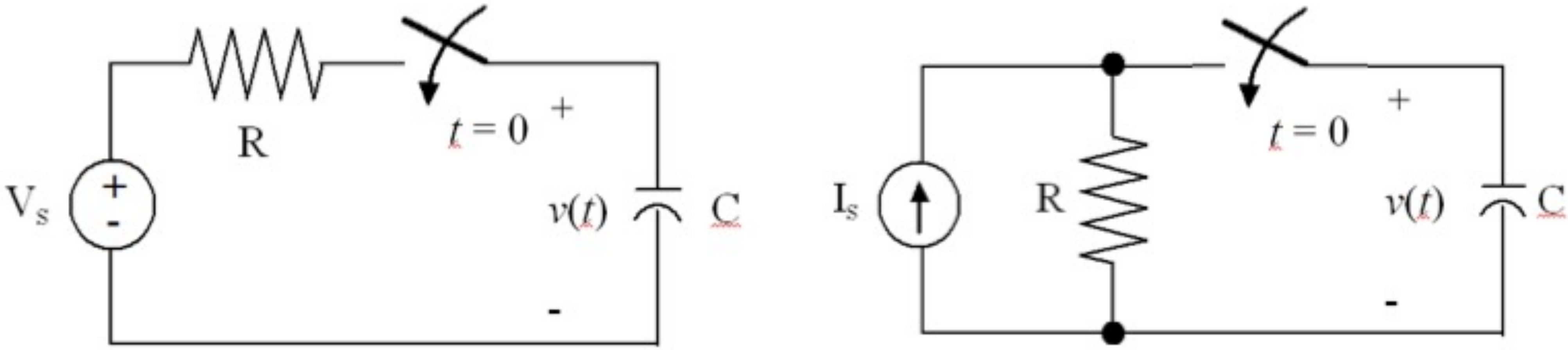
1^{st} Order Transients – 2

general solution

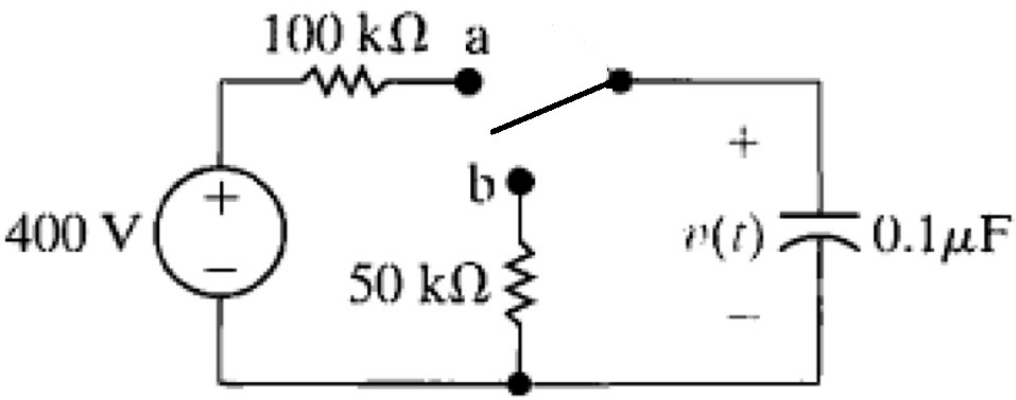
First Order RC Case



- Solution $$v(t) = (v_0 - v_\infty)\, e^{-t/RC} + v_\infty$$

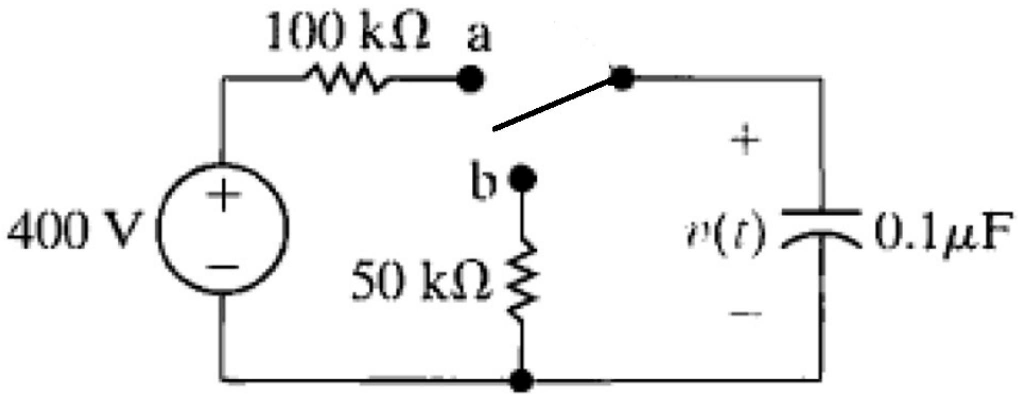
Example:

Switch changes a $\longrightarrow b$ at $t = 0$

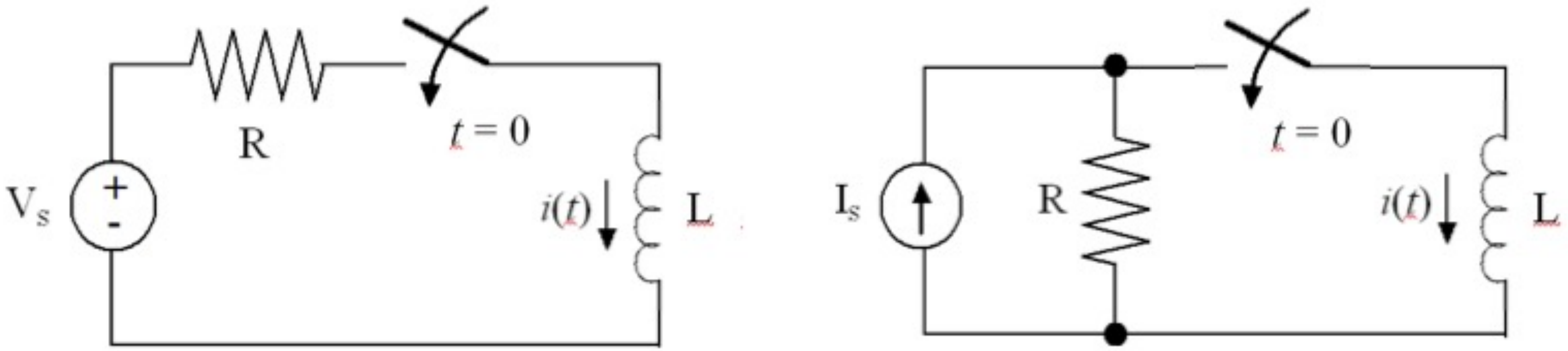


Example:

Switch changes b $\longrightarrow$ a at $t = 0$



First Order RL Case

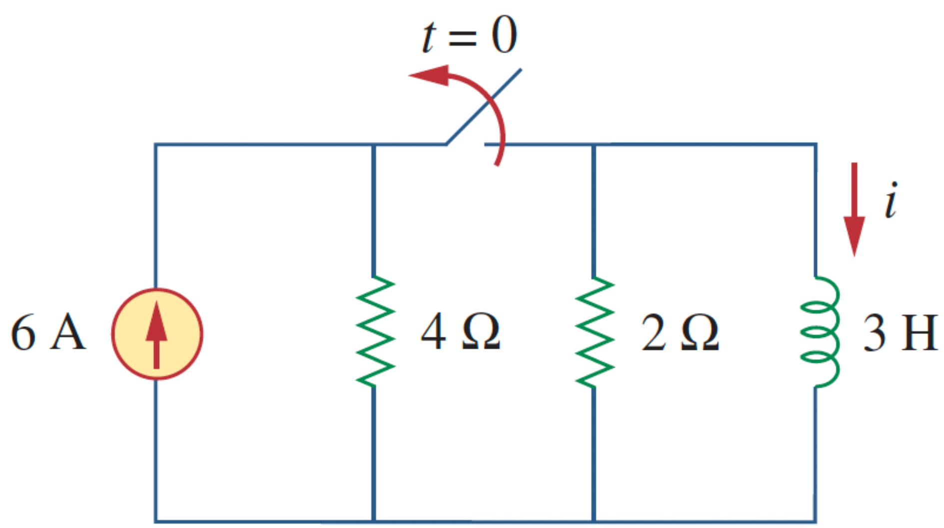


- Loop KVL equation: $\frac{di(t)}{dt} + \frac{R}{L}\, i(t) = \frac{1}{R} V_S$

- Solution: $i(t) = (i_0 - i_\infty)\, e^{-\frac{R}{L}t} + i_\infty$

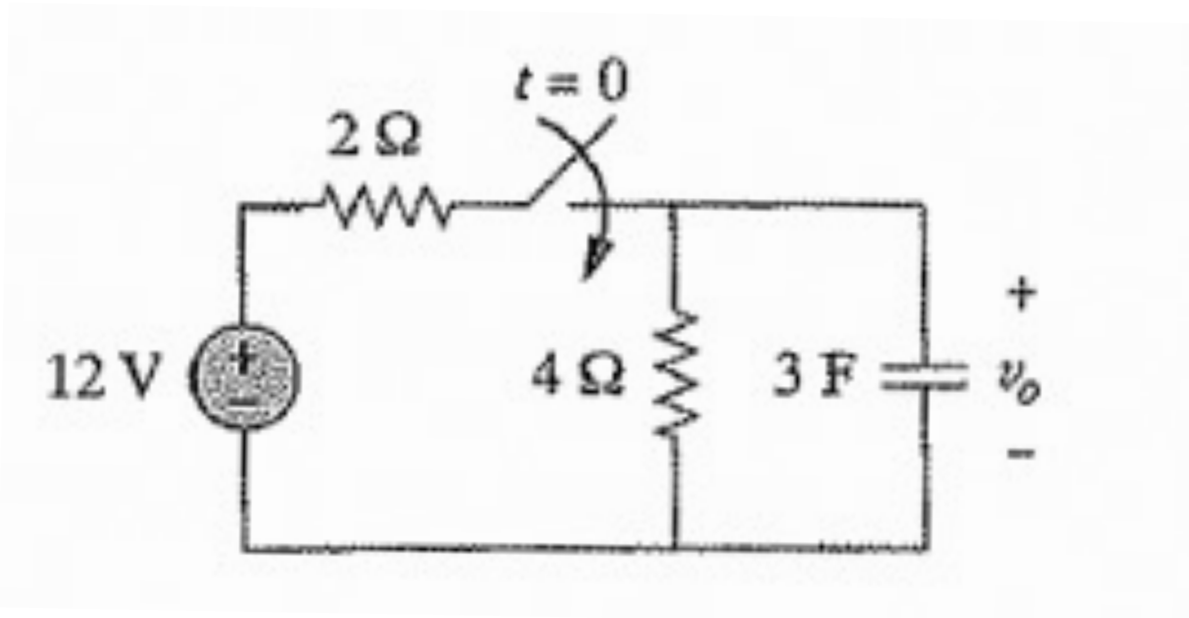
Example:

Switch opens at $t = 0$



Example:

Switch closes at $t = 0$



General Result – 1st Order

- Inductor current or capacitor voltage, $x(t)$ for $t > 0$

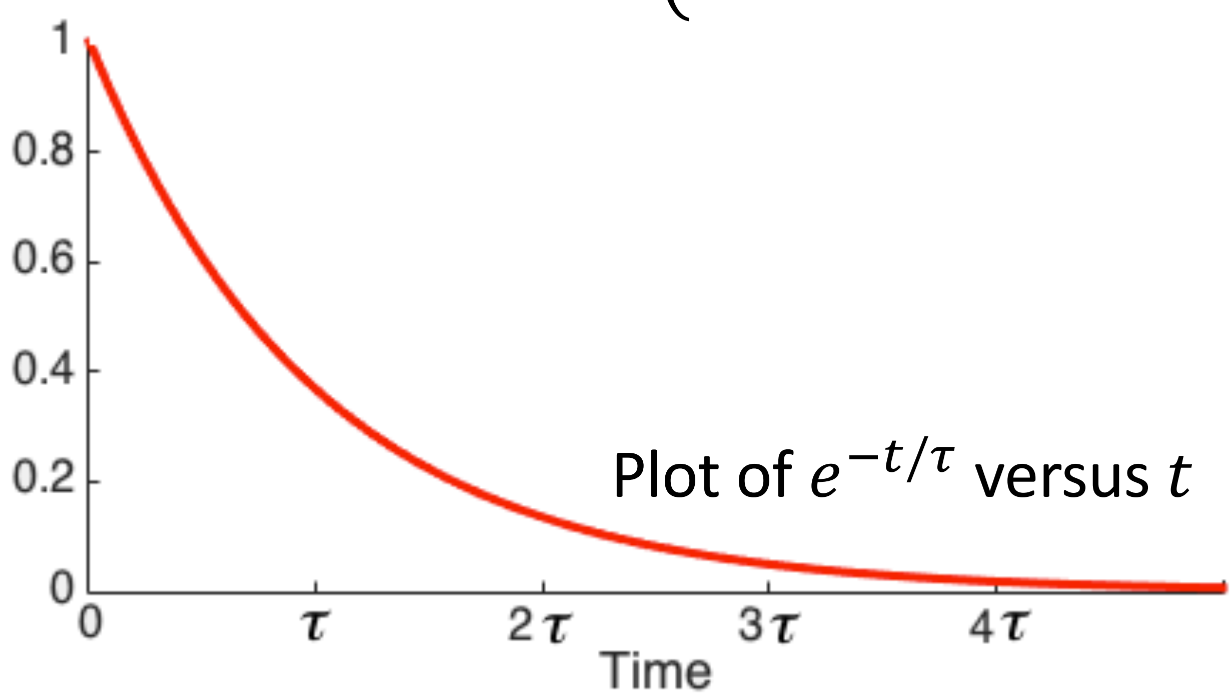
$$x(t) = [x(0) - x(\infty)]\, e^{-t/\tau} + x(\infty)$$

