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ECEn 380-002

Lab 2: Continuous-Time LTI Impulse Response & Convolution

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Task 1a TA Signature 9-24-19
Task 1b TA Signature 9/24/19
Task 2 TA Signature 10-1-19
Task 3 TA Signature 10-8-19

Prelab:

- Example 2.17: $Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = x(t)$

$\boxed{\Rightarrow}\left(Ri(t) + \frac{1}{CD}i(t) = x(t)\right)\cdot D \boxed{\Rightarrow} \left(DR + \frac{1}{C}\right)i(t) = Dx(t)$

$\boxed{a.\ \left(R + \frac{1}{CD}\right)i(t) = x(t)}$

$\boxed{b.\ \left(DR + \frac{1}{C}\right)\frac{D}{C}y(t) = x(t)}$

- Example 2.5: $(D^2 + 5D + 6)y(t) = (D+1)x(t)$

characteristic equation: $\lambda^2 + 5\lambda + 6 \qquad \lambda = -2, -3$

$h(t) = (c_1 e^{-2t} + c_2 e^{-3t})u(t)$

$x(t) = \delta(t) \qquad y(t) = h(t)$

$h''(t) + 5h'(t) + 6h(t) = \delta'(t) + \delta(t)$

$5k_1 + k_2 = 1$, make $k_1 = 1$, which means $k_2 = -4$

$t = 0^+ \Rightarrow h(t): c_1 + c_2 = 1$

$h'(t): -2c_1 - 3c_1 = -4$

$\rightarrow c_1 = -1, \quad c_2 = 2$

$\boxed{h(t) = (-e^{-2t} + 2e^{-3t})u(t)}$

Task 1 Summary: We used circuit analysis to calculate the capacitance we'd need. We then modeled the circuit using a differential equation which we derived, solved for its impulse response, & wrote a MATLAB script to plot the impulse response as recorded in figure 1.

Task 2 Summary: We built the crossover circuit we modeled in Task 1 using a 10 μF capacitor & an 82 Ω resistor for R_G. We then measured the impulse response using the O-scope & found it to be similar to the Task 1 MATLAB plot.

Task 3 Summary: We first used the convolution integral to calculate the RC response. Then we corroborated the calculations using O-scope measurements. These were done using a square pulse input signal. We then plotted the calculated RC response in MATLAB & used the conv function to make sure our calculation was correct.

Conclusion:

In this lab we learned how to model & work with LTI's using differential equations, convolutions, & impulse response. We were able to confirm the validity of each method using O-scope measurements & MATLAB plots. Specifically, we experimentally measured the properties of a speaker crossover circuit using these methods.

Task 1:

- Using 2.5 kHz for ω_c, which converted to rad/sec ~~2.5×10^3~~ becomes $5\pi \times 10^3$ rad/s

$C = \frac{1}{8(5\pi\times10^3)} = 8\,\mu F$. 10 μF is the closest value to 8 μF that we have in our kit.

- recalculating w_c using $10\,\mu F$:

$$w_c = \frac{1}{10e\text{-}6(8)} = \boxed{12.5e3 \text{ rad/s}}$$

- Picking R_G: needs to be ~10x bigger than R_L ($8\,\Omega$). So we chose $\boxed{82\,\Omega}$

- Making Equation (3) match Form of Equation (5):

$N = 1$

$M = 0$

$a_0 = \frac{1}{R_L} = 0.125$

$a_1 = C = 10\,\mu F$

$b_0 = 1$

$$(10e\text{-}6)\frac{d}{dt}V_L(t) + 0.125\,V_L(t) = i_g(t)$$

- $$i_g(t) = \frac{V_s(t) - V_L(t)}{R_s + R_g} = C\frac{d}{dt}V_L(t) + \frac{1}{R_L}V_L(t)$$

$$\frac{V_s(t)}{R_s + R_G} = C\frac{d}{dt}V_L(t) + \left(\frac{1}{R_L} + \frac{1}{R_s + R_G}\right)V_L(t)$$

