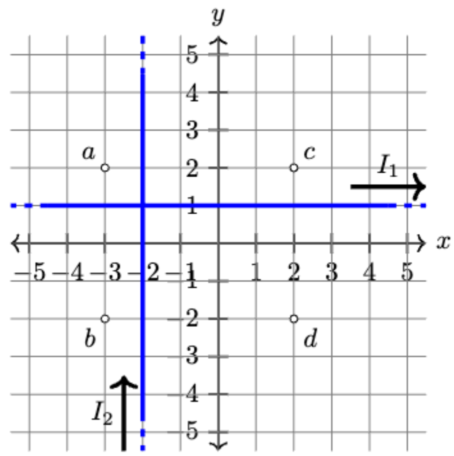
NAME and ID: ____ SCORE: _____ /4 points

PHY 274 PROBLEM SOLVING WORKSHOP VII

1. There are two infinite wires arranged in a plane as shown in the figure to the right, where x and y are measured in meters. Each wire carries a current, given by $I_1 = 3\,\text{A}$, $I_2 = 12\,\text{A}$, in the directions shown. Calculate the magnitude and direction of the total magnetic field ($\vec{\boldsymbol{B}}_a$, $\vec{\boldsymbol{B}}_b$, $\vec{\boldsymbol{B}}_c$, $\vec{\boldsymbol{B}}_d$) at each the indicated points a, b, c, d shown.



Part (a)

At point a both fields are directed out of the page ($\odot$). So the fields are in the same direction, resulting in a magnitude given by:

$$B_a = \frac{\mu_0}{2\pi}\left|\frac{I_1}{r_1} + \frac{I_2}{r_2}\right| = 2\left|\frac{3}{1} + \frac{12}{1}\right| \times 10^{-7} = \boxed{30 \times 10^{-7}\,\text{T}}$$

With a direction $\boxed{\odot}$.

Part (b)

At point b B_1 is into the page ($\otimes$) and B_2 is out of the page ($\odot$). So the fields are in the opposite directions, resulting in a magnitude given by:

$$B_a = \frac{\mu_0}{2\pi}\left|-\frac{I_1}{r_1} + \frac{I_2}{r_2}\right| = 2\left|-\frac{3}{3} + \frac{12}{1}\right| \times 10^{-7} = \boxed{22 \times 10^{-7}\,\text{T}}$$

With a direction $\odot$.

Part (a)

At point c B_1 is out of the page ($\odot$) and B_2 is into the page. So the fields are in opposite directions, resulting in a magnitude given by:

$$B_a = \frac{\mu_0}{2\pi}\left|\frac{I_1}{r_1} - \frac{I_2}{r_2}\right| = 2\left|\frac{3}{1} - \frac{12}{4}\right| \times 10^{-7} = \boxed{0 \times 10^{-7}\,\text{T}}$$

With no direction.

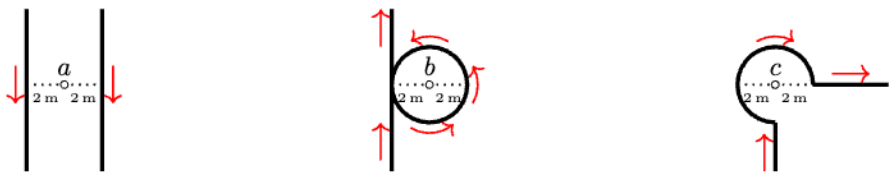
Part (a)

At point d both fields are directed into the page ($\otimes$). So the fields are in the same direction, resulting in a magnitude given by:

$$B_a = \frac{\mu_0}{2\pi}\left|-\frac{I_1}{r_1} - \frac{I_2}{r_2}\right| = 2\left|\frac{3}{3} + \frac{12}{4}\right| \times 10^{-7} = \boxed{8 \times 10^{-7}\,\text{T}}$$

With a direction $\boxed{\otimes}$.

2. There are three separate configurations of straight wires and circular loops shown in the figure below. In configuration a there are two infinite wires. In configuration b there is one infinite wire and one full circular loop. In configuration c there are two semi-infinite wires and one three-quarter circular loop. A current $I = 4\,\mathrm{A}$ flows in the directions indicated by the arrows. The radius of each circular segment is $R = 2\,\mathrm{m}$.



Find magnitudes B_a, B_b, B_c and direction ($\odot$, $\otimes$) of the magnetic field thus generated at points a, b, c. (For this problem, approximate the value of pi as $\pi \approx 3$, and the permeability constant as $\mu_0 \approx 12 \times 10^{-7}\,\mathrm{Tm/A}$.)

Part (a)

The left wire produces a field out of the page (+) and the right wire produces a field into the page (−), thus the total field is:

$$B_a = \frac{\mu_0 I}{2\pi R} - \frac{\mu_0 I}{2\pi R} = \boxed{0}$$

Part (b)

The straight wire produces a field into the page (−) and the circular loop produces a field out of the page (+).

$$B_b = -\frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2\pi R}(-1+\pi) \approx \frac{\mu_0 I}{\pi R} = \boxed{8 \times 10^{-7}\,\mathrm{T}}$$

