March 25, 2024 **REVIEW Quiz – Integrals** MTH 243 (Perović)

Max Score: **20 pts** **Name:** Evan O'Neill

GRADING:

- Completing this quiz is OPTIONAL, however it is *strongly* recommended.
- By completing this quiz, you will be able to earn up to 5% extra credit toward the second exam!
- The *deadline is sharp* so please make sure you do not miss it!!!

INSTRUCTIONS: Please read the following instructions **carefully**.

- You are allowed to use *any* "single variable calculus" textbook that you have access to (printed copy or online copy).
- For your convenience, here is a link to Paul's Online Notes for various Calculus courses.
- You should work on completing this quiz on *your own*, that is, you should *NOT* work with fellow students, roommates, friends, tutors, online chatting buddies, etc.
- You are NOT allowed to look for answers to these specific question on any of the online forums/platforms or apps.
- It is ok to use a calculator to *check* your work, BUT you should be able to these problems on the exam without a calculator!
- **Neatness and organization of your answers/submission matter!**
  - Your answers should be submitted as a *single pdf file*.
  - The uploaded pdf file should be titled `YOURLASTNAME-mth243-review-quiz2.pdf`.
  - Your answers should be *legible and with no scribbles*.
  - Your answers should be written on *the quiz itself* **OR** if writing on a separate paper, then each new problem should start on a separate page in your pdf submission.

  Failing to follow submission instructions will result in your final score being reduced.
- **All work must be shown for full credit!**
- Reviewing this material is essential to your understanding of integration of multivariable functions and related concepts. *Whether you complete this quiz or not, does not change the fact that you are responsible for this material!*

DEADLINE (sharp): **11:59pm on Monday, March 25, 2024, via Brightspace.**

1. Find the following *indefinite* integrals.

(a) $\int 7\,t\,e^{8-5t^2}\,dt$

$u = 8 - 5t^2$

$du = -10t\,dt$

$dt = \dfrac{du}{-10t}$

$7\int \cancel{t} e^u \cdot \dfrac{du}{-10\cancel{t}} = \dfrac{7}{10}\int e^u\,du = -\dfrac{7}{10} e^{8-5t^2} + C$

$\boxed{\int 7\,t\,e^{8-5t^2}\,dt = \dfrac{-7e^{8-5t^2}}{10} + C}$

(b) $\int (2x+5)\,e^{3x-7}\,dx$

$u = 2x+5 \qquad dv = e^{3x-7}\,dx$

$du = 2\,dx \qquad v = \frac{1}{3}e^{3x-7}$

$uv - \int v\,du$

$(2x+5)\left(\frac{1}{3}e^{3x-7}\right) - \frac{2}{3}\int e^{3x-7}\,dx$

$(2x+5)\left(\frac{1}{3}e^{3x-7}\right) - \frac{2}{3}\left(\frac{1}{3}e^{3x-7}\right)dx$

$\frac{2}{3}xe^{3x-7} + \frac{5}{3}e^{3x-7} - \frac{2}{9}e^{3x-7} + C$

$e^{3x-7}\left(\frac{2}{3}x + \frac{5}{3} - \frac{2}{9}\right) + C$

$\boxed{\int (2x+5)\,e^{3x-7}\,dx = \frac{2}{3}xe^{3x-7} + \frac{13}{9} + C}$

Turn over →

2. Find the following *definite* integrals.

$u = 2x^2 + 7$

$du = 4x\,dx$

$x\,dx = \frac{du}{4}$

(a) $\displaystyle\int_1^3 5\,x\,\sqrt{2x^2+7}\;dx$

$\frac{5}{4}\int_1^3 \sqrt{u}\;du = \left[\frac{5}{4}\cdot\frac{2}{3}(2x^2+7)^{3/2}\right]_1^3$

$\frac{10}{12}\left((2(3)^2+7)^{3/2} - (2(1)^2+7)^{3/2}\right) = \frac{10}{12}(125-27) = \frac{980}{12}$