$N = 1$

$M = 0$

$a_0 = \left(\frac{1}{R_L} + \frac{1}{R_s + R_G}\right)$

$a_1 = C$

$b_0 = \frac{1}{R_s + R_G}$

- Finding Equation (6)

as $R_s + R_G$ approaches infinity, $\frac{1}{R_s + R_G} = 0$.

a_0 becomes $\frac{1}{R_L}$. Divide above equation by C =>

$$\boxed{\text{Eq. 6} \quad \frac{V_s(t)}{C(R_s + R_G)} = \frac{d}{dt}V_L(t) + \frac{1}{R_L C}V_L(t)}$$

$$\boxed{\text{Eq. 7} \quad \frac{\delta(t)}{C(R_S+R_G)} = \frac{d}{dt}h(t) + \frac{1}{R_L C}h(t)}$$

- $\lambda = -\frac{1}{R_L C}$ since $\lambda + \frac{1}{R_L C} = 0$ is the characteristic equation

$$h(t) = C_1 e^{(-1/R_L C)t}(u(t))$$

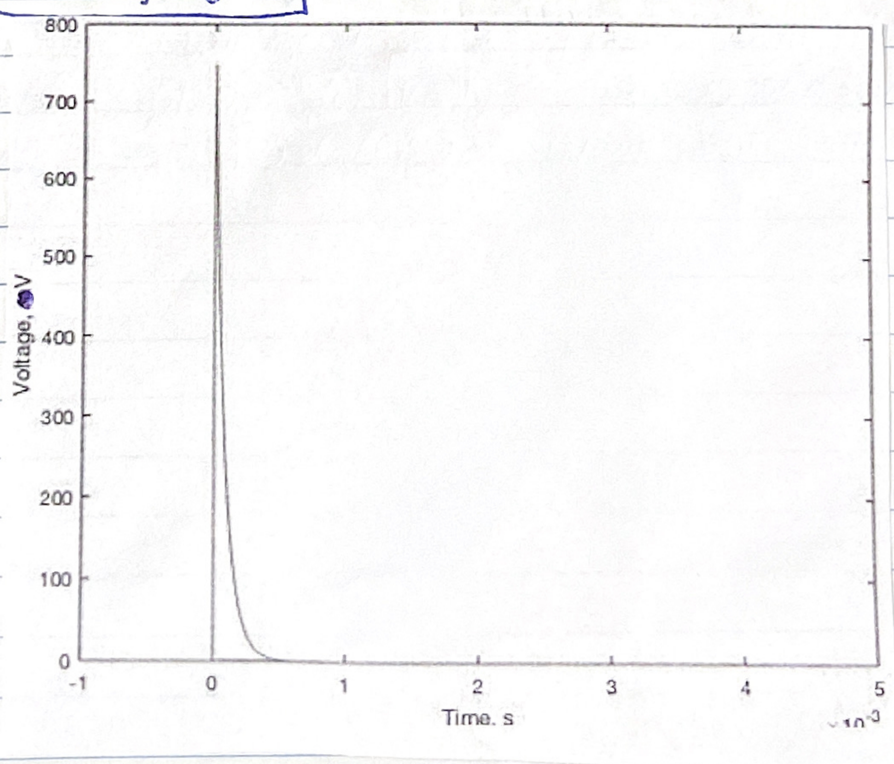
$h'(0) = K_1\,\delta(t)$, substituting into equation 7:

$$K_1\,\delta(t) = \frac{\delta(t)}{C(R_S+R_G)} \qquad K_1 = \frac{1}{C(R_S+R_G)}$$

at $t=0$ $K_1 = C_1$

$$\boxed{h(t) = \frac{e^{(-1/R_L C)t}\,u(t)}{C(R_S+R_G)}}$$

Figure 1
Calculated RC Impulse Response



- The plot looks like the natural response of an RC circuit because it is an RC circuit. We put a ton of voltage through the circuit, which charges the capacitor to a peak. Then we remove the voltage source & the capacitor begins to discharge.

