Light

The Speed of Light and Index of Refraction

ELECTROMAGNETIC WAVES

Disturbances on the ocean's surface are mechanical waves that propagate in various directions and at a variety of wavelengths, amplitudes, and speeds. Sound is a mechanical wave, too, that propagates through various substances and at a variety of wavelengths, amplitudes, and speeds. All mechanical waves, by definition, require a medium through which to travel.

Electromagnetic waves, on the other hand, require no medium. Electromagnetic waves are waves of varying electric and magnetic fields, which propagate at high speed through various media—but also though a vacuum, which is evident when we look up at the night sky and see celestial objects through the vacuum of space. Electromagnetic waves are literally everywhere—permeating space and moving in all directions at every location. If you need evidence for this, simply look around you. Visible light is coming at you from all directions, but visible light is only a narrow band of the full electromagnetic spectrum, with gamma rays, X-rays, ultraviolet light, infrared waves, and radio waves also coming at you from all directions.

Space is literally a sea of electromagnetic disturbances, and humans have learned to manipulate these disturbances in ways that affect almost all of modern technology. (In physics, the terms "light" and "electromagnetic waves" are often used interchangeably, even though "light" is visible. When the distinction is important, however, we will say "visible light.")

ELECTROMAGNETIC WAVE PRODUCTION

How are electromagnetic waves produced? We have seen that an electric charge creates an electric field and that moving electric charge creates a magnetic field. To create a wave of electric and magnetic fields, however, charges must be accelerated. The following animation illustrates one way to produce an electromagnetic wave. In it, the terminals of a sinusoidal AC generator are connected to two straight metal wires, which are parallel to the y axis, as shown. This setup is called a dipole antenna.

[See animation: antenna]

Initially the wires are uncharged. When the generator is turned on, however, positive charge begins to accumulate on the upper wire, while negative charge begins to accumulate on the bottom wire. This separation of charge creates an electric field that points in the negative y direction at the origin. After half of a generator cycle, the wires are uncharged again, but only momentarily. Charge then begins to accumulate on the wires again, but this time the top wire becomes negative and the bottom wire becomes positive, creating an electric field in the positive y direction. After a full generator cycle, the wires are uncharged again, and the next cycle begins. In this way, an electric field wave is created that propagates in all directions (for simplicity only the part on the positive x axis is shown).

A magnetic field wave (not shown) travels along with the electric wave. The presence of the magnetic wave can be understood by remembering that electric current creates a magnetic field, in this case, the current in the antenna wire. This current is parallel to the y axis. According to the magnetic field right-hand rule, the magnetic field at points on the x axis will be into or out of the screen (the $\pm z$ direction).

The electric and magnetic fields close to the antenna are called the near field. The near field is complex and decreases rapidly as one moves away from the antenna. Far from the antenna, however, the electromagnetic wave propagates as illustrated in the following animation. Here, the magnetic wave is in phase with the electric wave and the magnetic and electric fields are everywhere perpendicular to each other. The wavelength λ is the distance between the peaks of the wave. The electric and magnetic fields are perpendicular to the wave velocity, so electromagnetic waves are transverse waves.

[See animation: EMwave]

There are many ways to create ripples on a pond. There are many ways to create electromagnetic waves, too. Their production by a dipole antenna, while easy to visualize, is just one of many ways to produce electromagnetic waves. There is no way, however, to create isolated electric or magnetic waves, which follows from Faraday's law of electromagnetic induction—namely, a changing magnetic field creates an electric field (an induced emf) and a changing electric field creates a magnetic field. Thus, no matter how the wave is produced, an electric wave is always accompanied by a magnetic wave.

THE SPEED OF LIGHT

Light travels at a very high speedâ€"it is so fast, in fact, that it was long thought by many that the speed of light is infinite. In 1676, however, while studying the motion of one of Jupiterâ€™s moons, Danish astronomer Ole Romer discovered that the speed of light is finite, and he estimated its speed to be 200,000 km/s. For the next two centuries, the speed of light was measured with ever-increasing accuracy, when in 1865, Scottish physicist James Clerk Maxwell made a groundbreaking theoretical prediction: Light is an electromagnetic wave that travels through a vacuum at a speed given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

where $\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{T \cdot m}}{\mathrm{A}}$ is the magnetic permeability of free space and $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$ is the electric permittivity of free space. Substituting the numerical values,

$$c = \frac{1}{\sqrt{\left(4\pi \times 10^{-7} \frac{\mathrm{T \cdot m}}{\mathrm{A}}\right)\left(8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}\right)}} = 3.00 \times 10^{8}\,\mathrm{m/s}$$

