A No-Tanking Draft Allocation Policy *

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Abstract

To promote competition among all teams in the league, the National Basketball Association (NBA) allocates draft picks with a higher probability to teams with worse records in a season. This causes some teams to 'tank', or to lose games intentionally to secure a valuable draft pick with higher probability. We provide a theoretical model of team decision-making, which identifies under what conditions teams choose to lose purposely. We prove that in this setting, any lottery based on end of season rankings that is not uniform will provide incentives for some teams to tank. We relax the constraint that the rule depends on the final rankings only, and show how to design a lottery that favors the worst teams in a season, but eliminates the incentive to tank.

Keywords: Institutional Design, Competition Policy, Tournaments

JEL Codes: D47, Z28, D82

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1 Introduction

Sports leagues want to promote competition amongst teams to increase fan engagement and revenues. Teams who repeatedly finish near the bottom of the league are likely to suffer financially over time. As a result, the league would like to allocate draft picks to the worst teams in the league. However, they do not want to provide incentives for teams to purposely lose games in order to have a better chance of receiving a draft pick.

In an attempt to meet this objective, both the NHL and the NBA currently allocate top draft picks through a weighted lottery system, but have changed systems frequently in recent years. We focus on the NBA for the applications in this paper, but the analysis is easily adapted to other sports leagues. In the NBA, the system for awarding the top draft picks was changed in 1966, 1985, 1990, 1993, 2005, and 2019. Most allocation systems favored the worst team at the end of the season. However, the system from 1985-1989 awarded the top draft pick with equal probability to all teams that didn't make the playoffs. Despite the frequent changes in the allocation rule, many fans still believe teams purposely lose games at the end of the season to improve their odds of receiving a good draft pick. This shirking behavior has been documented empirically in Taylor and Trogdon (2002).

There is existing work which has proposed alternative allocation systems. Gold (2010) proposes allocating the top pick to the team with the highest the number of wins after elimination from the playoffs while Lenten et al. (2018) suggests the team that is eliminated first from playoff contention. Though the authors of existing work have provided empirical evidence that their proposals would reduce tanking, there has not been a systematic theoretical evaluation of draft allocation policies. This work provides that theory to determine if an optimal rule for draft allocation exists that satisfies the league's parity objectives while eliminating incentives for teams to lose.

Our work is most closely related to the existing literature on tournament theory. Lazear and Rosen (1981) design a prize allocation system that incentivizes optimal effort for workers, where a worker's ranking in output, rather than their actual output, determines their prize allocation. The prizes decrease as rankings decrease. The workers have a single choice of effort, which depends on their ability and the the prize allocation system. he setting of the NBA is different in two ways. First, under a re-distributive lottery, the value of the lottery assignment for the bottom-ranked team is greater than that of the second-to-last team. In addition, there is a dynamic aspect to effort choice. At each game, teams know their past performance and make a choice to exert effort which is conditional both on their past performance and the prize allocation.

Currently, the league bases the draft probability lotteries on final rankings only. We show that under the current policy, unless the draft probabilities are equal for every team that does not make the playoffs, there is always some history of a season such that teams would have the incentive to purposely lose games after they are eliminated from the playoffs. We argue that instead, the lottery draft probabilities should depend on how few games a team wins in a subset of the season, rather than ranking over the full season. We show how to dynamically adjust the lottery probabilities as a season progresses in a way that removes incentives for teams to lose purposely, and gives the draft pick to the worst team in a season for a subset of games in that season.

For readers who are not familiar with U.S. sports leagues, we provide some brief background of the draft allocation system in the NBA in Section 2. Section 3 defines the theoretical model of team decision making that we use to prove that the current allocation rule is not optimal and to introduce our draft allocation system. A less mathematically-inclined reader can skip some of the notation and proofs in Section 3 and still follow the main arguments given in the text. In Section 4, we show the results of our draft policy applied both to simulated seasons, and to real data from the NBA in the 1980s.