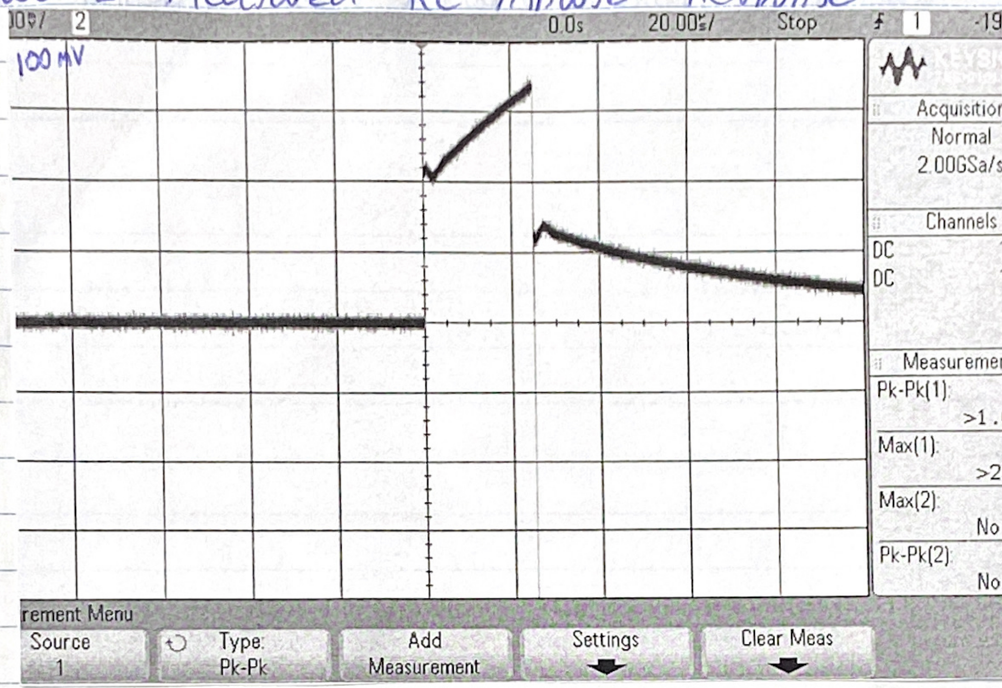
Task 2: Measure an RC Impulse Response

- Measured impulse response vs. Calculated:

  The first thing we noticed was the max voltage difference. Our calculated impulse response had a max voltage around 760mV while our measured max voltage was around 330mV.

  We also noticed that the measured response jumps up steeply (when the signal is turned on), then slowly increases from there until the peak (normal behavior for a capacitor). Once the signal turns off, the voltage drops steeply, then slowly decreases towards zero (also in accordance with the discharge properties of a capacitor).

- Figure 2: Measured RC Impulse Response



- calculating scale factor using 25µs as pulse width & a 10 Vpp square wave. $10\int_0^{25e^{-6}} dt = 10\left[t\right]_0^{25e^{-6}} = \boxed{25e-5}$

So we must multiply our Matlab plot from Task 1 by 25e-5 in order to have that plot match our measured impulse response.

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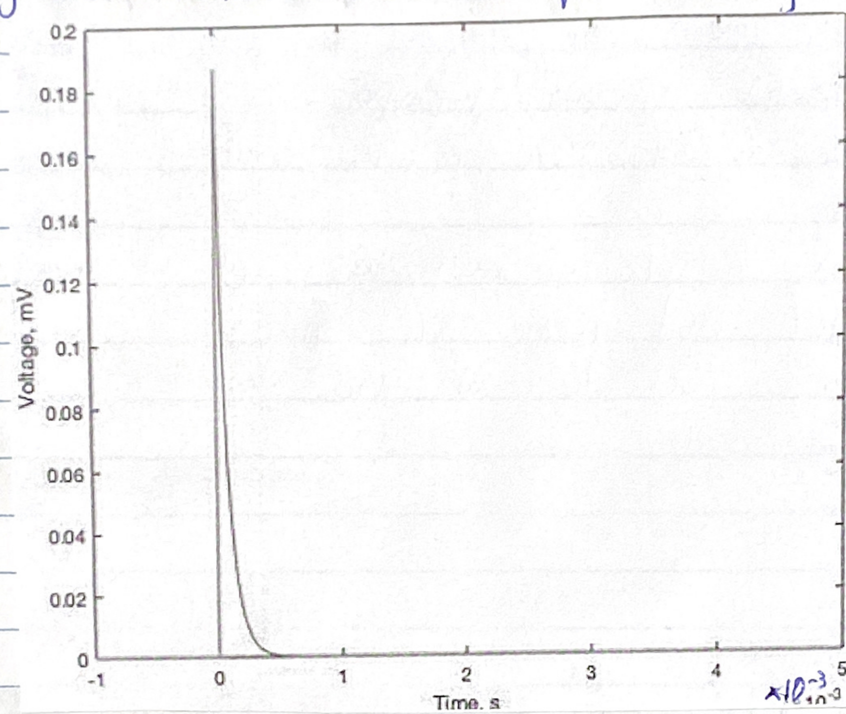
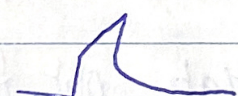
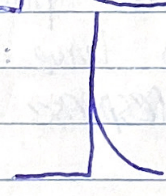
Figure 3: New MatLab plot using Scaling Factor