$= \frac{245}{3}$

$$\int_1^3 5\,x\,\sqrt{2x^2+7}\;dx \;=\; \frac{245}{3}$$

(b) $\displaystyle\int_{-\pi}^{\pi} \sin(3y)\,dy$

$u = 3y$

$du = 3dy$

$dy = \frac{du}{3}$

$\int_{-\pi}^{\pi} \sin(u)\cdot\frac{1}{3}\,du$

$\frac{1}{3}\int_{-\pi}^{\pi} \sin(u)\,du = \left[-\frac{1}{3}\cos(3y)\right]_{-\pi}^{\pi}$

$\left(-\frac{1}{3}\cos(3\pi)\right) - \left(-\frac{1}{3}\cos(-3\pi)\right)$

$= 0$

$$\int_{-\pi}^{\pi} \sin(3y)\,dy \;=\; 0$$

3. Let $\displaystyle\int_0^3 f(x)\,dx = 8$. What is the *average value of* $f(x)$ on the interval $0 \le x \le 3$?

$\frac{1}{b-a}\cdot\int_a^b f(x)\,dx$

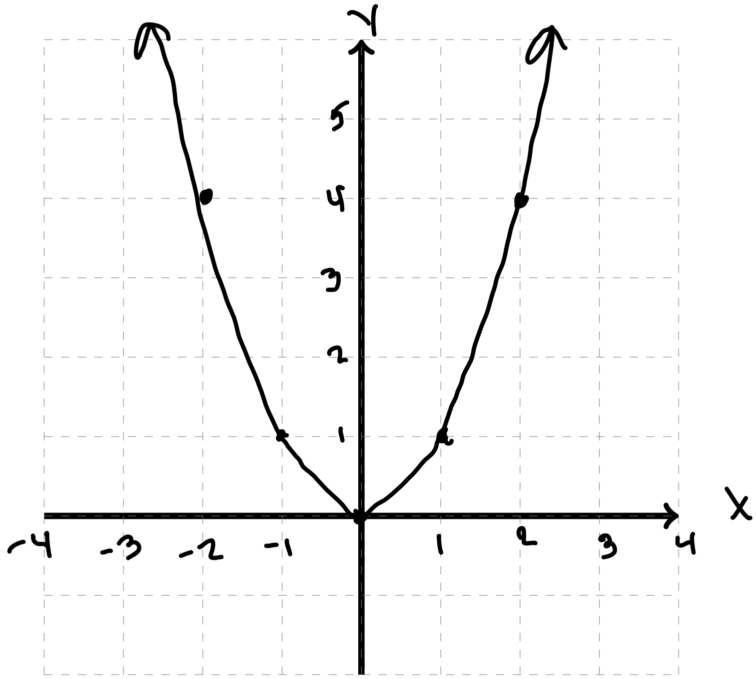
$\frac{1}{3-0}\cdot 8 = \frac{8}{3}$

Answer : $\frac{8}{3}$

Turn over →

4. Consider the function $f(x) = x^2$.

(a) *Sketch* the graph of $f(x)$ and make sure to label axes and increments clearly.



(b) Calculate the *LEFT Riemann sum* of $f(x)$ on the interval $-2 \leq x \leq 2$ with TWO subintervals. *Make sure to include the expression for the sum leading to you answer.*

$\Delta x = \dfrac{2-(-2)}{2} = 2$ $\quad [-2,0] \quad [0,-2]$

$\Delta x \left[(-2)^2 + 0^2\right] = 2(4) = 8$

Answer : 8

(c) Calculate the *RIGHT Riemann sum* of $f(x)$ on the interval $-2 \leq x \leq 2$ with TWO subintervals. *Make sure to include the expression for the sum leading to you answer.*

$\Delta x = 2$ $\quad [-2,0]\ [0,2]$

$\Delta x \left[0^2 + 2^2\right] : (2)(4)$

Answer : 8

Turn over $\longrightarrow$

(d) Calculate the *LEFT Riemann sum* of $f(x)$ on the interval $-2 \leq x \leq 2$ with FOUR subintervals. *Make sure to include the expression for the sum leading to you answer.*

$$\Delta x = \frac{2-(-2)}{4} = 1$$

$[-2,-1]\ [-1,0]\ [0,1]\ [1,2]$

$$\Delta x\left((-2)^2 + (-1)^2 + 0^2 + 1^2\right) = 4 + 1 + 1 = 6$$

Answer : 6

(e) Calculate the *RIGHT Riemann sum* of $f(x)$ on the interval $-2 \leq x \leq 2$ with FOUR subintervals. *Make sure to include the expression for the sum leading to you answer.*