This theoretical prediction of the speed of light agrees with experimental measurements, providing solid evidence that light is, indeed, an electromagnetic wave. The preceding equation gives the speed of electromagnetic waves in a vacuum, but light travels at a speed somewhat less than $3.00 \times 10^{8}\,\mathrm{m/s}$ when it passes through a transparent medium such as glass or water. In air and other gases, it travels at nearly the same speed as it does in a vacuum. Unless stated otherwise, we will use $c = 3.00 \times 10^8\,\mathrm{m/s}$ for the speed of electromagnetic waves both in gases and through a vacuum.

INDEX OF REFRACTION

A light ray that passes from one transparent medium into another may change direction. This phenomenon, called refraction, is responsible for numerous natural phenomena, but also governs the behavior of many practical devices such as lenses.

Light travels at a speed of $c = 3.00 \times 10^8\,\mathrm{m/s}$ in a vacuum. In transparent materials, like glass or water, light travels at a lower speed because the light is continually absorbed and reemitted by atoms as it passes through the material. The amount by which the lightâ€™s speed v in a transparent medium is different from its speed c in a vacuum is expressed as the ratio c/v, which is called the index of refraction n of the material:

$$n = \frac{c}{v}$$

The index of refraction is a dimensionless quantity whose value is always greater than 1 (because $v < c$). The following table lists the indices of refraction of various substances. The index of refraction depends weakly on wavelength, and the values listed in the table correspond to a wavelength of $\lambda = 589\,\mathrm{nm}$ in a vacuum. Unless otherwise specified, we will use the indices of refraction listed in the table.

Medium	Index of refraction n
Gases at $0\,^\circ\mathrm{C}$ *and* $1\,\mathrm{atm}$	
Helium	1.000036
Hydrogen	1.000132
Air	1.000293
Carbon dioxide	1.00045
Liquids at $20\,^\circ\mathrm{C}$	
Water	1.333
Ethyl alcohol	1.361
Carbon tetrachloride	1.461
Benzene	1.501
Solids at $20\,^\circ\mathrm{C}$ (*except ice*)	
Ice at $0\,^\circ\mathrm{C}$	1.309
Soda-lime glass	1.518
Crown glass	1.523
Quartz crystal	1.544
Diamond	2.419

The index of refraction n increases as v decreases. For the values listed in the table, then, light travels slowest in diamond and fastest in helium gas. The speed of light in gases is very close to c, and for problems in this course a value of $n = 1.00$ may be used for all gases.