2 Background

The National Basketball Association (NBA) is one of main professional sports leagues in the United States. In the current system, the regular season begins in the last week of October and ends in the middle of April. There are 30 teams competing in each season. Each team plays 82 games in a single regular season, and each team plays every other team during the regular season, at least once at home and once away. At the end of the regular season, the 16 teams with the highest number of wins advance to the playoffs, and the winner of the playoffs wins the championship. The remaining 14 teams participate in the annual NBA draft lottery.

During the draft, teams can select players who are eligible and wish to join the league. An eligible player is at least 19 years old and one year removed from their high school graduation date. The teams pick sequentially, according to a prescribed ordering, the player they value the most out of the remaining pool of eligible draftees. The first to fourth draft picks are considered the most valuable, and those are allocated using a lottery system. In Appendix A we include the lottery system for the first pick as of 2019.

After the first four picks are allocated according to the outcome of the lottery. The remaining picks are allocated according to the following reverse order system: the lowest ranked team that didn't get one of the first four picks will get the fifth pick, the lowest ranked team that didn't get one of the first five picks will get the sixth pick, and so on. Once all thirty teams have received a draft pick in the first round, the second round begins and an additional thirty players are drafted according to the inverse ranking in the regular season. Players that go undrafted in the two rounds of draft picks are free to try out for any team. One caveat to the system described is that teams are allowed to sell their draft picks to other teams.

In our model we will simplify the environment to consider only the problem of allocating the first draft pick to make the exposition more clear, but it is not difficult to extend our results to the allocation of multiple picks.

3 Designing a Draft Allocation Mechanism

3.1 Defining League and Team Objectives

First, we define some variables and notation required for our basic model for a single season S. The season is made up by a total of m games, each between two teams. There are a total of n teams in the league. $S_{t1} \in \{1, \ldots, n\}$ denotes the home team for game t, and $S_{t2} \in \{1, \ldots, n\}$ denotes the away team for game t. $\vec{h}^t \in \{0, 1\} \times t$ is a t-length vector that denotes the history of the season up to and including game t. It is defined recursively as follows:

$$\vec{h}^0 = \emptyset$$

If the home team wins game t, then:

$$\vec{h}^t = \vec{h}^{t-1} \cup 1$$

If the away team wins game t, then:

$$\vec{h}^t = \vec{h}^{t-1} \cup 0$$

The possible outcomes of a season with m games are $\vec{h}^m \in \{0, 1\} \times m$. For some history h^t, the total number of wins for team i for that history is:

$$v_i(\vec{h}^t) = \sum_{s=1}^{t} h_s^t \mathbb{1}(S_{s1} = i) + (1 - h_s^t)\mathbb{1}(S_{s2} = i)$$

The team with the most wins given a season's result is:

$$v^{(1)}(h^m) = \arg\max_i v_i(\vec{h}^m)$$

The team with the least wins given a season's result is :

$$v^{(n)}(h^m) = \arg\min_i v_i(\vec{h}^m)$$

The team with the k-th most wins after a season is played is $v^{(k)}$. We assume for now that there is a primary prize π^V which is allocated to the top k^* teams. For example, currently in the NBA, the top sixteen teams make the playoffs[1]. We assume the value of making the playoffs is constant for every team. There is also a single secondary prize[2] $\pi^D < \pi^V$, which is allocated based on a league policy $y_i(\vec{h}^t)$, which assigns a probability that team i gets the draft pick given a history $\vec{h}^t$. Currently, $y_i(\vec{h}^m)$ depends on a team's final ranking only:

$$r_i(h^m) = k \text{ s.t. } v^{(k)}(h^m) = i$$

[1]It is actually the top eight teams in each conference that make the playoffs, but we do not model divisions and conferences in this paper.

[2]In this paper, we focus on the allocation problem for the first draft pick only, although in reality the league may allocate multiple draft picks based on the outcomes of a season.