  - Final and initial values, $x(\infty)$ and $x(0)$:
    - From a DC analysis based on “open” or “short” models for C and L
    - Initial value exploits the continuity of capacitor voltages and inductor currents at $t = 0$

$$x(t) = [x(0) - x(\infty)]\, e^{-t/\tau} + x(\infty)$$

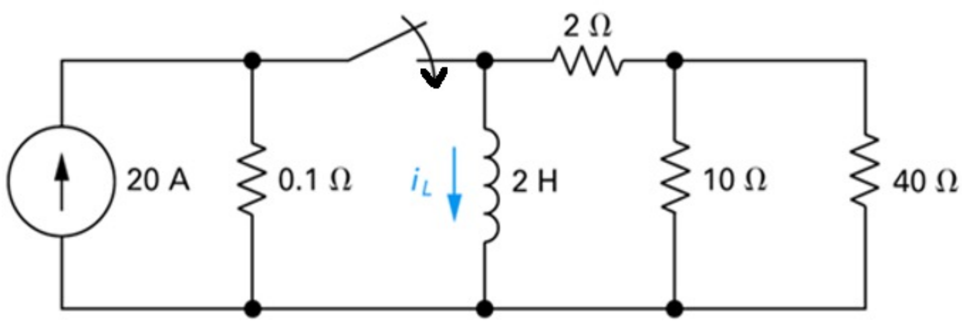
- Time constant τ (= L/R or RC)
- Why this form?

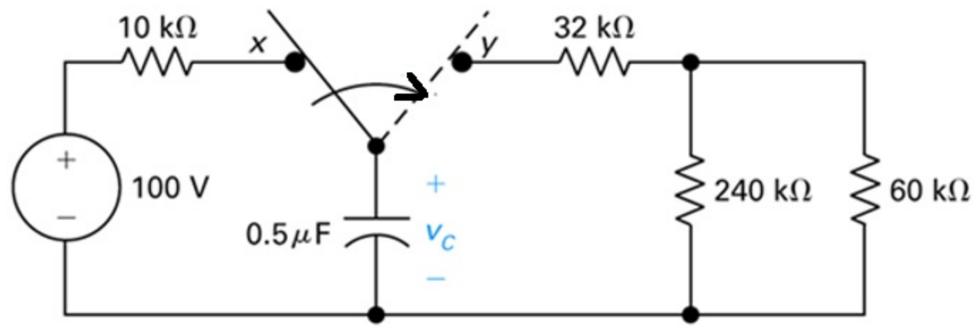
$$e^{-t/\tau} = \begin{cases} 1 & t = 0 \\ 0.369 & t = \tau \\ 0.002 & t = 4\tau \end{cases}$$



Plot of $e^{-t/\tau}$ versus t

What if the Circuit is more Complex?