- Differences between Figures 2 & 3.
  - shape: our measured plot has a shape like



while the calculated plot still looks like:



This is due to the limitations of a normal capacitor. It cannot charge & discharge as quickly as the model would like.

  - Amplitude: Our measured plot saw an amplitude of about 330mV while our calculated plot had an amplitude of about 190mV
  - Time Scale: the time scale is very different. Figure 2 is working with a 25 μS period while Figure 3 is showing about .5mS.

- Other sources of error
  1. We are not inputting an infinite voltage
  2. The internal resistance of the capacitor causes a sudden drop in voltage once the signal is turned off as opposed to the nice decay curve in the MATLAB plot. This is due to the capacitor not being ~~ideal~~ ideal.
  3. Since capacitors resist changes in voltage, the physical capacitor could not reach the max voltage as instantly as the matlab model shows.

Task 3: RC Output & Convolution

We want to find $V_L(t) = \text{rect}(t, t_0) * h(t)$

Where $h(t)$ is the equation from Task 1 &

$$\text{rect}(t, t_0) = \begin{cases} 1 & 0 \le t \le t_0 \\ 0 & \text{otherwise} \end{cases} \qquad t_0 = 1\,ms$$

$$\frac{1}{C(R_s+R_G)} \int_0^t e^{-\frac{\tau}{R_L C}}\, d\tau \Rightarrow \boxed{\frac{R_L}{R_s+R_G}\left[1 - e^{\frac{-t}{R_L C}}\right], \quad 0 < t < 1\,ms}$$

$$\frac{1}{C(R_s+R_G)} \int_{t-t_0}^{t} e^{-\frac{\tau}{R_L C}}\, d\tau \Rightarrow \cancel{\frac{R_L}{R_s+R_G}\left[e^{\frac{-t}{R_L C}} - e^{\frac{-t+t_0}{R_L C}}\right]}$$

$$\boxed{\frac{R_L}{R_s+R_G}\left[e^{\frac{-t+t_0}{R_L C}} - e^{\frac{-t}{R_L C}}\right], \quad t \ge 1\,ms}$$