See Table 4 in Appendix A for the current probabilities for 2019, and Table 3 for a history of how the lottery mechanism has changed since 1966. Each team makes a single strategic choice in each game, which is whether or not not to exert effort.[3] $e_{it} \in [0,1]$ is the choice of effort by team i. A team exerting full effort in a game has $e_{it} = 1$, while a team that is tanking has $e_{it} = 0$. For our basic model specification, we assume that win probabilities for a team in game t against opponent j are generated by the following process:

- Each team draws a realization from the random variable z_{it} distributed according to
$$z_{it} \sim N(e_{it}, \sigma^2)$$
- the probability that a team i wins against opponent j in game t is:
$$p_{it}(e_{it}, e_{jt}) = Pr(z_{it} > z_{jt}) = 1 - \Phi\left(\frac{e_{it} - e_{jt}}{\sigma}\right)$$
where Φ is the standard normal cumulative distribution function

Lastly, we define the probability of winning the main prize in the season for team i given a history $\vec{h}^t$. This probability can be defined recursively.

$$q_i(\vec{h}^m) = \mathbb{1}(r_i \geq k^*)$$

In the current NBA system, a team makes the playoffs if it is in the top 16 teams at the end of the season. If the end of the season has not yet been reached, then the probability a team makes the playoffs depends on what has happened so far, and the probability of every possible outcome in future games:

$$q_i(\vec{h}^t) = p_{1,t+1} q_i(\vec{h}^t \cup 1) + (1 - p_{1,t+1}) q_i(\vec{h}^t \cup 0)$$

We suppress the dependence of $p_{1,t+1}$ on $e_{1,t+1}$ and $e_{2,t+1}$ because we focus on incentive compatible mechanisms for draft allocation, where the effort chosen is always 1 and therefore the dependence on effort choice of each team disappears. In order to derive the incentive compatible draft mechanisms, we first explicitly derive the team and league objectives:

Team Objective The team chooses an effort level in game t to maximize their expected payoff given the results in the games played so far $\vec{h}^{t-1}$. Let $W = 1$ if the team i wins and is the home team, and $W = 0$ if team i wins and is the away team. $L = 0$ if team i loses and is the home team, and $L = 1$ if team i loses and is the away team. Team j is the opposing team.

Problem 1.

$$\begin{aligned}\max_{e_{it}} \quad & [p_{it}(e_{it}, e_{jt})(q(\vec{h}^{t-1} \cup W)\pi^V + y(\vec{h}^{t-1} \cup W)\pi^D) \\ & + (1 - p_{it}(e_{it}, e_{jt}))(q(\vec{h}^{t-1} \cup L)\pi^V + y(\vec{h}^{t-1} \cup L)\pi^D)]\end{aligned}$$

[3] For the current paper, we assume ability is constant. An in-progress extension of the paper assumes that ability is unknown by the league at the start of the season, but is learned over the course of the season.

No-Tanking Condition (NTC) Taking the first-order conditions of Objective 1 with respect to e_{it} and ensuring it is positive everywhere, we arrive at the following condition:

$$\left[q_i(\vec{h}^{t-1} \cup W) - q_i(\vec{h}^{t-1} \cup L)\right] \frac{\pi^V}{\pi^D} \geq \left[y_i(\vec{h}^{t-1} \cup L) - y_i(\vec{h}^{t-1} \cup W)\right] \tag{NTC}$$

NTC has a clear interpretation. If the team's increase in probability of receiving the draft pick when they lose a certain game is less than the change in probability of receiving the primary prize, scaled by the ratio of the value of winning the primary vs the secondary prize, then the team will exert maximum effort in a game ($e_{it} = 1$). If this condition does not hold for some team in some game, then the team will not exert maximum effort in that game ($e_{it} < 1$). We want to find a draft allocation mechanism that ensures NTC holds in every game.