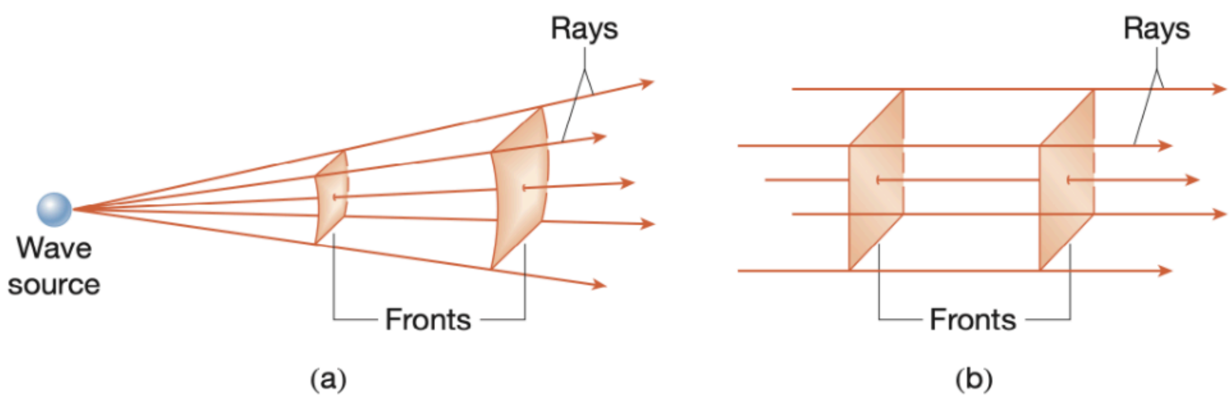
WAVE FRONTS AND RAYS

Consider the waves that are created when a stone is dropped into a still body of water, as illustrated in the following animation. The disturbance creates a series of concentric ripples that propagate outward from the source. If we draw lines connecting all points on a wave peak, we form the yellow circles. These circles are called the wave fronts. (Strictly speaking, wave fronts are lines connecting points of any particular phase—not necessarily wave peaks, but the distinction is not important here.) The black lines are called rays. They are perpendicular to the wave fronts and point in the direction of the wave velocity.

The wave fronts can be thought of as physical parts of the wave (e.g., the wave peaks), but the imaginary rays are actually more helpful in describing wave propagation. Specifically, the physical laws governing reflection (and other optical phenomena) are more easily stated in terms of what happens to the rays than what happens to the wave fronts. The animation uses water waves to illustrate these concepts, but wave fronts and rays can also be used to characterize light waves. A laser creates a narrow, directional beam of light, which serves as a good model of a light ray.

[See animation: FrontsRays]

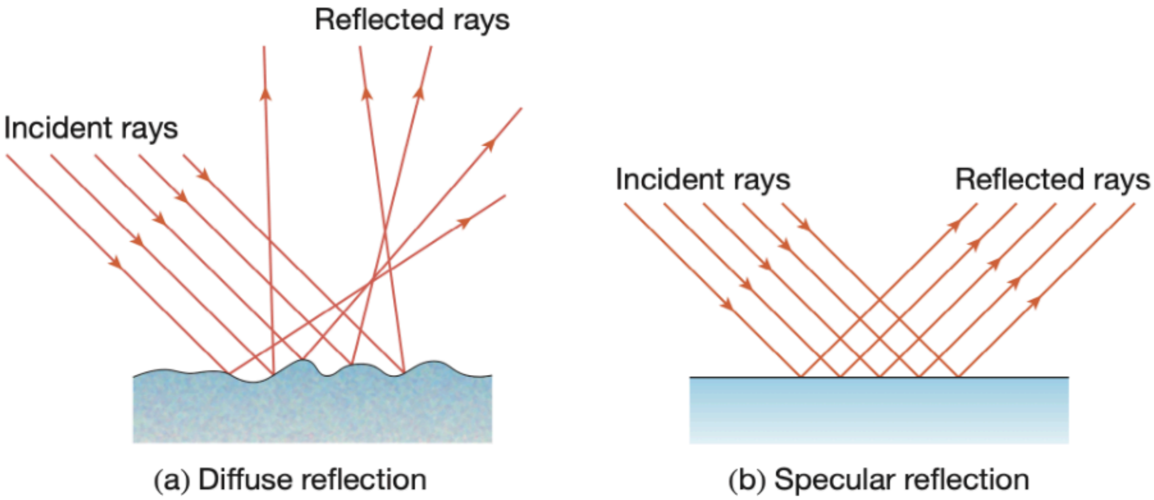
Now consider a source of waves in three dimensions, like a light bulb or a sound speaker. The wave fronts are now concentric spheres (instead of circles), and the rays propagate radially outward from the source. Part (a) of the following figure illustrates the situation near the source, where the wave fronts are curved. Far from the source, the wave fronts are flat and form planes, or plane waves, and the rays associated with plane waves are parallel to each other, as illustrated in part (b).



