ECEn 390
Statistical Distance Experiment Report
Evan Nuss

I. Introduction

Data was reported on the ability of 2 laser tag systems to transmit and detect hits at 20’, 40’, 60’, 80’, and 100’. This task is an exercise of statistical analysis in which we will estimate hit probability and confidence intervals as well as observe and analyze the randomness of the data.

II. Probability Estimation and Confidence Interval Calculation

An estimation of the probability for this type of variable is found using the following equation:

$$p = \frac{X}{n} \qquad \text{(Eq. 1)}$$

Where X is the number of successful trials (hits detected in our case) and n is the total number of trials. Because p is just an estimation, we must use a different approach to calculating the confidence intervals. We modify n and p using the following equations:

$$\tilde{n} = n + 4 \qquad \text{(Eq. 2)}$$

$$\tilde{p} = \frac{X+2}{\tilde{n}} \qquad \text{(Eq. 3)}$$

To get the following as the equation for a level 100(1- α) confidence interval:

$$\tilde{p} \pm z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \qquad \text{(Eq. 4)}$$

A. Board 1 (with Cole as the shooter)

Using Eq. 1:
p_20 = 40/40 = **1.0**

Using Eq. 2 (becomes new n for all distances):
n_new = n + 4 = **44**

Using Eq. 3:
p_20_new = 42/44 = **0.955**

Using Eq. 4:
α/2 = 0.025, zielding a z score of 2.021
0.955 ± 2.021*sqrt((0.955*0.045)/44)
Which gives us an interval of **(1.0, 0.892)**

Using the same equations and n_new, I repeat the process with each distance:
p_40 = 35/40 = **0.875**
p_40_new = 37/44 = **0.841**
0.841 ± 2.021*sqrt((0.841*0.159)/44)
Which gives us an interval of **(0.952, 0.730)**

p_60 = 35/40 = **0.875**
p_60_new = 37/44 = **0.841**
0.841 ± 2.021*sqrt((0.841*0.159)/44)
Which gives us an interval of **(0.952, 0.730)**

p_80 = 29/40 = **0.725**
p_80_new = 31/44 = **0.705**
0.705 ± 2.021*sqrt((0.725*0.295)/44)
Which gives us an interval of **(0.850, 0.564)**

p_100 = 24/40 = **0.60**
p_100_new = 26/44 = **0.591**
0.591 ± 2.021*sqrt((0.591*0.409)/44)
Which gives us an interval of **(0.741, 0.441)**

B. Board 2 (with Kris as the shooter)
We follow the exact same process as with Board 1:
p_20 = 40/40 = **1.0**
p_20_new = 42/44 = **0.955**
0.955 ± 2.021*sqrt((0.955*0.045)/44)
Which gives us an interval of **(1.0, 0.892)**

p_40 = 40/40 = **1.0**
p_40_new = 42/44 = **0.955**
0.955 ± 2.021*sqrt((0.955*0.045)/44)
Which gives us an interval of **(1.0, 0.892)**

p_60 = 39/40 = **0.975**
p_60_new = 41/44 = **0.932**
0.932 ± 2.021*sqrt((0.932*0.068)/44)
Which gives us an interval of **(1.0, 0.855)**

p_80 = 28/40 = **0.70**
p_80_new = 30/44 = **0.682**
0.682 ± 2.021*sqrt((0.682*0.318)/44)
Which gives us an interval of **(0.824, 0.540)**

p_100 = 0/40 = **0**
p_100_new = 2/44 = **0.045**
0.045 ± 2.021*sqrt((0.045*0.955)/44)
Which gives us an interval of **(0.108, 0)**

III. Hypotheses Concerning Probabilities

Based on the rating we are given that our system works at 40’, I hypothesize that the probability for a hit at 20’ would be 100%, then slightly below 100%, maybe 95%, at 40’, 60% at 60’, 30% at 80’, and 5% at 100’.

IV. Summarizing Results and Conclusion

Table 1: Data from Results, Probability Estimates, and Confidence Intervals

	Board 1			Board 2		
	# of Hits (n = 40)	Probability Estimate	95% Level Confidence Interval	# of Hits (n = 40)	Probability Estimate	95% Level Confidence Interval
20 feet	40	0.955	(1.0, 0.892)	40	0.955	(1.0, 0.892)
40 feet	35	0.841	(0.952, 0.73)	40	0.955	(1.0, 0.892)
60 feet	35	0.841	(0.952, 0.73)	39	0.932	(1.0, 0.855)
80 feet	29	0.705	(0.85, 0.564)	28	0.682	(0.824, 0.54)
100 feet	24	0.591	(0.741, 0.441)	0	0.045	(0.108, 0)

Factors leading to the randomness of the outcomes:

- Noise from ceiling lights.
- Differences in the design of the transmitter and receiver boards.
- The aim of the shooter.
- Inconsistencies in the signal being transmitted.
- Hardware limitations.

The total probability of a hit at any distance for Board 1 is 0.7866 while for Board 2 is 0.7138. So even though Board 2 performs better at the first 3 distances, Board 1 has a slight advantage overall since the performance of Board 2 sharply drops off for the last 2 distances.

If I were to sell either of these systems, I would market it for a range of 60 feet, since they both perform very well up to, and at 60 feet, as well as marginally well past 60 feet.