Note that we assume that if the team is indifferent between exerting effort and tanking, they will exert effort, since winning a game should be preferred to losing a game in the short-term, in the absence of long term incentives. This simple condition indicates that any draft allocation policy that satisfies NTC should not change the probability that a team receives a draft pick in games where they are already eliminated from playoff contention, since at that point the change in probability of receiving π^V from winning in any game is always zero. That is not the case for any of the weighted lottery rules that the NBA has implemented that depend only on the final season ranking. This leads us to our first result:

Theorem 1. *The only draft allocation policy that is a function only of a team's final ranking r_i and satisfies NTC for any history $\vec{h}^m$ is a uniform lottery, which assigns equal probability to every team that does not receive π^V.*

Proof. Suppose that $y_i(\vec{h}^m)$ is chosen before the season starts, and assigns a fixed probability of getting the draft to every position in the final ranking. Suppose that the probability of getting the draft pick for team ranked $b + 1$ is greater than the probability for team ranked b. Consider a history such that before the very last game where they play each other, two teams are tied at position $b + 1$ and they are mathematically eliminated from the playoffs. It is easy to see that in this scenario, NTC cannot hold. Since both teams are already eliminated from playoff contention:

$$q_i(\vec{h}^{t-1} \cup W) - q_i(\vec{h}^{t-1} \cup L) = 0$$

. However,

$$y_i(\vec{h}^{t-1} \cup L) - y_i(\vec{h}^{t-1} \cup W) \neq 0$$

If team i loses the game and clinches the $b + 1$-th ranking, then he receives a higher probability of getting the draft pick than if he wins the game and is ranked b. The only way to restore NTC is if the the draft probabilities are equal for the b-th and $b + 1$-th teams. Since b and $b + 1$ were chosen arbitrarily, this must hold for every position $b > k$, where teams in position 1 to k receive π^V. Therefore, only a uniform lottery satisfies (NTC) for every possible history in a season.

□

This means that no matter how carefully the league chooses the weighted lottery used to assign the first draft pick, unless the weights are equal for all teams that did not make the playoffs that year, there will sometimes be incentives for certain teams to tank once they have been eliminated from the playoffs. We also briefly comment on a related incentive issue with the two proposed rules from Gold (2010) and Lenten et al. (2018). For the rules to satisfy NTC in every possible game in a season, they implicitly require that teams will always exert effort pre-elimination. However, under our model, for example for teams that lose many games at the beginning of a season and have a high probability of elimination, NTC may be violated before the teams are actually mathematically eliminated. For example, under Gold (2010)'s rule that counts wins pre-elimination, a team with a high probability of elimination may have an incentive to purposely lose and eliminate themselves earlier to increase their expected number of wins that count towards the draft allocation rule.

In order to design a better rule, we need to define what the league's preferred draft allocation policy is. If teams were not able to tank and attempt to lose games purposely, we assume the league would assign the draft pick to the team with the lowest rank at the end of the season with probability one. In this paper, we do not provide a theoretical model for why a league might want to assign the first draft pick to the worst team in a season. In general, redistributive policies enacted in sports leagues aim to maintain competitiveness of all teams over multiple seasons to increase fan engagement and revenues.

League Objective The league objective is to maximize the probability that the league assigns the draft to the team with the worst record at the end of the season, while maintaining NTC in every game.

Problem 2.

$$\min_{y} \left(y_{v(n)}(\vec{h}^m) - 1 \right)^2 \qquad \forall h^m$$

s.t.

$$\left[q_i(\vec{h}^{t-1} \cup W) - q_i(\vec{h}^{t-1} \cup L) \right] \frac{\pi^V}{\pi^D} \geq \left[y_i(\vec{h}^{t-1} \cup L) - y_i(\vec{h}^{t-1} \cup W) \right] \qquad \forall i, t \tag{NTC}$$

$$y_i(\vec{h}^{t-1}) = p_{it} y_i(\vec{h}^{t-1} \cup W) + (1 - p_{it}) y_i(\vec{h}^{t-1} \cup L) \qquad \forall i, t \tag{DC}$$

$$y_i(\vec{h}^t) \in [0, 1], \qquad \sum_{i=1}^{n} y_i(\vec{h}^t) = 1 \qquad \forall i, t \tag{PROB}$$

$$y_i(\vec{h}^0) = \frac{1}{n} \qquad \forall i \tag{FAIR}$$

The league wants to maximize the probability that the lowest performing team that year gets the draft pick, while holding true the NTC condition so that no teams have an incentive to tank. Furthermore, the probability that any given team receives the draft pick should be equal at the beginning of any given season (FAIR) and the probabilities should be dynamically consistent (DC), or equivalently, satisfy Bayes' rule.