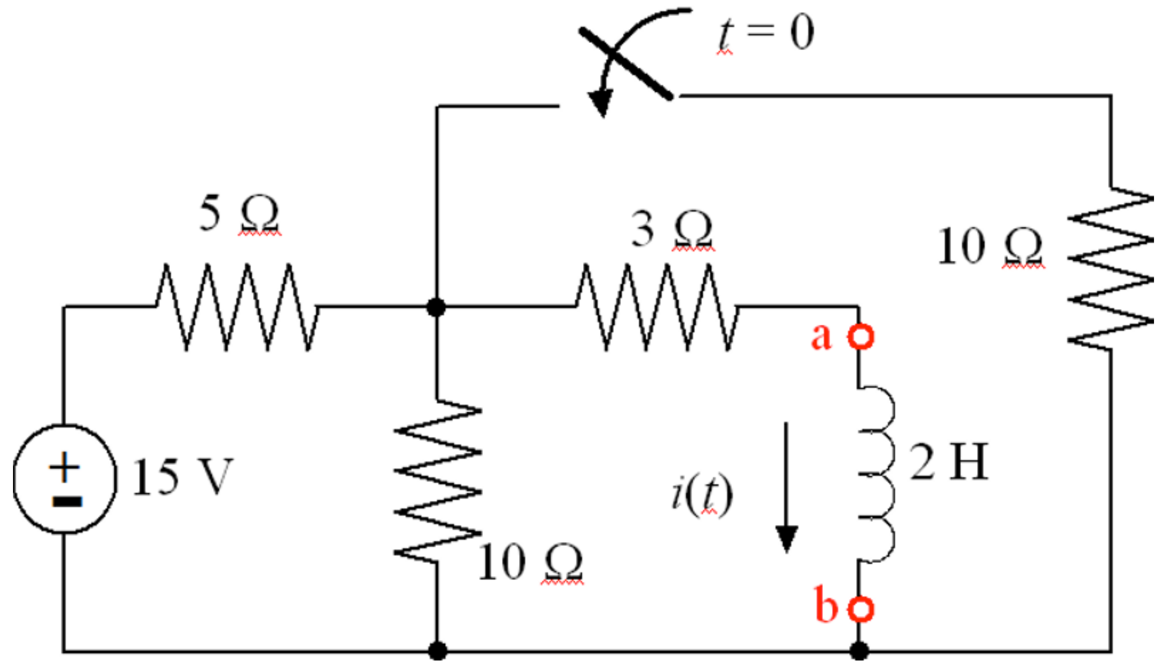




- Use the Thevenin equivalent circuit seen by L or C
  - Time constant $\tau = L/R_{Th}$ or $R_{Th}C$

