NAME: ______________________________ ID: ____________________

EXAM 3

PHY 204: Elementary Physics II

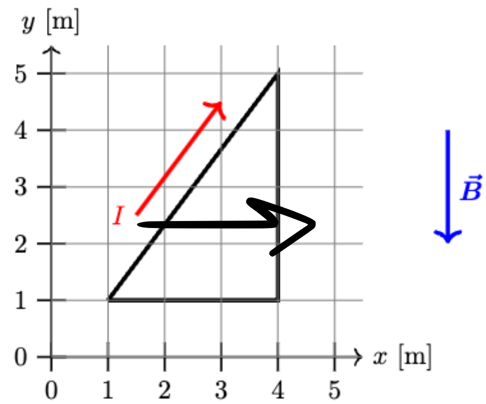
You have 50 minutes to complete this exam. Explain your reasoning, include sketches where appropriate, and clearly indicate your final answer. All vectors must have clearly identified magnitude and direction (or be written in unit-vector notation) for full credit. All final answers must have units for full credit.

1. The conducting loop shown in the figure to the right has $N = 3$ turns and carries a clockwise current $I = 1\,\text{A}$. A constant magnetic field $\vec{B} = -2\,\text{T}\,\hat{j}$ is present throughout the region, as shown. Calculate the magnitude and direction of each of the following:

   (a) The magnetic dipole moment $\vec{\mu}$ of the loop.

   (b) The net force $\vec{F}$ on the loop.

   (c) The net torque $\vec{\tau}$ on the loop.



Part (a)

The area of the loop is that of a right-triangle:

$$A = \frac{1}{2}(3\,\text{m})(4\,\text{m}) = 6\,\text{m}^2$$

The magnitude of the magnetic dipole moment is then simply:

$$\mu = NIA = (3)(1\,\text{A})(6\,\text{m}^2) = 18\,\text{Am}^2$$

By the right hand rule the magnetic moment is directed into the page (-z direction), thus the final answer is:

$$\boxed{\vec{\mu} = -18\,\text{Am}^2\,\hat{k}}$$

Part (b)

The net force on a closed loop in a constant magnetic field is always $\boxed{0}$. This can also be proven by calculating the force on the bottom (1) the right (2) and the hypotenuse (3):

$$\vec{F}_1 = (1\,\text{A})(-3\,\text{m}\,\hat{i}) \times (-2\,\text{T}\,\hat{j}) = 6\,\text{N}\,\hat{k}$$
$$\vec{F}_2 = (1\,\text{A})(-4\,\text{m}\,\hat{j}) \times (-2\,\text{T}\,\hat{j}) = 0\,\text{N}$$
$$\vec{F}_3 = (1\,\text{A})(3\,\text{m}\,\hat{i} + 4\,\text{m}\,\hat{j}) \times (-2\,\text{T}\,\hat{j}) = -6\,\text{N}\,\hat{k}$$

Such that the total is $F_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \boxed{0}$

Part (c)

The torque can be calculated directly from:

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (-18\,\text{Am}^2\,\hat{k}) \times (-2\,\text{T}\,\hat{j}) = \boxed{-36\,\text{Nm}\,\hat{i}}$$

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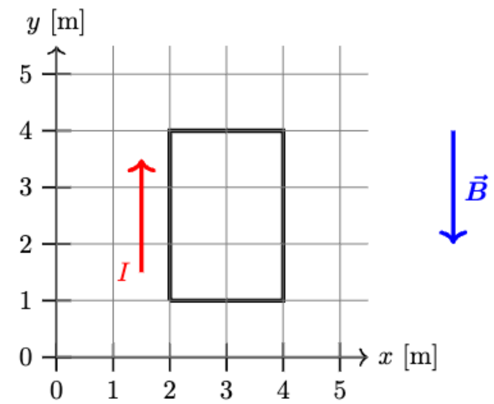
1. The rectangular conducting loop shown in the figure to the right has $N = 3$ turns and carries a clockwise current $I = 1\,\mathrm{A}$. A constant magnetic field $\vec{B} = -\frac{1}{2}\,\mathrm{T}\,\hat{\jmath}$ is present throughout the region, as shown. Calculate the magnitude and direction of each of the following:

   (a) The magnitude and direction of the magnetic dipole moment $\vec{\mu}$ of the loop.

