MTH 243: Multivariable Calculus

Instructor: Vasilije Perović **Date:** November 2023

Exam #2B

Problem	**Max Points**	**Your Score**
1	6	
2	2	
3	2	
4	2	
5	2	
6	3	
7	4	
8	8	
9	4	
10	4	
11	3	
12	3	
13	4	
Total	47	

Name: ______________________________________

Please PRINT *clearly*

Name: ______________________________________

Please SIGN

$\boxed{6}$ 1. Determine which statement is **TRUE** and which is **FALSE**. *No justification is required.*

(a) ______ For any function $f(x, y)$ and an arbitrary rectangular region $\mathcal{R}$, the integral $\int_{\mathcal{R}} f(x, y)\, dA$ is either positive or zero.

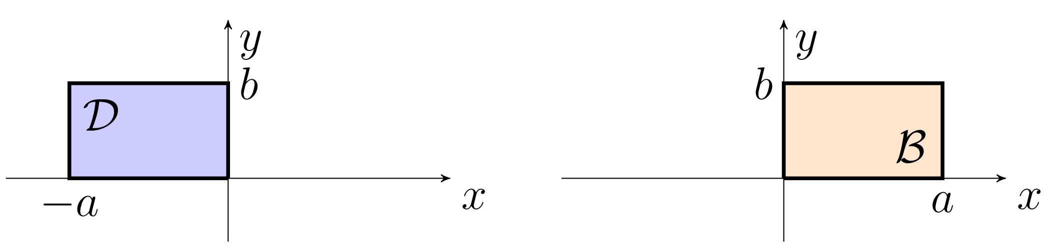
(b) ______ If $f(x, y)$ has a local maximum at the point (a, b), then so does the function $g(x, y) = f(x, y) - 3$.

(c) ______ If (a, b) is a critical point of $f(x, y)$, then (a, b) is either a local maximum or a local minimum of $f(x, y)$.

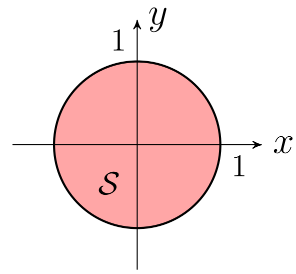
(d) ______ If $\vec{u}$ is tangent to the level curve of f at some point, then $\nabla f \bullet \vec{u} = 0$ at that point.

(e) ______ If k is constant, then $\int_{\mathcal{R}} k \cdot g(x, y)\, dA = k \cdot \int_{\mathcal{R}} g(x, y)\, dA$.

(f) ______ $\int_{\mathcal{B}} x^2 y^2 dA = \int_{\mathcal{D}} x^2 y^2 dA$, where $\mathcal{B}$ and $\mathcal{D}$ are the rectangles given below.



$\boxed{2}$ 2. Let $\mathcal{S}$ be the region inside the unit disc as shown below.



Determine (*without calculation*) whether the integral $\int_{\mathcal{S}} e^{xy}\,dA$ is:

(A) Negative

(B) Positive

(C) Zero

(D) Sign of the integral $\int_{\mathcal{S}} e^{xy}\,dA$ can not be determined with the given information.

$\boxed{2}$ 3. Find a *direction of the greatest increase* for $f(x,y) = x^3y + 11x^2 - 8y$ at $(-1,4)$.

(A) $10\vec{i} + 9\vec{j}$

(B) $-10\vec{i} + 9\vec{j}$

(C) $10\vec{i} - 9\vec{j}$

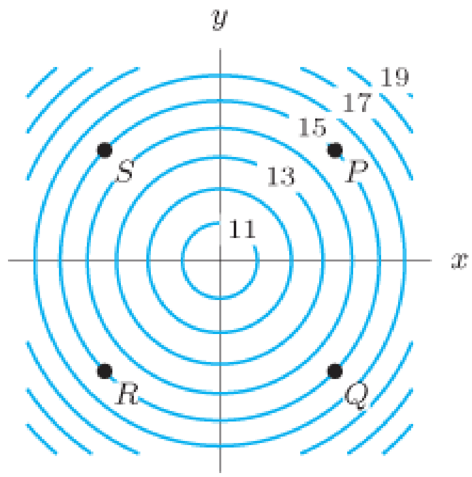
(D) $-10\vec{i} - 9\vec{j}$

(E) $-\sqrt{181}$

(F) $\sqrt{181}$

(G) None of the above

2 4. Use the following contour diagram of $f(x,y)$ to determine signs of f_x and f_y at the specified points.