Worked Example

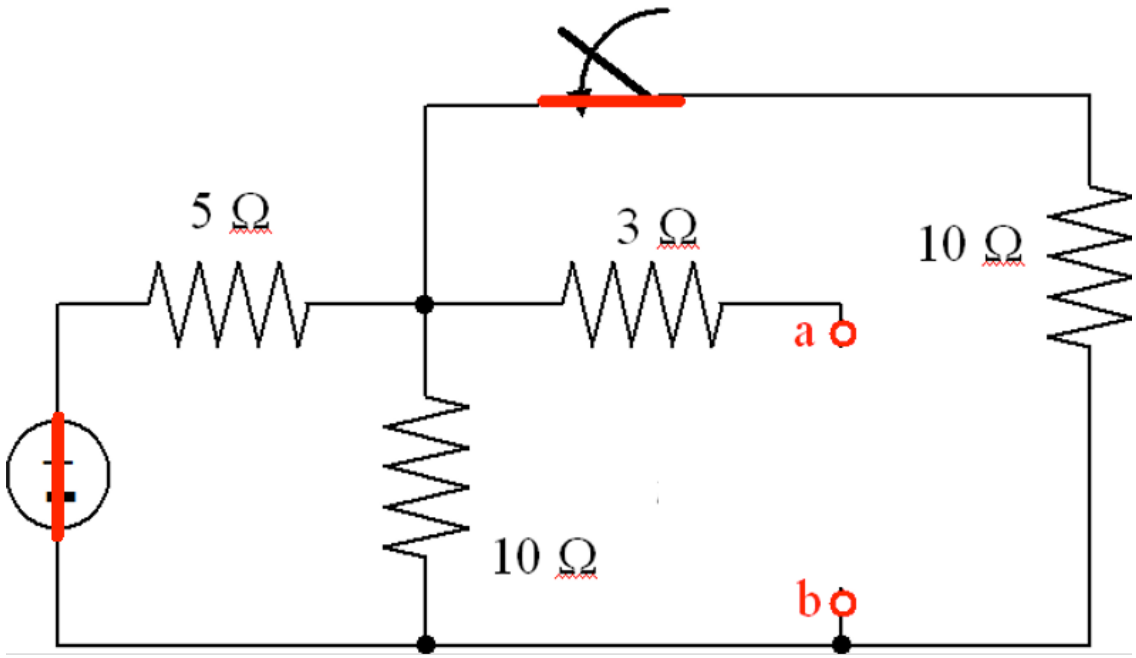
- Find $i(t)$



$$i(t) = (i_0 - i_\infty)\, e^{-t/\tau} + i_\infty$$

- Need: τ, i_∞, and i_0

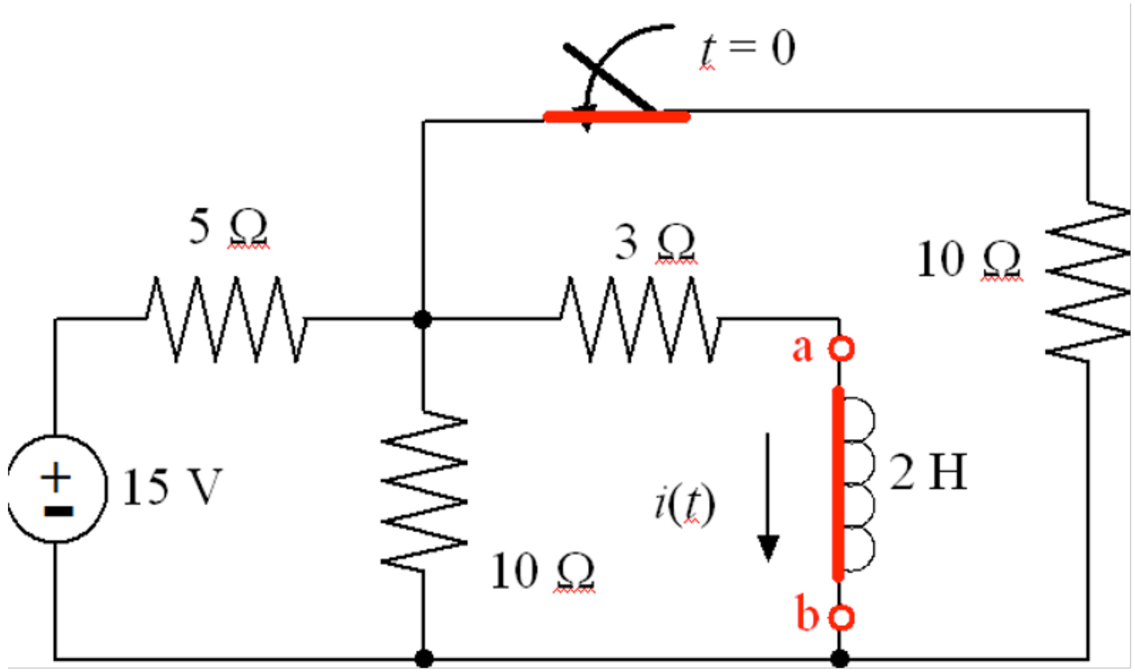
Step 1 – time constant $\tau = \frac{L}{R_{Th}}$



