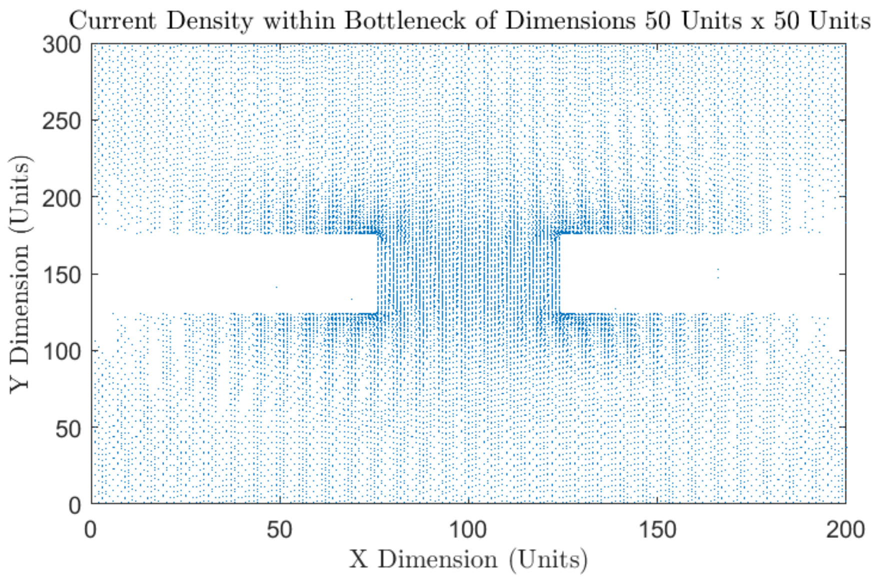

Figure 7: Current density in rectangular region containing bottleneck.

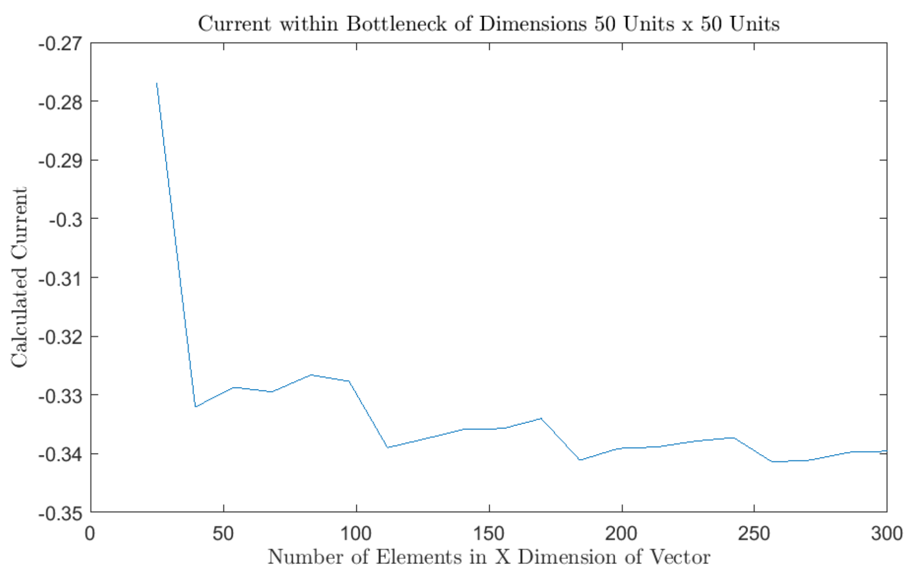

Figure 8: Calculated current through a rectangular region with a bottleneck of area that remained constant relative to the number of elements in the X dimension of the mesh. Current Density was integrated across the center of the bottleneck to determine the current.

It can be seen from Figure 8 that as the mesh density is increased, variations in the calculated current are drastically reduced from one simulation to the next. It is expected that as the mesh size would continue to increase, the current through the bottleneck would converge to some ”correct” physical value. This convergence then raises the problem of how accurate the calculated value needs to be in order to optimize result accuracy and compuation time. Similar to Figure 8, the conduction of the material inside the bottleneck can be investigated. With a constant meshing and bottleneck area the current calculated through the bottleneck as a function of the conduction can be found in Figure 9.