   (b) The magnitude and direction of the net torque $\vec{\tau}$ on the loop.

   (c) The magnitude and direction of the force $\vec{F}$ on each of the four sides of the loop (top, bottom, left, right), as well as the net force $\vec{F}_{\mathrm{net}}$ on the loop.



Part (a)

The area of the loop is that of a rectangle:

$$A = (2\,\mathrm{m})(3\,\mathrm{m}) = 6\,\mathrm{m}^2$$

The magnitude of the magnetic dipole moment is then simply:

$$\mu = NIA = (3)(1\,\mathrm{A})(6\,\mathrm{m}^2) = 18\,\mathrm{Am}^2$$

By the right hand rule the magnetic moment is directed into the page (-z direction), thus the final answer is:

$$\boxed{\vec{\mu} = -18\,\mathrm{Am}^2\,\hat{k}}$$

Part (b)

The torque can be calculated directly from:

$$\vec{\tau} = \vec{\mu}\times\vec{B} = (-18\,\mathrm{Am}^2\,\hat{k})\times(-\frac{1}{2}\,\mathrm{T}\,\hat{\jmath}) = \boxed{-9\,\mathrm{Nm}\,\hat{\imath}}$$

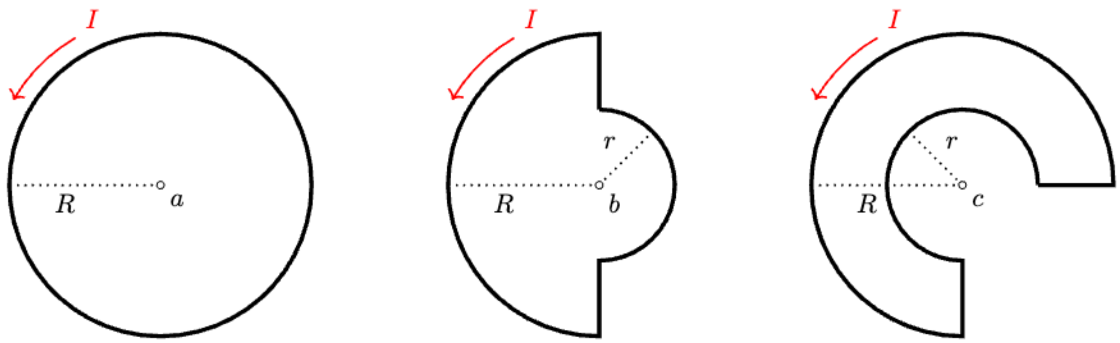
Part (c)

The total force is N times the force on a single wire ($\vec{F} = N\vec{F}_0 = NI\vec{L}\times\vec{B}$), such that the force for the top (1), bottom (2), left (3), and the right (4) are:

$$\vec{F}_1 = (3)(1\,\mathrm{A})(2\,\mathrm{m}\,\hat{\imath})\times(-\frac{1}{2}\,\mathrm{T}\,\hat{\jmath}) = -3\,\mathrm{N}\,\hat{k}$$
$$\vec{F}_2 = (3)(1\,\mathrm{A})(-2\,\mathrm{m}\,\hat{\imath})\times(-\frac{1}{2}\,\mathrm{T}\,\hat{\jmath}) = 3\,\mathrm{N}\,\hat{k}$$
$$\vec{F}_3 = (3)(1\,\mathrm{A})(3\,\mathrm{m}\,\hat{\jmath})\times(-\frac{1}{2}\,\mathrm{T}\,\hat{\jmath}) = 0\,\mathrm{N}$$
$$\vec{F}_4 = (3)(1\,\mathrm{A})(-3\,\mathrm{m}\,\hat{\jmath})\times(-\frac{1}{2}\,\mathrm{T}\,\hat{\jmath}) = 0\,\mathrm{N}$$

Such that the total is $F_{\mathrm{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \boxed{0}$, which is consistent with the fact that <u>the net force on a closed loop in a constant field is always zero.</u>

2. A single loop of wire is bent into semi-circular segments connected by straight segments of wire in three different configurations, as shown in the figures below. There is a current $I = 4\,\mathrm{A}$ flowing through each loop in the direction shown. The smaller radius is $r = 1\,\mathrm{m}$ and the larger radius is $R = 2\,\mathrm{m}$. The center of each loop is indicated by the point a, b, and c in each figure. Use the approximation $\pi \approx 3$ such that $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Tm/A} \approx 12 \times 10^{-7}\,\mathrm{Tm/A}$.



