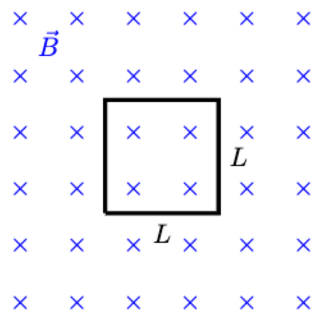
3. A square conducting loop of side-length $L = 3\,\text{m}$ is in a region with a time-varying magnetic field ($\vec{B}$). The field is directed into the page, and its magnitude as a function of time is shown in the graph to the right. The total resistance of the loop is $R = 3\,\Omega$

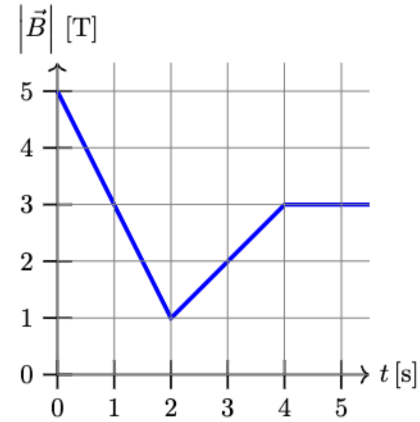
(a) Calculate the area of the loop, and specify the direction of its normal vector ($\odot$, $\otimes$).

(b) Find the magnetic flux (Φ_B) through the loop at time $t = 1\,\text{s}$.

(c) Find the induced EMF ($\mathcal{E}$) in the loop at time $t = 3\,\text{s}$.

(d) Find the induced current (I) in the loop at time $t = 5\,\text{s}$.





Part (a)

The loop is square, such that the area is $A = L^2 = \boxed{9\,\text{m}^2}$. The direction can be either into or out of the page, but your choice affects the answer to subsequent parts of this problem.

Part (b)

The flux is given by $\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = BA\cos\theta$. The value of the field at $t = 1$ is $B = 3\,\text{T}$, such that the product of the field by the area yields $BA = 27\,\text{Tm}^2$. The angle θ depends on your choice of area vector. If $\vec{\boldsymbol{A}}||\odot$ then $\theta = 180°$ and

$$\Phi_B = \boxed{-27\,\text{Tm}^2}$$

Otherwise, if $\vec{\boldsymbol{A}}||\otimes$ then $\theta = 0°$ and

$$\Phi_B = \boxed{+27\,\text{Tm}^2}$$

Part (c)

The EMF is given by the derivative of the flux, which here depends only on the derivative of the magnetic field since all other values are constant:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(\frac{dB}{dt}\right) A\cos\theta$$

The slope of the B-vs-t graph at $t = 3$ is $dB/dt = 1\,\text{T/s}$, such that if $\vec{\boldsymbol{A}}||\odot$ then $\theta = 180°$ and

$$\mathcal{E} = \boxed{+9\,\text{V}}$$

Otherwise, if $\vec{\boldsymbol{A}}||\otimes$ then $\theta = 0°$ and

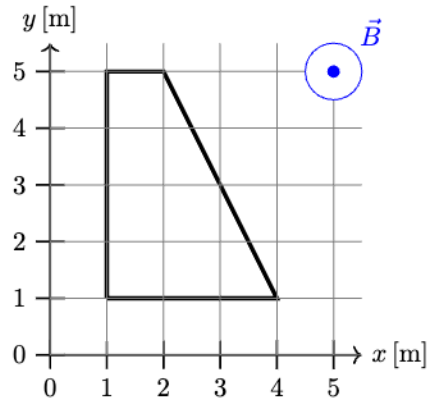
$$\Phi_B = \boxed{-9\,\text{V}}$$

Part (d)

By contrast, here the slope of the B-vs-t graph at $t = 5\,\text{s}$ is $dB/dt = 0$, such that the EMF is 0, therefore the answer in all cases is:

$$I = \frac{\mathcal{E}}{R} = \boxed{0}$$

3. A conducting loop of total resistance $R = 4\,\Omega$ is shown in the figure to the right. A time-dependent magnetic field is present throughout the region. The field has a magnitude given by the equation $B(t) = B_0 t + B_1$, where $B_0 = 3\,\mathrm{T/s}$ and $B_1 = 2\,\mathrm{T}$, and is directed out of the plane.



(a) Calculate the area of the loop, and specify the direction of its normal vector ($\odot$, $\otimes$).

(b) Find the magnetic flux through the loop at $t = 0$.

(c) Find the induced emf $\mathcal{E}$ at time $t = 1\,\mathrm{s}$.

(d) Find the magnitude and direction (cw/ccw) of the induced current I at $t = 2\,\mathrm{s}$

Part (a)

The area is the sum of a triangle (width $2\,\mathrm{m}$, height $4\,\mathrm{m}$) and a rectangle (width $1\,\mathrm{m}$, height $4\,\mathrm{m}$).

$$A = \frac{1}{2}(2\,\mathrm{m})(4\,\mathrm{m}) + (1\,\mathrm{m})(4\,\mathrm{m}) = \boxed{8\,\mathrm{m}^2}$$

And either direction $\boxed{\odot}$ or $\boxed{\otimes}$ is acceptable.