This minimization problem is a convex program, and is theoretically solvable by backward induction. However, the optimal backward induction solution is not at all computationally feasible, since for a standard NBA season the game tree is of size 2^m, where $m = 1,230$. For a comparison, the number of atoms in the universe is estimated to be less than 2^{273}. We propose a rule that is analytically feasible and satisfies NTC in any possible history while still targeting the optimum (2) directly.

3.2 An Incentive-Compatible Allocation Mechanism

Theorem 1 indicates that a lottery that has a higher weight to teams with the worst records in a full season will not satisfy NTC. We propose instead a weighted lottery based on teams' win-loss records for the first t^* games in a season; both the subset of games that count and the weights are determined dynamically as the season progresses. We call this mechanism the No Tanking Draft Allocation Rule (R-NTD).

Let $Pr(v^{(n)} = i|\vec{h}^t)$ be team i's probability of having the lowest number of wins after all m games have been played, given the results from t games. The probability team i receives the draft after game t in R-NTD is simply the probability team i ends up last given the results of all games up to $t^*(\vec{h}^t)$. We suppress the dependence of t^* on the history in the following equation for notational convenience:

$$y_i(\vec{h}^t) = Pr(v^{(n)} = i|\vec{h}^{t*})$$

$t^*(\vec{h}^t)$ is the smallest $s \leq t-1$ such that (1) holds true: given a history $\vec{h}^s$, NTC would be violated at time $s+1$ if the draft probabilities continued to be adjusted based on the probability that a team comes last. If the following condition does not hold true for any $s \leq t-1$, then $t^*(\vec{h}^t) = t$.

$$\begin{aligned} &\left[q_i(\vec{h}^s \cup W) - q_i(\vec{h}^s \cup L)\right] \frac{\pi^V}{\pi^D} \\ &< \left[Pr(v^{(n)} = i|\vec{h}^s \cup L) - Pr(v^{(n)} = i|\vec{h}^s \cup W)\right] \end{aligned} \tag{1}$$

A team's probability of receiving the draft before the season begins is $\frac{1}{n}$, since in our model each team has an equal probability of coming last before any games are played. As wins and losses are recorded in each game, a team's probability of getting the first draft pick adjusts based on how their probability of coming last changes. For example, when a team loses multiple games early in the season, they increase their probability of coming last in the season and their probability of receiving the draft pick increases. This adjustment process permanently stops the first time NTC would be violated for any team playing in any game.

If we didn't take into account the wins and losses from any games, then the policy that would minimize (2) in expectation is a uniform lottery. The more wins and losses we can take into account in a season, the more we can minimize (2) in expectation; if we take into account every game played ignoring NTC, then we could assign the draft pick with probability 1 to the team with the least number of wins, and (2) would be minimized (this is done in the NFL). Taking into account NTC, we end up somewhere in the middle between the no information case and the full information case; we minimize (2) in expectation, based on as much information as we can take into account without violating NTC.

Theorem 2. *R-NTD satisfies NTC, DC, FAIR and PROB. Moreover, the allocation rule minimizes the expected value of the league objective, conditional on the history up to time $t^*(\vec{h}^m)$.*

Proof. First, we show that the rule satisfies each of the constraints on the league objective.

- PROB is satisfied by definition, since $y_i(\vec{h}^t)$ are the probabilities that team i is ranked last in the league, which satisfy PROB.
- DC: Also satisfied by definition. At time t, the probability an individual is last at the end of a season conditional on a possibly limited history satisfies DC.
- FAIR: In our model, at the start of the season, the prior probability that any team wins a given game is $\frac{1}{2}$. So, every team has an equal chance of coming last in a given season and $y_i(\vec{h}^0) = \frac{1}{n}$.
- NTC: If NTC is violated at some time t, then it must be that inequality (1) is satisfied. But if inequality (1) is satisfied at any time t, then the draft probabilities were fixed based on some history before t, the RHS of inequality (1) is zero. Since the LHS of inequality (1) is made up of non-negative terms, then inequality (1) cannot hold and NTC is satisfied at every time t.

