MATH 243: Multivariable Calculus

Instructor: Vasilije Perović

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EXAM #1B

Problem	**Max Points**	**Your Score**
1	9	
2	3	
3	3	
4	3	
5	5	
6	3	
7	2	
8	2	
9	2	
10	6	
11	7	
12	4	
Total:	49	

Name: ______________________________________

Please PRINT *clearly*

Name: ______________________________________

Please SIGN

1. Determine which statements are **TRUE** and which are **FALSE** (circle your answer).
No justification is required. (9 pts)

(a) **True/False** Vector $\vec{i}-\vec{j}$ is perpendicular to both vectors $\vec{i}+\vec{j}-\vec{k}$ and $2\vec{i}-2\vec{k}$.

(b) **True/False** For *any two* vectors $\vec{u}$ and $\vec{v}$, the quantity $\vec{u}\times(\vec{u}\times\vec{v})$ is the zero vector.

(c) **True/False** $||\,\vec{a}+\vec{b}\,|| = ||\,\vec{a}\,|| + ||\,\vec{b}\,||$ for *any two* vectors $\vec{a}$ and $\vec{b}$.

(d) **True/False** The graph of function $z = f(x,y) = e^{-x^2-y^2}$ is a surface of revolution.

(e) **True/False** If $f(x,y)$ is a linear function, then contours of $f(x,y)$ are parallel lines.

(f) **True/False** The graph of function $z = f(x,y) = \sin(x\,y)$ is a cylinder.

(g) **True/False** For *any two* vectors $\vec{a}$ and $\vec{b}$, we have $\vec{a}\times\vec{b} = \vec{b}\times\vec{a}$.

(h) **True/False** For *any two* vectors $\vec{u}$ and $\vec{v}$, the value of $\vec{u}\cdot(\vec{u}\times\vec{v})$ is always zero.

(i) **True/False** The cross product of *any two* vectors is also a unit vector.

Turn over $\longrightarrow$

2. Find the *best description* of the **domain** of $f(x, y) = \sqrt{36 - 4x^2 - 9y^2}$. (3 pts)

(A) region between two concentric circles

(B) region above the x-axis

(C) region inside a circle

(D) region inside an ellipse

(E) region inside a square

(F) the first quadrant

(G) the entire (x, y) plane

(H) region outside a circle

(I) region outside a square

(J) region outside an ellipse

3. Find the **center**, O, and the **radius**, r, of the sphere (3 pts)

$$x^2 + y^2 + z^2 + 2x + 8y - 4z = -12.$$

(A) $O = (2, -8, -4)$ $r = \sqrt{12}$

(B) $O = (1, 1, -1)$ $r = 1$

(C) $O = (2, 8, -4)$ $r = \sqrt{2}$

(D) $O = (-2, -8, 4)$ $r = \sqrt{3}$

(E) $O = (1, -4, -2)$ $r = 1$

(F) $O = (-1, -4, -2)$ $r = 2$

(G) $O = (-1, -4, 2)$ $r = 3$

(H) $O = (-1, -4, 2)$ $r = 4$

(I) $O = (1, 4, 2)$ $r = 4$

(J) $O = (1, 1, 1)$ $r = -\sqrt{12}$

Turn over $\longrightarrow$

4. Which three of the following planes are parallel? (3 pts)

(i) $4x - 2y + 6z = 3$

(ii) $4x + 2y + 6z = 5$

(iii) $2x - y + 3z = 6$

(iv) $y = 2x + 3z - 11$

(v) $z = \frac{1}{3}y + \frac{2}{3}x + 1$

(A) (ii), (iii), (iv)

(B) (ii), (iii), (v)

(C) (ii), (iv), (v)

(D) (iii), (iv), (v)

(E) (i), (ii), (iii)

(F) (i), (ii), (iv)

(G) (i), (iii), (v)

(H) (i), (iii), (iv)

(I) (i), (ii), (v)

(J) (i), (iv), (v)

Turn over ⟶

5. **(NO PARTIAL CREDIT)**

Consider two vectors $\vec{a}$ and $\vec{b}$ given by: $\vec{a} = \vec{i} + 3\vec{j} + 2\vec{k}$ and $\vec{b} = 4\vec{i} + 2\vec{j} - 3\vec{k}$. (5 pts)

(a) Find $3\,\vec{a} - 2\,\vec{b}$.

$3\,\vec{a} - 2\,\vec{b} =$

(b) Find $\vec{a} \bullet \vec{b}$.

$\vec{a} \bullet \vec{b} =$

(c) Find the angle θ between $\vec{a}$ and $\vec{b}$. You may leave your answer as the inverse of an appropriate trig function.

$\theta =$

Turn over $\longrightarrow$

For problems 6 – 12, show all of your work for the full credit!

