

Quiz 6

Name: Evan O'Neill

Show all work and necessary steps. Answers without sufficient work will not be graded.

1. (4pts) Use the definition of even and odd functions to rewrite the following expression such that each argument, x , is positive:

$$\cos(-x) + \tan(-x) \sin(-x)$$

$$\cos(x) - \tan(x)(-\sin(x))$$

$$\boxed{\cos(x) + \tan(x) \sin(x)}$$

2. Give the **period** and **domain** (in set notation) of:

(a) $f(x) = \tan(2x)$

$$\frac{\pi}{2} = \pi$$

(2pts) Period:

$$\frac{\pi}{2}$$

(3pts) Domain:

$$\left\{ x \mid x \neq \frac{\pi}{4} + \frac{\pi}{2}k \right\}$$

(b) $g(x) = \csc(2x)$

(2pts) Period:

$$\frac{2\pi}{2} = \pi$$

$$\boxed{\pi}$$

(3pts) Domain:

$$\left\{ x \mid x \neq \frac{\pi}{2}k \right\}$$

3. Give the exact value of:

(a) (4pts) $\cos^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\cos(x) = -\frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{5\pi}{6}}$$

(b) (4pts) $g(x) = \cos\left(\tan^{-1}\left(\frac{6}{5}\right)\right)$

$$\cos\left(\arctan\left(\frac{6}{5}\right)\right)$$

↓

$$\tan^{-1}(x) = \frac{6}{5}$$

$$\boxed{\cos(\tan(x)) = \frac{5}{5}}$$

4. (6pts) Simplify fully (your answer should be a single term using sin or cos):

$$(1 + \cos(x))(1 - \cot(x) \sin(x))$$

$$(1 + \cos(x))(1 - \cot(x) \sin(x))$$

$$(1 + \cos(x))(1 - \frac{\cos(x)}{\sin(x)} \sin(x))$$

$$(1 + \cos(x))(1 - \cos(x))$$

$$(1 - \cos^2(x))$$

$$\boxed{\sin^2(x)}$$

5. Verify the following using identities:

(a) (6pts) $\frac{\cos^2(x)}{\sin(x)} - \sin(-x) = \csc(x)$

$$\frac{\cos^2(x)}{\sin(x)} - \sin(-x) = \csc(x)$$

$$\frac{\cos^2(x)}{\sin(x)} + \sin(x) = \csc(x)$$

$$\frac{\cos^2(x) + \sin^2(x)}{\sin(x)} = \csc(x)$$

$$\frac{\cos^2(x) + \sin^2(x)}{\sin(x)} = \frac{1}{\sin(x)}$$

$$\frac{\cos^2(x) + \sin^2(x)}{\sin(x)} = \frac{1}{\sin(x)} = \csc(x)$$

$$\frac{\cos^2(x) + \sin^2(x)}{\sin(x)}$$

$$\frac{1}{\sin(x)} = \boxed{\csc(x)}$$

(b) (6pts)

$$\frac{4\sec^2(x) + 4\sec(x) + 1}{2\sec(x) + 1} = \frac{2}{\cos(x)} + 1$$

$$\frac{4\sec^2(x) + 4\sec(x) + 1}{2\sec(x) + 1}$$

$$\frac{(2\sec(x) + 1)^2}{2\sec(x) + 1}$$

$$\frac{(2\sec(x) + 1)(2\sec(x) + 1)}{2\sec(x) + 1}$$

$$2\sec(x) + 1$$

$$2 \cdot \frac{1}{\cos(x)} + 1$$

$$\boxed{\frac{2}{\cos(x)} + 1}$$