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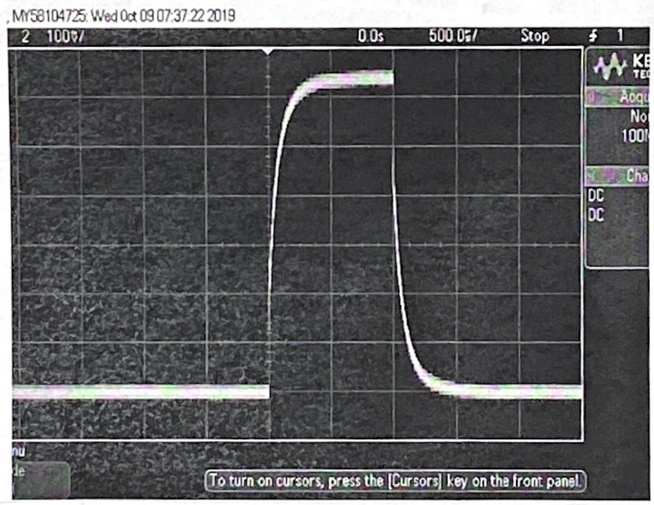
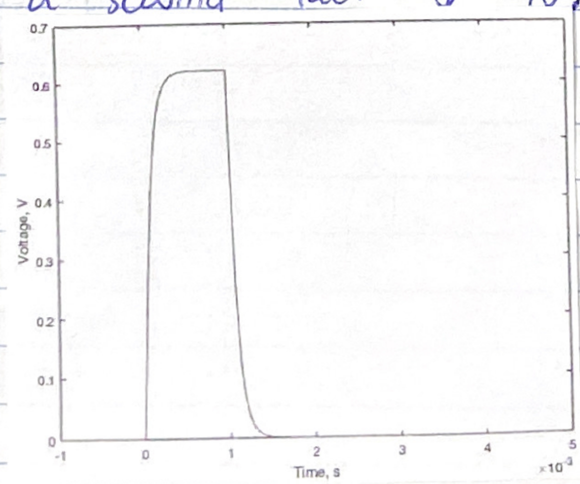
- Physical circuit measurements using square pulse input signal:
  We built the speaker crossover circuit & measured the output using the O-scope.

Figure 4: O-scope measurement of RC output. Square pulse input from function generator with amplitude of 10V



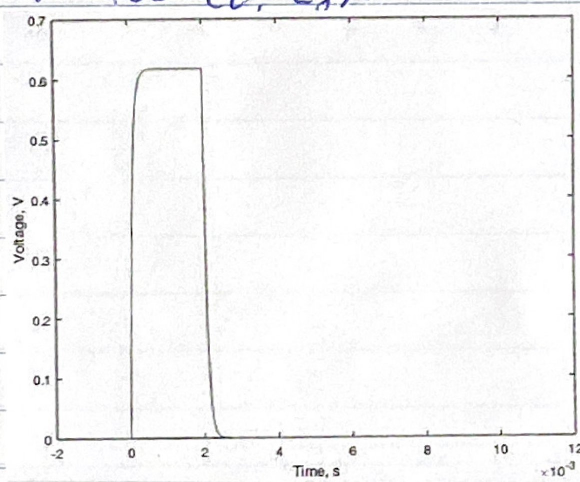
- MATLAB script to plot $V_L(t)$.

Figure 5: MATLAB plot of $V_L(t)$. We had to use a scaling factor of 10. This very well matched our measured result from Figure 4.



- We then wrote a script to convolve $h(t)$ with the square pulse input from task 2, adjusted our time sampling since convolutions have a length of the 2 vector lengths added together plus 1, & multiply by 0.00002 as well as the scaling factor 10.

Figure 6: MATLAB plot of the convolution of $h(t)$ & $rect(t, t_n)$.



This plots lines up almost exactly with Figures 4 & 5, which is very important in making sure all calculations & measurements were done correctly.

Figure 7: MATLAB script used to create all plots found in this lab report. However, for task 2, h was multiplied by 25e-5 for scaling. That was taken out for task 3.

Lab2_Task1.m

```
C = 10e-6;
Rs = 50;
Rg = 82;
Rl = 8.2;

t = -.001:0.000001:.005;

h = exp((-t/(Rl*C)))/(C*(Rs+Rg)).*(t>0);

plot(t, h); xlabel('Time, s'); ylabel('Voltage, mV');

vL1 = Rl/(Rs+Rg)*(1-exp(-t/(Rl*C))) .* ((0 < t) .* (t < .001));
vL2 = Rl/(Rs+Rg)*(exp((-t+.001)/(Rl*C))-exp(-t/Rl/C)) .* (t >= .001);

figure;
plot(t,(vL1+vL2)*10); xlabel('Time, s'); ylabel('Voltage, V');

input = 1 .* (t<.001);

Ts = .000001;

t2 = -.001:0.000001:.011;

convolution = conv(h,input)*Ts*10;

figure;
plot(t2,convolution); xlabel('Time, s'); ylabel('Voltage, V');
```