In the following **TWO** questions, **circle** the correct answer. *No justification is required.*

(i) Sign of f_x at the point S is:

(A) positive

(B) negative

(C) zero

(ii) Sign of f_y at the point Q is:

(A) positive

(B) negative

(C) zero

$\boxed{2}$ 5. Let $g(x, y, z)$ be a differentiable function and let x, y, and z be given by

$$x = x(r, s, t)\,, \qquad y = y(r, s)\,, \qquad z = z(r, t)\,.$$

How MANY nonzero terms does the expression for $\dfrac{\partial g}{\partial t}$ have?

(A) One

(B) Two

(C) Three

(D) Four

(E) Six

(F) Nine

For the remaining problems show all of your work for the full credit!

$\boxed{3}$ 6. For a differentiable function $h(x, y)$ we are told that $h(500, 100) = 100$, $h_x(500, 100) = 8$, and $h_y(500, 100) = 15$. Estimate $h(510, 97)$.

$\boxed{4}$ 7. The temperature at a point (x, y) is $H(x, y)$ and is measured in Celsius. A bug crawls so that its position after t seconds is given by

$$x(t) = \sqrt{1+t}\,, \qquad y(t) = 2 + \frac{1}{3}t\,,$$

were x and y are measured in centimeters. The temperature function satisfies

$$H_x(2,3) = 4\,, \qquad H_y(2,3) = 6\,.$$

How fast is the temperature rising on the bugs path after 3 seconds? Make sure to include units and any relevant computations.

$\boxed{8}$ 8. Let $f(x, y) = x^2 + x^2y + y^2$. Suppose that you are standing at the point $(x, y) = (1, 1)$.

(a) If you move along a line from $(1, 1)$ toward the point $(2, 3)$, find the directional derivative of $f(x, y)$ at $(1, 1)$ in your direction of motion.

(b) What is the maximum rate of increase of $f(x, y)$ at $(1, 1)$?

(c) Suppose that you are standing at the point $(1, 1)$. Give a concrete example of a direction in which you should move so the value of f does not change?

$\boxed{4}$ 9. Let $f(x,y)$ be a differentiable function and assume the following information is known:

$$\begin{aligned} f(3,-1) &= 7 & \qquad f_{xx}(3,-1) &= 6 \\ f_x(3,-1) &= 0 & \qquad f_{yy}(3,-1) &= 8 \\ f_y(3,-1) &= -3 & \qquad f_{xy}(3,-1) &= 0 \end{aligned}$$

(a) Find an equation of the plane tangent to the graph of $f(x,y)$ at the point $(3,-1)$.

(b) Find the quadratic Taylor polynomial for $f(x,y)$ near $(3,-1)$.
You do not have to simplify your answer just write down the main expression.

(c) Which of the following statements is **TRUE**? Only one choice is true!

(A) Point $(3,-1)$ is a local minimum of f.

(B) Point $(3,-1)$ is a saddle point of f.

(C) Point $(3,-1)$ is a local maximum of f.

(D) $(3,-1)$ is not a critical point.

(E) None of the above statements is true.

$\boxed{4}$ 10. **Find** *all* critical points of $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$.

Do NOT try to classify critical points!

$\boxed{3}$ 11. Suppose $f(x, y)$ is function that has TWO critical points $(1, 2)$ and $(-1, 5)$.

Furthermore, assume that values of f and its various derivatives at those critical points, labeled as (a, b), are known and given by the table below.

(a,b)	$(1,2)$	$(-1,5)$
$f(a,b)$	1	5
$f_x(a,b)$	0	0
$f_y(a,b)$	0	0
$f_{xx}(a,b)$	3	6
$f_{yy}(a,b)$	5	4
$f_{xy}(a,b)$	2	5

Classify each of the critical points $(1, 2)$ and $(-1, 5)$ as a local minimum, local maximum, or a saddle point. *Briefly explain your reasoning and provide any relevant supporting computations.*

Classification :

$\boxed{3}$ 12. Evaluate the following iterated integral $\displaystyle\int_0^1 \int_0^2 xy\,dy\,dx$.

Answer: $\displaystyle\int_0^1 \int_0^2 xy\,dy\,dx \;=$

$\boxed{4}$ 13. Evaluate the following integral $\displaystyle\int_0^1 \int_0^{8x} e^{4x^2}\,dydx$.

$\displaystyle\int_0^1 \int_0^{8x} e^{4x^2}\,dydx \;=$

END

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