$\Delta x = 1$

$[-2,-1]\ [-1,0]\ [0,1]\ [1,2]$

$$\Delta x\left((-1)^2 + (0)^2 + 1^2 + 2^2\right) = 1 + 1 + 4$$

Answer : 6

(f) Calculate the *MIDPOINT Riemann sum* of $f(x)$ on the interval $-2 \leq x \leq 2$ with FOUR subintervals. *Make sure to include the expression for the sum leading to you answer.*

$\Delta x = 1$

$$\Delta x\left[f\left(\frac{-2-1}{2}\right) + f\left(\frac{-1+0}{2}\right) + f\left(\frac{0+1}{2}\right) + f\left(\frac{1+2}{2}\right)\right)$$

$$f(-1.5) + f(-0.5) + f(0.5) + f(1.5)$$

$f = x^2$

Answer : 5

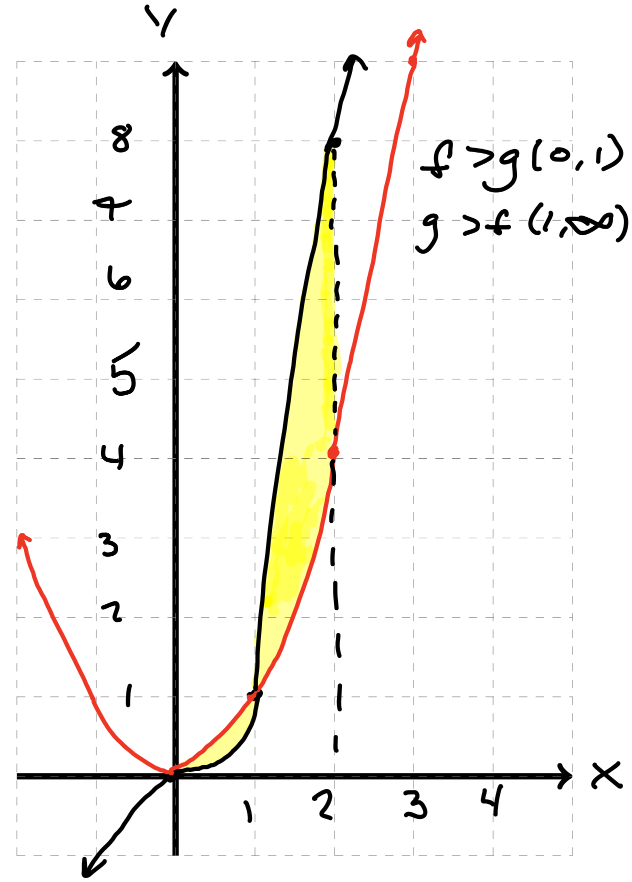
$$2.25 + 0.25 + 0.25 + 2.25$$

$$4.5 + 0.5 = 5$$

Turn over →

5. Consider the functions $f(x) = x^2$ and $g(x) = x^3$.

(a) **Sketch the region** bounded by the graphs of $f(x)$ and $g(x)$ on the interval $0 \leq x \leq 2$. *Make sure to clearly label axes, increments, and which graph is which.*



(b) **Set up** the integral(s) that can be used to determine the *area of the region* from part (a).

$$\int_0^1 (x^2 - x^3) + \int_1^2 (x^3 - x^2)$$

Answer :

$$\int_0^1 (x^2 - x^3) + \int_1^2 (x^3 - x^2)$$

(c) **Determine** the area of the region from part (a) by evaluating the appropriate integral(s) from part (b).