Now we turn to optimality. When the set of information is restricted up to the history $t^*(\vec{h}^m)$, the league can change the draft probabilities until the game before NTC is first violated. For a history of a season with m games, $t^*(\vec{h}^m) = \tilde{t}$. If $\tilde{t} = m$, then $y_{v(n)}(\vec{h}^m) = 1$ and the league objective is minimized. If $\tilde{t} < m$, then the draft probabilities are cutoff at time $\tilde{t}$, and are equal to the probability that each team will come last after m games, conditional on the record only after $\tilde{t}$ games are played. But this, by definition of the draft probabilities at time $\tilde{t}$, optimizes the league objective in expectation : the highest draft probability is assigned to the team that has the highest probability of ending up last after m games.

□

We do not claim that this rule is globally optimal for the league objective. There may be other rules, which allow the league to change draft probabilities even after $\tilde{t}$, or that prescribe smaller changes in probabilities in the first games of the season. The only way to find this optimum would be to solve league objective exhaustively by backward induction, but this is impossible as discussed previously. Our rule, which approximates the optimum, performs well in practice and most importantly has theoretical guarantees that each team has the incentive to exert effort in every game. The one downside is that for certain histories the game after which the rule stops taking results into account can be quite early in the season. In the next section we show how the mechanism works in practice and find that in simulations and in real data, it takes a large proportion of the games in a season into account before freezing the draft lottery probabilities.

4 Examining the Rule in Practice

4.1 Simulations

We examine some of the incentive issues described in theory in the previous section through simple simulations. First, we simulate a very simple setting with two teams and 50 games. We assume for the rest of this section that $\frac{\pi^V}{\pi^D} = 10$, so that the value of winning is at least 10x the value of receiving the draft pick. Figure 1 shows how incentives to win change when Team 1 loses all of the remaining games in a season after winning the first 10 games. Closer to the beginning of the season, when there is still substantial uncertainty about which team will win or lose the tournament, the incentives to win are strong for Team 1. They peak at the most crucial games when Team 1 and Team 2 have fairly even records. As the season progresses, and it becomes clear that Team 1 will not win the overall season, then his incentives to win reach zero. Our draft allocation rule takes advantage of this incentive structure for teams. It adjusts the probability a team receives the draft during the initial period in the season when the incentive to win is high; as the incentive to win decreases, the draft probability adjustment is eventually stopped to avoid giving teams incentives to lose intentionally. The red line in Figure 1 is the difference in a team's probability of coming first if he wins compared to if he loses game g. The blue line is this difference multiplied by $\frac{\pi^V}{\pi^D}$, which is the upper limit imposed by NTC that a draft probability can change for any game in a season. In this season, for example, the draft allocation probabilities cannot change after game 40.