(a) What is the magnitude and direction ($\odot$, $\otimes$) of the magnetic field $\vec{\boldsymbol{B}}_a$ at point a?

(b) What is the magnitude and direction ($\odot$, $\otimes$) of the magnetic field $\vec{\boldsymbol{B}}_b$ at point b?

(c) What is the magnitude and direction ($\odot$, $\otimes$) of the magnetic field $\vec{\boldsymbol{B}}_c$ at point c?

Part (a)

The magnetic field is given by

$$B_a = \frac{\mu_0 I}{2R}$$

$$B_a = \boxed{12 \times 10^{-7}\,\mathrm{T}}$$

And is directed $\boxed{\odot}$

Part (b)

The magnetic field is given by

$$B_b = \frac{1}{2}\frac{\mu_0 I}{2R} + \frac{1}{2}\frac{\mu_0 I}{2r}$$

$$B_b = \frac{\mu_0 I}{4}\left(\frac{1}{R} + \frac{1}{r}\right)$$

$$B_b = \boxed{18 \times 10^{-7}\,\mathrm{T}}$$

And is directed $\boxed{\odot}$

Part (c)

The magnetic field is given by

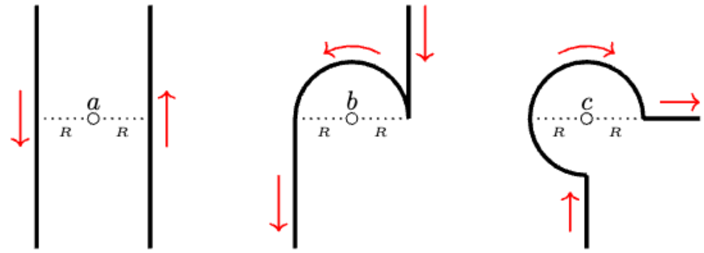
$$B_c = \frac{3}{4}\frac{\mu_0 I}{2R} - \frac{3}{4}\frac{\mu_0 I}{2r}$$

$$B_c = \frac{3\mu_0 I}{8}\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$B_c = \boxed{-9 \times 10^{-7}\,\mathrm{T}}$$

And is directed $\boxed{\otimes}$

2. Two semi-infinite straight wires are connected to a curved wire in the form of either a half-circle or three-quarters of a circle with radius $R = 1\,\mathrm{m}$ in three different configurations (except for configuration a, where instead there are simply two infinite straight wires). A current $I = 2\,\mathrm{A}$ flows in the directions indicated by the arrows. Find magnitudes B_a, B_b, B_c and direction ($\odot$, $\otimes$) of the magnetic field thus generated at points a, b, c. Recall that $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Tm/A}$, and use the approximation $\pi \approx 3$.



Part (a)

Each infinite wire produces a magnetic field directed out of the page with magnitude $B_{\mathrm{IW}} = \mu_0 I/2\pi R$, such that the net field is:

$$B = \left|\frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2\pi R}\right| = \boxed{8 \times 10^{-7}\,\mathrm{T}}$$

And the direction is $\boxed{\odot}$

Part (b)

Each semi-infinite wire produces a magnetic field with magnitude $B = \mu_0 I/4\pi R$, one of which is directed into the page, and one of which is directed out of the page, which cancel with one another. Thus the only contribution is from the half-loop, which produces a field directed out of the page with magnitude $\frac{1}{2}\frac{\mu_0 I}{2R}$

$$B = \left|\frac{1}{2}\frac{\mu_0 I}{2R}\right| = \boxed{6 \times 10^{-7}\,\mathrm{T}}$$

And the direction is $\boxed{\odot}$

Part (c)

The only non-zero contribution is from the three-quarters loop, which produces a field directed into the page with magnitude $B = \frac{3}{4}\frac{\mu_0 I}{2R}$.

$$B = \left|\frac{3}{4}\frac{\mu_0 I}{2R}\right| = \boxed{9 \times 10^{-7}\,\mathrm{T}}$$

And the direction is $\boxed{\otimes}$