Part (b)

The magnetic field at $t = 0$ is

$$\vec{\boldsymbol{B}}_0 = B_1\,\hat{\boldsymbol{k}}$$

And the flux is therefore given by:

$$\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = B_1 A \cos\theta$$

Where here the angle is $\theta = 0°$ if $A || \odot$ and the angle is $\theta = 180°$ if $A || \otimes$. This results in a flux of:

$$\Phi_B = \boxed{\pm 16\,\mathrm{Tm}^2}$$

Part (c)

Since the area is constant, the derivative of the magnetic flux is:

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt} A\cos\theta$$

The derivative of the magnetic field is $\frac{dB}{dt} = 3\,\mathrm{T/s}$ such that the derivative will be:

$$\frac{d\Phi_B}{dt} = \pm 24\,\mathrm{Tm}^2/\mathrm{s}$$

And the emf will be:

$$\mathcal{E} = \boxed{\mp 24\,\mathrm{V}}$$

Note that this does not depend on time.

Part (d)

The emf is constant with respect to time, so the current is simply:

$$I = \frac{\mathcal{E}}{R} = \boxed{6\,\mathrm{A}}$$

And the direction will be such that it opposes changes in flux. Thus the current must produce an inwards magnetic field, which corresponds to a clockwise current.

NAME and ID: ______________________________ SCORE: _____ /4 points

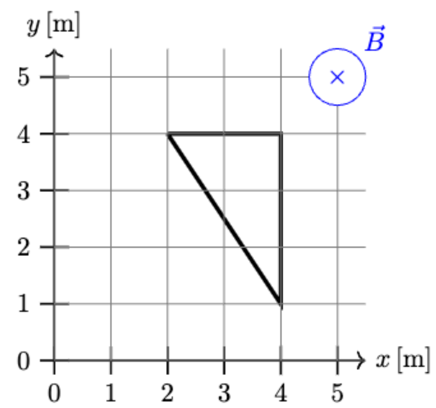
PHY 274 PROBLEM SOLVING WORKSHOP X

1. A triangular conducting loop is shown in the figure to the right. A time-dependent magnetic field is present throughout the region. The field has a magnitude given by the equation $B(t) = B_0 t^3 + B_1$, where $B_0 = 1\,\mathrm{T/s^3}$ and $B_1 = 3\,\mathrm{T}$, and is directed into the plane.

   (a) Calculate the area of the loop, and specify the direction of its normal vector ($\odot$, $\otimes$).

   (b) Find the magnetic flux through the loop at $t = 0$.

   (c) Find the magnitude and direction (cw/ccw) of the induced emf $\mathcal{E}$ at time $t = 2\,\mathrm{s}$.



Part (a)

The area of the loop is that of a triangle:

$$A = \frac{1}{2}bh = \boxed{3\,\mathrm{m}^2}$$

The direction can be either $\boxed{\odot}$ or $\boxed{\otimes}$, but the choice made here affects the sign of the answers you get in parts (b) and (c).

Part (b)

The magnetic flux is given by:

$$\Phi = \vec{B}\cdot\vec{A} = BA\cos\theta$$

Here the value of theta depends on your choice of area vector, if $A||\otimes$ then $\theta = 0°$ and if $A||\odot$ then $\theta = 180°$. The answer will then be either positive or negative, based on your choice.

$$\Phi = \pm BA$$

The magnetic field at $t = 0$ is $B(0) = B_0(0)^3 + B_1 = B_1 = 3\,\mathrm{T}$, such that the final result is:

$$\Phi = \boxed{\pm 9\,\mathrm{Wb}}$$

Part (c)

The emf is related to the derivative of the magnetic flux, and the only variable that is changing is the magnetic field.

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{dB}{dt}A\cos\theta$$

The derivative of the magnetic field is $3B_0t^2$

$$\mathcal{E} = \mp(3B_0t^2)3 = \boxed{\mp 36\,\mathrm{V}}$$

Where the sign is opposite of whatever the result from (b) was. But regardless of the sign, the direction is <u>counter-clockwise</u>

2. A conducting loop has a time-dependent area vector given by $\vec{\boldsymbol{A}}(t) = (A_0 t^2 - A_1)\,\hat{\boldsymbol{k}}$, where $A_0 = 1\,\mathrm{m^2/s^2}$ and $A_1 = 4\,\mathrm{m^2}$ are constants. A constant magnetic field $\vec{\boldsymbol{B}} = 2\,\mathrm{T}\,\hat{\boldsymbol{k}}$ is present throughout the region.

(a) Find the magnetic flux Φ_B through the loop at $t = 1\,\mathrm{s}$.

(b) Find the induced emf $\mathcal{E}$ in the loop at time $t = 2\,\mathrm{s}$.

Part (a)

The magnetic flux is given by

$$\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = B_z A_z = B(A_0 t^2 - A_1)$$

Which, at $t = 1\,\mathrm{s}$ becomes

$$\boxed{\Phi_B = -6\,\mathrm{Wb}}$$

Part (b)

The emf is given by Faraday's law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B_z \frac{dA_z}{dt} = -2BA_0 t$$

The derivative of the area is $dA/dt = 2A_0 t$, such that

$$\boxed{\mathcal{E} = -8\,\mathrm{V}}$$