$$R_{Th} = 3 + 5||10||10$$
$$= 3 + 5||5$$
$$= 5.5\ \Omega$$

$$\tau = \frac{2}{5.5} = \frac{1}{2.75}\ \text{sec}$$

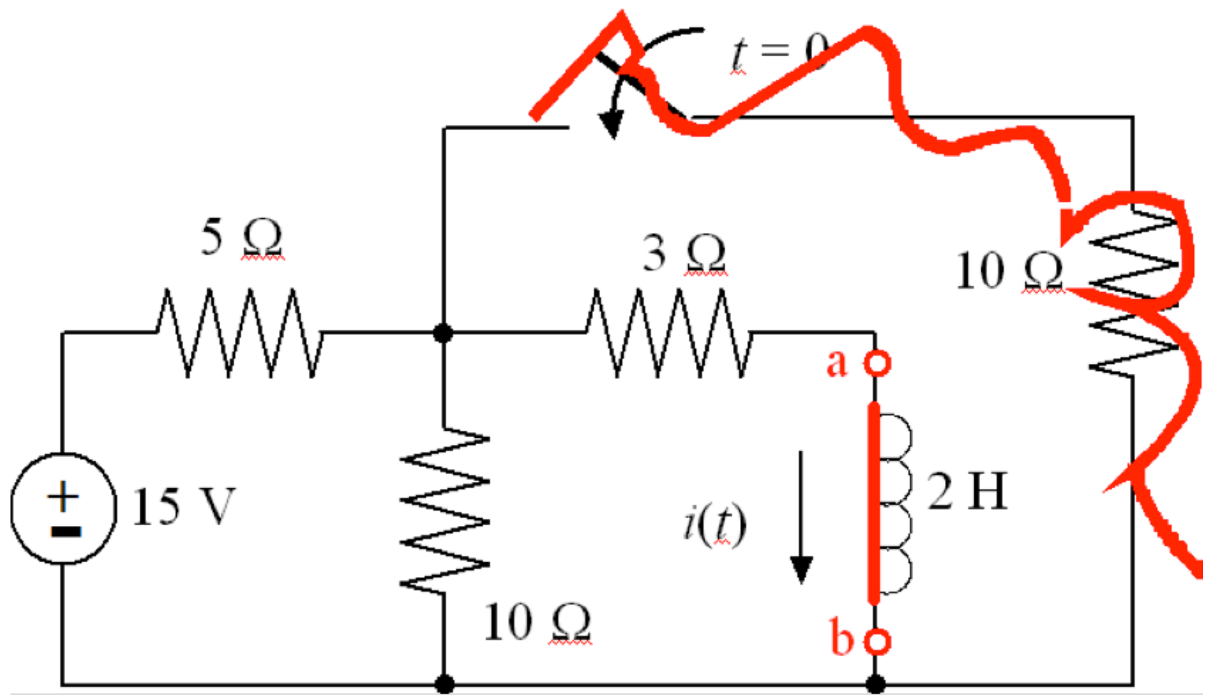
Step 2 – final value i_∞; as $t \to \infty$



$$\frac{v-15}{5}+\frac{v}{10}+\frac{v}{3}+\frac{v}{10}=0 \quad \Rightarrow \quad v=\frac{45}{11}$$

$$i_\infty = \frac{v}{3}=\frac{15}{11}=1.36 \text{ amps}$$

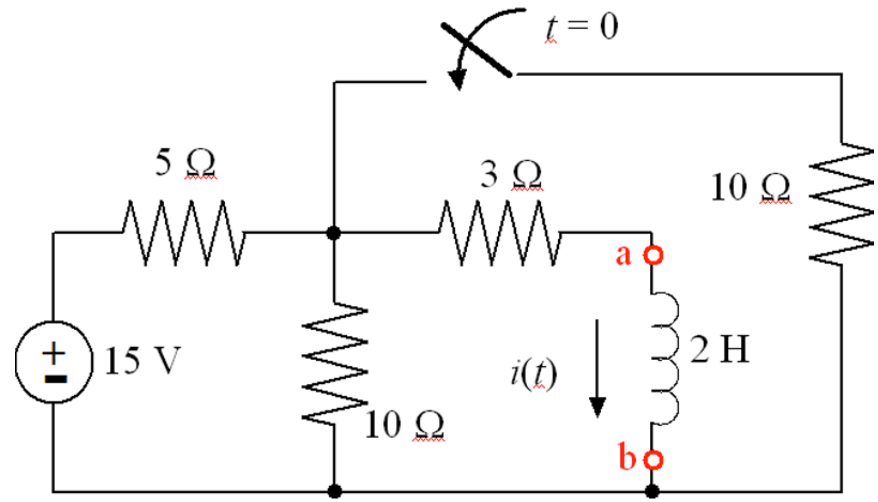
Step 3 – initial value i_0



$$\frac{v-15}{5}+\frac{v}{10}+\frac{v}{3}=0 \quad \Rightarrow \quad v=\frac{90}{19}$$