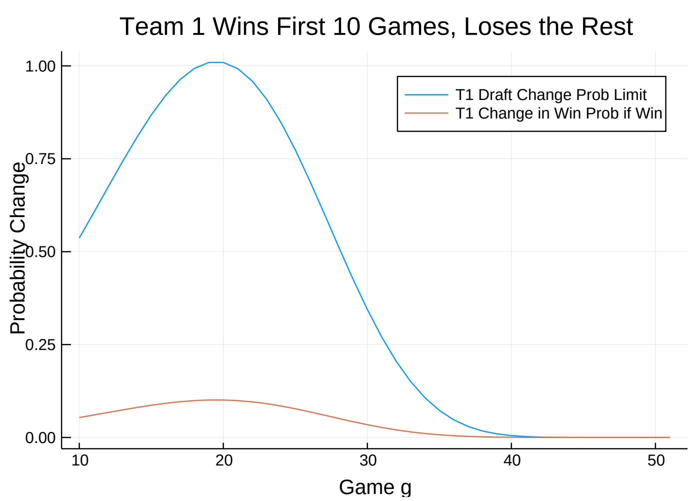
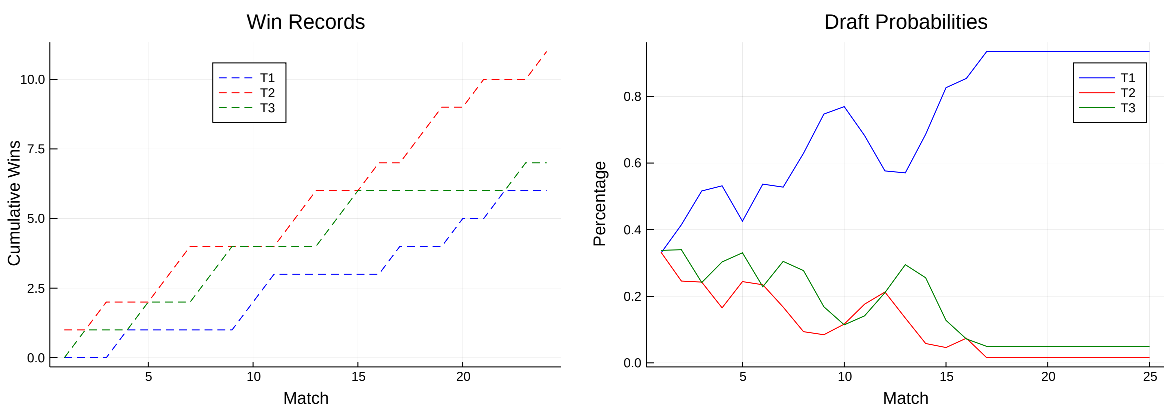


Figure 1: Change in Incentives for Example Season, Assume $\frac{\pi^V}{\pi^D} = 10$

Our next simulated setting has $n = 3$ teams. Each team plays 16 games over the course of a season (there are 24 total games in the season), and only the first place team receives π^V. We randomly simulate a single season and show how the draft probabilities change for each team compared to their win records in Figure 2. The final ranking of the season is Team 2 (11), Team 3 (7) , Team 1 (6). Their respective probabilities of receiving the first draft pick are 93.5%, 4.5% and 1.5%.

Examining Figure 2 shows how these probabilities are derived. At the beginning of the season, each team has an equal probability of coming third. After 9 games, Team 1 has lost all games but one, so has a nearly 80% chance of coming last. Since NTC still holds for all teams, he has an 80% chance of getting the first draft pick at this point. After 16 games, then Team 1 has nearly been eliminated mathematically, with a 93.5% chance of coming last, so no longer has an incentive to win if draft probabilities continue to adjust. So, the draft probabilities are fixed at that point going forward. Team 2 and Team 3 have similar records at this point so their draft probabilities are both small. Team 3 then has a lengthy losing streak at the end of the season that results in him ending the season with only 1 win more than Team 1, but the losing streak does not affect his draft probability since it comes after NTC has been violated for Team 1.



(a) Draft Probability

(b) Win Record

Figure 2: Allocation Mechanism for Simulated Season

4.2 Empirical Studies

From 1985 to 1989, the NBA had a uniform lottery system, where each non-playoff team received the first draft pick with equal probability. Under our model, teams under this system did not have incentives to lose games purposely. As a result, the league records in these seasons provide a good case study for the performance of our rule, which also should not provide incentives for teams to lose purposely.

We calculate the draft probabilities based on our rule in 1987. In Table 1 we show the draft probabilities for our rule compared to the uniform lottery in place at the time. Teams are ordered by their final ranking. The draft probabilities in this season were not adjusted after the 266th game, which is when, based on simulations of the remainder of the season to approximate the probabilities q_i, that we estimated that NTC began to fail. The lowest ranked team, the L.A. Clippers, receives the pick with a 28.5% probability. This is much higher probability than in the uniform case, but the allocation mechanism does not violate NTC. The draft probabilities are mostly decreasing in the ultimate final ranking, except that the Kings have a sightly higher probability than the Spurs. This is because the probabilities were determined based on each team's probability of coming last conditional on the first

266 games of the season, when the Kings were ranked lower than the Spurs. Overall, though, in 1987 the teams that had performed badly after 266 games were largely the same teams that performed poorly over the whole season.