6. Find the **equation of the plane** that has slope 2 in the x-direction, has slope -5 in the y-direction and passes through the point $(-1, 3, 5)$. Write you answer in the form $ax + by + cz = d$. (3 pts)

Answer:

7. Find the **equation of the plane** that passes through the point $(-1, 3, 2)$ and is parallel to the plane $-2y + 5z = 12$. (2 pts)

Answer:

8. Find the vector $\vec{w}$ of magnitude 8 in the *same direction* as the vector $\vec{v} = 1\vec{i} + 4\vec{k}$ (2 pts)

$\vec{w} =$

Turn over $\longrightarrow$

9. Consider the plane $\quad 3x + 6y - 9z = 36$. (2 pts)

Find the x**-intercept,** y**-intercept,** and z**-intercept** of this plane and *write them as ordered triples.*

$x -$ **intercept:**

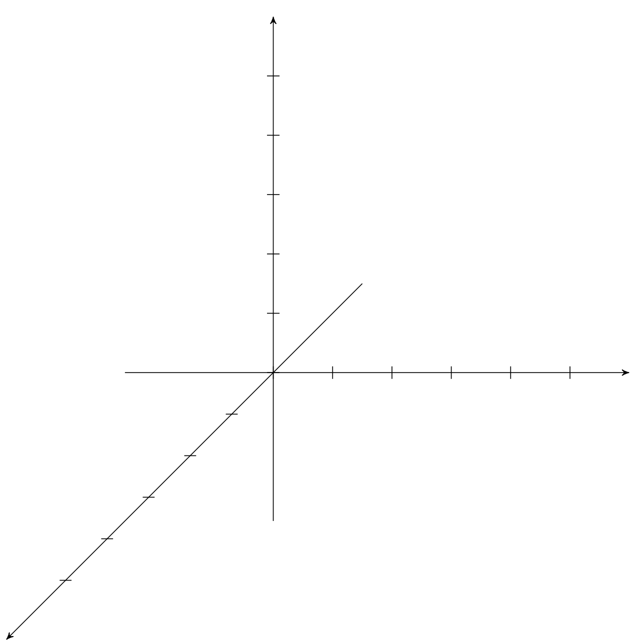
$y -$ **intercept:**

$z -$ **intercept:**

10. Consider the equation $\quad z^2 + y^2 = 1$.

Sketch the surface corresponding to this equation and **briefly describe** it in words. (6 pts)

Note: What you will be graded on this problem is the *general shape* of the surface, *properly labeled axes*, and *description of the surface.* The sketch should somewhat resemble the surface.



Turn over

11. Consider the following three points $P = (1, 2, 2)$, $Q = (5, 2, -2)$, and $R = (1, 3, 3)$. (7 pts)

(a) Find the **midpoint** M of the line segment connecting points P and Q.

$M =$

(b) Find the cross product of displacement vectors $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$\overrightarrow{PQ} \times \overrightarrow{PR} =$

Turn over →

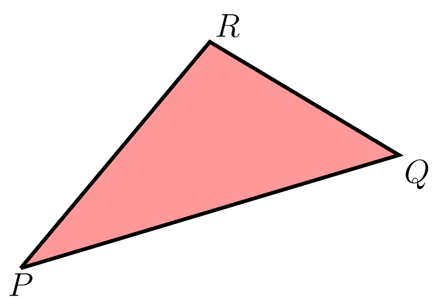
(parts (c) and (d) below are continuation of problem #11)

(c) Find the *equation of the plane* containing the points P, Q, and R.

Answer:

(d) Find the **area of the triangle** that has P, Q, and R as its vertices.

Note: Picture below is not drawn to scale, it is just for illustrative purposes.



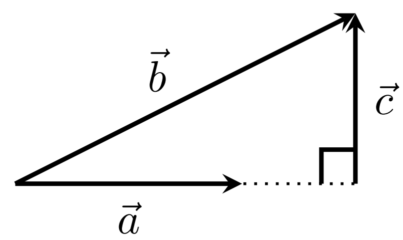
$Area\,(\triangle PQR)\ =$

Turn over ⟶

12. In the picture below vectors $\vec{a}$ and $\vec{b}$ are represent $\quad \vec{a} = 2\vec{i} + 2\vec{j} - 1\vec{k} \quad$ and $\quad \vec{b} = 4\vec{i} + 6\vec{j} - 7\vec{k}$.

Find the vector $\vec{c}$ in the picture below. (4 pts)

Note: The picture below is not drawn to scale or accurately, but it is used to describe *relative relation between vectors* $\vec{a}$, $\vec{b}$, and $\vec{c}$.



$\vec{c} =$

END

SCRAP PAPER: **Name:** ____________________

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