The positive sign implies that the direction is $\boxed{\odot}$

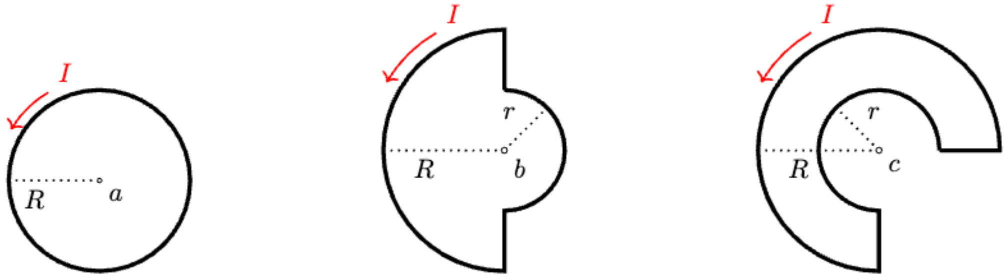
Part (c)

Neither of the semi-infinite wires contribute to the field, because point c is directly along their central axis. The net field is thus caused entirely by the semi-circular loop (which is directed into the page (−) such that

$$B_c = -\frac{3}{4}\frac{\mu_0 I}{2R} \approx \boxed{-9 \times 10^{-7}\,\mathrm{T}}$$

The negative sign implies that the direction is $\boxed{\otimes}$

3. A single loop of wire is bent into semi-circular segments connected by straight segments of wire in three different configurations, as shown in the figures below. There is a current $I = 4\,\text{A}$ flowing through each loop in the direction shown. The smaller radius is $r = 1\,\text{m}$ and the larger radius is $R = 2\,\text{m}$. The center of each loop is indicated by the point a, b, and c in each figure. Use the approximation $\pi \approx 3$ such that $\mu_0 = 4\pi \times 10^{-7}\,\text{Tm/A} \approx 12 \times 10^{-7}\,\text{Tm/A}$.



(a) What is the magnitude and direction ($\odot$, $\otimes$) of the magnetic field $\vec{\boldsymbol{B}}_a$ at point a?

(b) What is the magnitude and direction ($\odot$, $\otimes$) of the magnetic field $\vec{\boldsymbol{B}}_b$ at point b?

(c) What is the magnitude and direction ($\odot$, $\otimes$) of the magnetic field $\vec{\boldsymbol{B}}_c$ at point c?

Part (a)

The magnetic field is given by

$$B_a = \frac{\mu_0 I}{2R}$$

$$B_a = \boxed{12 \times 10^{-7}\,\text{T}}$$

And is directed $\boxed{\odot}$

Part (b)

The magnetic field is given by

$$B_b = \frac{1}{2}\frac{\mu_0 I}{2R} + \frac{1}{2}\frac{\mu_0 I}{2r}$$

$$B_b = \frac{\mu_0 I}{4}\left(\frac{1}{R} + \frac{1}{r}\right)$$

$$B_b = \boxed{18 \times 10^{-7}\,\text{T}}$$

And is directed $\boxed{\odot}$

Part (c)

The magnetic field is given by

$$B_c = \frac{3}{4}\frac{\mu_0 I}{2R} - \frac{3}{4}\frac{\mu_0 I}{2r}$$

$$B_c = \frac{3\mu_0 I}{8}\left(\frac{1}{R} - \frac{1}{r}\right)$$

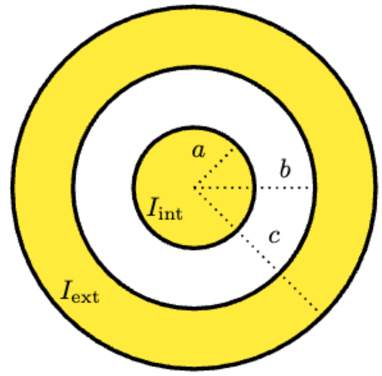
$$B_c = \boxed{-9 \times 10^{-7}\,\text{T}}$$

And is directed $\boxed{\otimes}$

4. A large coaxial cable shown has surfaces at radii $a = 1\,\text{m}$, $b = 3\,\text{m}$, and $c = 5\,\text{m}$, as labeled in the figure to the right. The current in the interior wire is $I_{\text{int}} = 3\,\text{A}$ directed into the page, and current in the exterior wire is $I_{\text{ext}} = 9\,\text{A}$ directed out of the page. Find the magnitude and direction (clockwise / counterclockwise) of the magnetic field ($\vec{\mathbf{B}}$) at the following radii:

   (a) $r = 2\,\text{m}$

   (b) $r = 6\,\text{m}$



Part (a)

At $r = 2\,\text{m}$ we are calculating the magnetic field between the interior wire and the exterior wire. An Amperian loop at this radius contains only the interior wire, so the magnitude of the magnetic field is given by:

$$B_2 = \frac{\mu_0 I_{\text{int}}}{2\pi(2\,\text{m})} = \boxed{3.0 \times 10^{-7}\,\text{T}}$$

And, by the right hand rule, is directed clockwise

Part (b)

At $r = 6\,\text{m}$ we are calculating the magnetic field outside the coaxial cable. An Amperian loop at this radius contains both the interior wire and exterior wire (which produce fields in opposite directions), so the magnitude of the magnetic field is given by:

$$B_2 = \frac{\mu_0 |I_{\text{ext}} - I_{\text{int}}|}{2\pi(6\,\text{m})} = \boxed{2.0 \times 10^{-7}\,\text{T}}$$

The contribution from the exterior wire is stronger than that of the interior wire (due to the larger current). Thus the total field will be in the same direction as the field generated by the exterior wire which, by the right hand rule, is counterclockwise