In this particular season, after 266 games, some of the teams that ultimately made the playoffs still had a non-zero probability of being last in the season. As a result, the mechanism described in Section 3 assigns small probability in the draft to teams that are playoff teams. We can address this by adjusting the rule described in Section 3 without violating NTC; in Table 1 and Table 2 we redistribute the small probability assigned to teams that do make the playoffs uniformly to the teams that ultimately do not make the playoffs. This removes all probability from teams that make the playoff and increase the draft probability for each of the non-playoff teams by around 1%. To keep the theoretical analysis simple, the adjustment is simply mentioned in this section rather than described formally in Section 3.

Rank	Team	Wins	NBA Lottery	Our Lottery
23	Los Angeles Clippers	12	14%	28.5%
22	New Jersey Nets	24	14%	25.5%
21	New York Knicks	24	14%	21.0%
20	San Antonio Spurs	28	14%	9.0%
19	Sacramento Kings	29	14%	12.0%
18	Cleveland Cavaliers	31	14%	2.0%
17	Phoenix Suns	36	14%	2.0%

Table 1: Allocation Policy from 1987 Season, Adjustment Cutoff at 266th Game

We repeat the analysis for 1989, when there were two more teams in the NBA than in 1987, and summarize in Table 2. This time, the draft probabilities are adjusted until the 332nd game. The Miami Heat, the team ranked last in the season, had a poor enough start to the season that their draft probability is 45.5% when the NTC condition binds and the probability adjustment stops. Some other teams, though, performed better in the latter half of the season than the first half. As a result, the team ranked 24th has a lower draft probability than the team ranked 20th, since more of the Charlotte Hornets' losses occurred after draft probabilities were frozen compared to the Indiana Pacers' losses, which occurred closer to the beginning of the season.

Rank	Team	Wins	NBA Lottery	Our Lottery
25	Miami Heat	15	11%	45.5%
24	Charlotte Hornets	20	11%	7.5%
23	San Antonio Spurs	21	11%	9.5%
22	Los Angeles Clippers	21	11%	3.5%
21	New Jersey Nets	26	11%	2.0%
20	Sacramento Kings	27	11%	12.0%
19	Indiana Pacers	28	11%	18.5%
18	Dallas Mavericks	38	11%	0.5%
17	Portland Trail Blazers	39	11%	0.5%

Table 2: Allocation Policy from 1989 Season, Adjustment Cutoff at 332nd Game

5 Conclusion

Implementing our proposed decoupling of the final ranking from the draft allocation rule results in a rule that dynamically adjusts each season. It ensures that teams do not have an incentive to purposely lose games but still aims to allocate the draft pick to the team with the worst record in a season. This results in a draft allocation rule that optimizes both competition and parity in a league. To our knowledge, this is the first paper to examine the draft allocation problem using theory of tournaments and incentives in a mathematical model of team decision making.

One important extension to our paper would be to incorporate heterogeneous abilities for the teams. In this setting, the results from games played in the season reveal information about which teams have a lower ability, and lowest ranked teams are also teams with lower estimated skill.

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A Existing Draft Allocation Policies

Year	Mechanism
1966-1984	Coin flip between conference losers
1985-1989	Uniform lottery
1990-Present	Weighted lottery, modified in 1993, 2005, 2009

Table 3: A History of NBA Draft Allocation Policies

Rank	Probability (%)
30	14
29	14
28	14
27	12.5
26	10.5
25	9
24	7.5
23	6
22	4.5
21	3.0
20	2.0
19	1.0
18	1.5
17	0.5

Table 4: 2019 NBA Draft Lottery for First Draft Pick