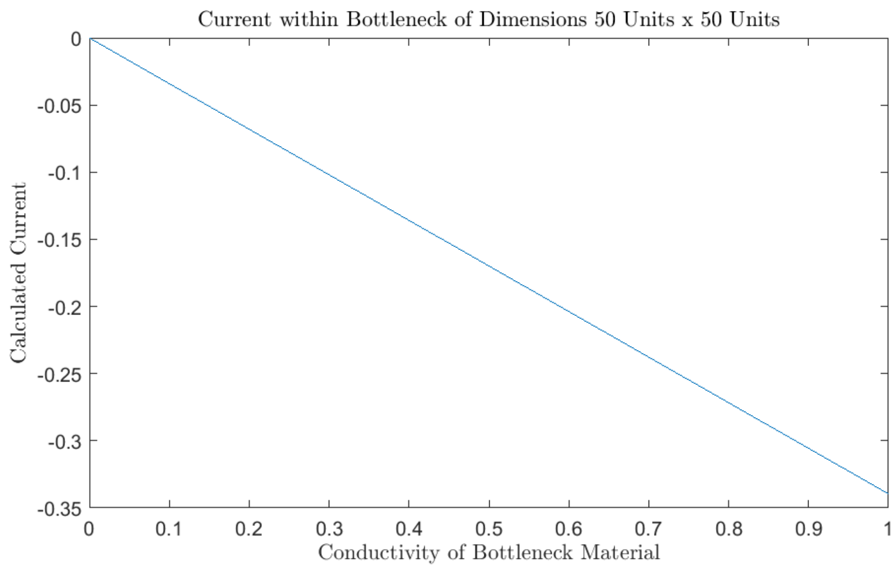

Figure 9: Calculated current through a rectangular region with a constant bottleneck area and varied conductivity in the bottleneck region. Current Density was integrated across the center of the bottleneck to determine the current.

The linear increase in the absolute value of the current seen in Figure 9 can be attributed to the linear relation of the current density with the conductivity, assuming the electric fields in the regions stay the same, which does hold true as the geometry is held constant for each iteration. The final variation carried out for the current calculations was the width of the bottleneck region. The current as a function of the width can be found in Figure 10.

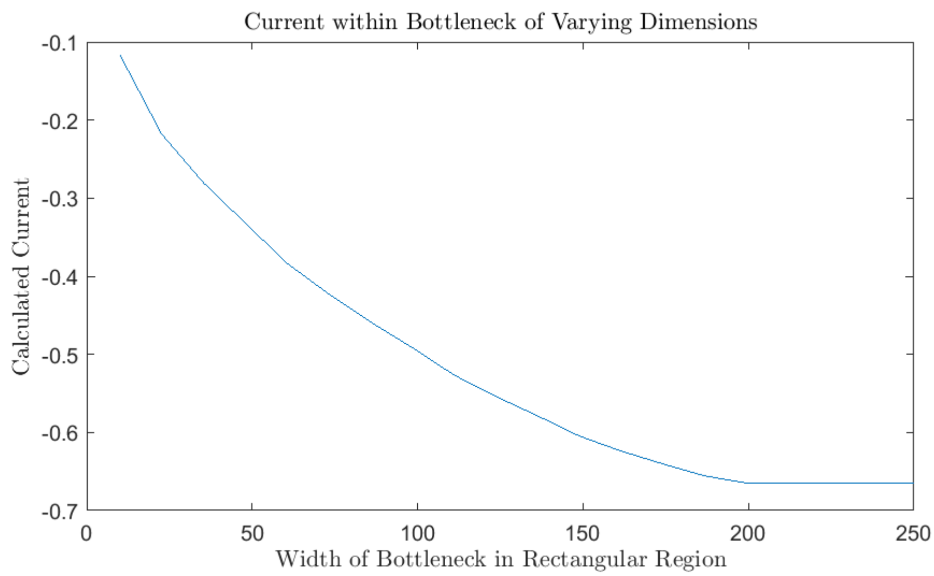

Figure 10: Calculated current through a rectangular region with a constant bottleneck area and varied conductivity in the bottleneck region. Current Density was integrated across the center of the bottleneck to determine the current.