$$\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2$$

$$\left[\left(\frac{1^3}{3} - \frac{1^4}{4}\right)\right] + \left[\left(\frac{2^4}{4} - \frac{2^3}{3}\right) - \left(\frac{1^4}{4} - \frac{1^3}{3}\right)\right]$$

$$\left(\frac{1}{3} - \frac{1}{4}\right) + \left(\left(4 - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right)$$

$$\left(\frac{1}{12}\right) + \left(\frac{4}{3} + \frac{1}{12}\right)$$

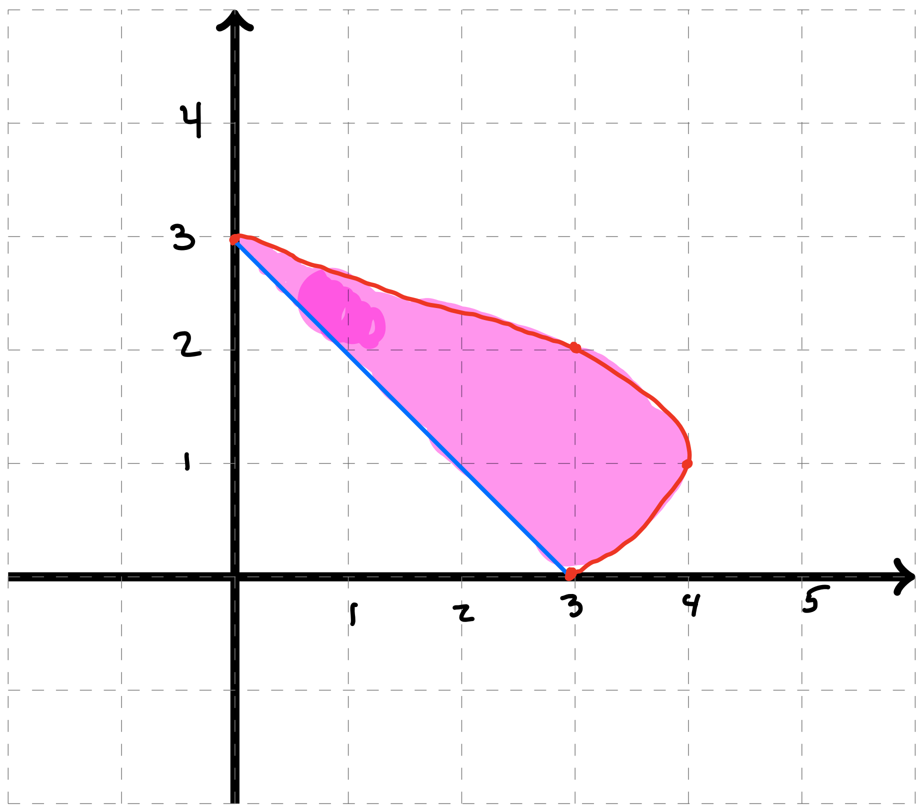
$$\frac{1}{12} + \frac{17}{12} = \frac{18}{12} = \frac{3}{2}$$

Area $= \frac{3}{2}$

Turn over →

6. Consider the *relations*

(a) **Sketch the region** *in the first quadrant* ($x \geq 0, y \geq 0$) bounded by the graphs of equations $y = -x + 3$ and $x = -(y-1)^2 + 4$. *Make sure to clearly label axes, increments, and which graph is which.*



(b) **Set up (but do not evaluate)** the integral(s) that can be used to determine the *area of the region* from part (a).

At all points $x = (0,3)$ and $y = (0,3)$, Red Dominates Blue

$x = -y + 3$

$x = -(y-1)^2 + 4$

$$\int_0^3 \left(-(y-1)^2 + 4 \right] - (-y+3)\, dy$$

Answer :

$$\int_0^3 \left(-(y-1)^2 + 4 \right] - (-y+3)\, dy$$

Turn over ⟶

7. Suppose that f and g are integrable on any finite interval and that

$$\int_1^2 f(x)\,dx = -4\,, \qquad \int_1^5 f(x)\,dx = 6\,, \qquad \int_1^5 g(x)\,dx = 8\,.$$

Find the following quantities:

(a) $\displaystyle\int_2^2 g(x)\,dx =$ 0

(b) $\displaystyle\int_5^1 g(x)\,dx =$ -8

(c) $\displaystyle\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = -12$

(d) $\displaystyle\int_1^5 \big[f(x) - g(x)\big]\,dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = -2$

(e) $\displaystyle\int_1^5 \big[4f(x) - g(x)\big]\,dx = 4\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 16$

(f) $\displaystyle\int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx = 10$

END