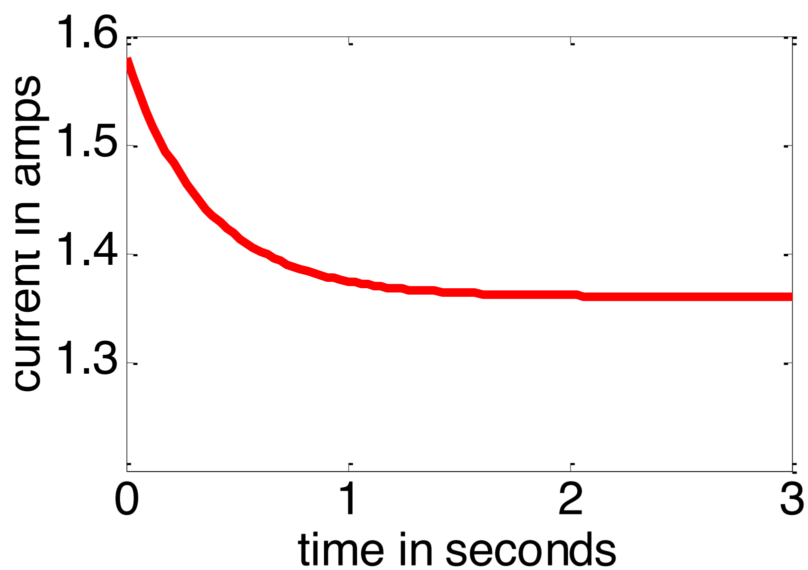
$$i_0 = \frac{v}{3} = \frac{30}{19} = 1.58 \text{ amps}$$

Combining



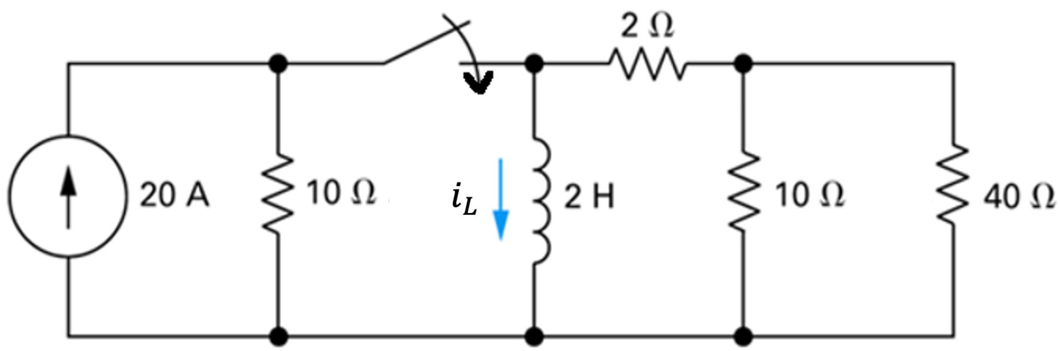
$$i(t) = (i_0 - i_\infty)\, e^{-2.75\,t} + i_\infty$$

$$= 0.22\ e^{-2.75\,t} + 1.36\ \text{amps}$$



Practice Problem:

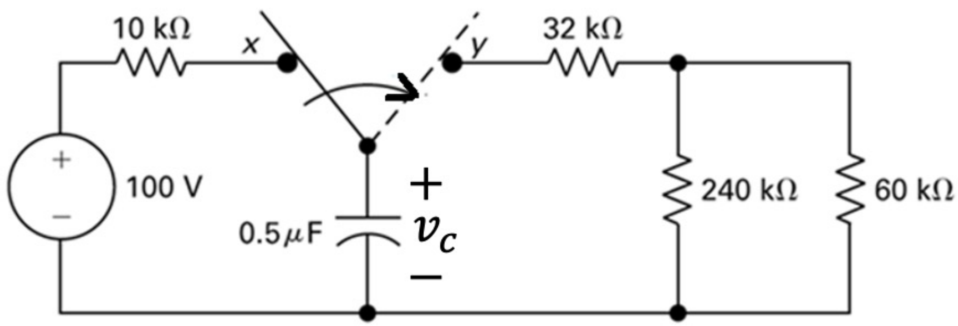
Find $i_L(t)$



$20 - 20e^{-2.5\,t}\ A$

Practice Problem:

Find $v_c(t)$



$100\, e^{-25\, t}\; V$