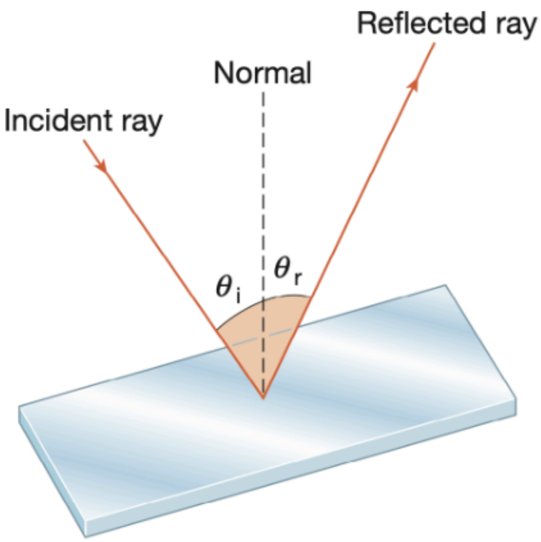
The Law of Reflection

SPECULAR AND DIFFUSE REFLECTION

The reflection of light from a surface can be categorized as either diffuse or specular. The following figure, which shows parallel light rays incident on two surfaces, helps distinguish between the two. In part (a), the surface is rough, resulting in diffuse reflection. The rays reflect in almost random directions because of surface irregularities. In part (b), the surface is smooth, resulting in specular reflection. If the incident rays are parallel, then the reflected rays all travel in the same direction in specular reflection. Examples of diffuse reflection include the reflection of light from unfinished wood or from a wall painted with a flat finish. Specular reflection occurs when light reflects from a mirror or the smooth surface of calm water.



The following figure shows the specular reflection of a light ray from a plane mirror (a plane mirror is simply a flat mirror). The normal is a line drawn perpendicular to the surface at the point of reflection. The incident ray makes an angle θ_i with the normal, while the reflected ray makes an angle θ_r with the normal.

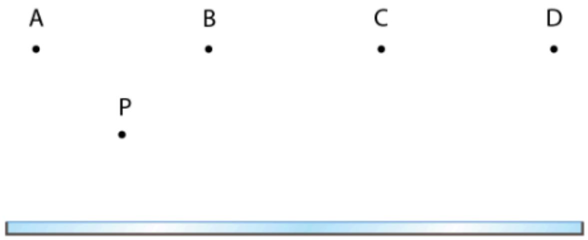


The angles θ_i and θ_r are called the angle of incidence and the angle of reflection, respectively, and are related by the law of reflection:

- For specular reflection, the incident and reflected rays lie in the same plane. The angle of incidence θ_i and the angle of reflection θ_r (measured with respect to the normal) are equal: $\theta_\mathrm{i} = \theta_\mathrm{r}$

EXAMPLE

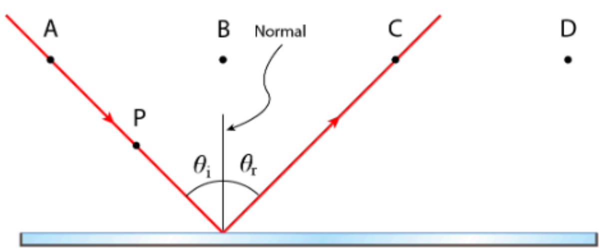
Consider the situation illustrated in the figure below, in which a plane mirror lies with the reflecting surface upward. Four points above the mirror (A, B, C, D, and P) are indicated. Consider a light ray that is incident on the mirror from above.



If the incident ray passes though points A and P, then through which point does the reflected ray pass?

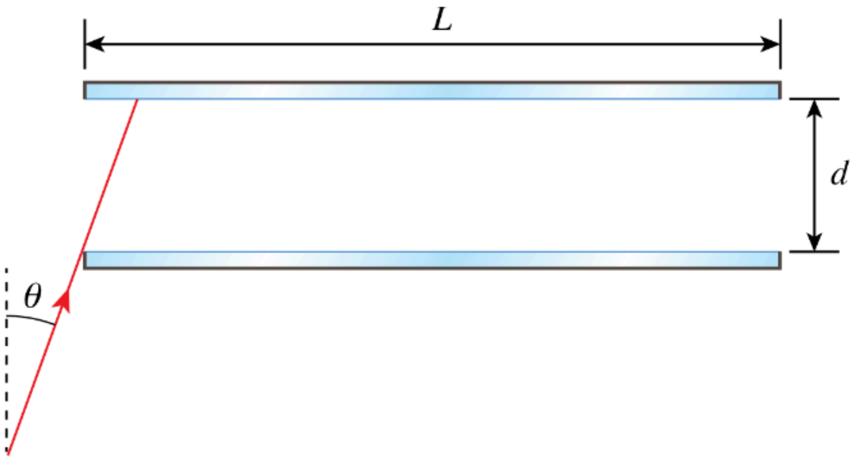
SOLUTION

The following diagram illustrates the solution, where the incident ray is drawn passing through points A and P. The reflected ray is drawn so that θ_i and θ_r make the same angle to the normal (which should always be drawn passing through the reflection point). In this case, the reflected ray passes through point C.



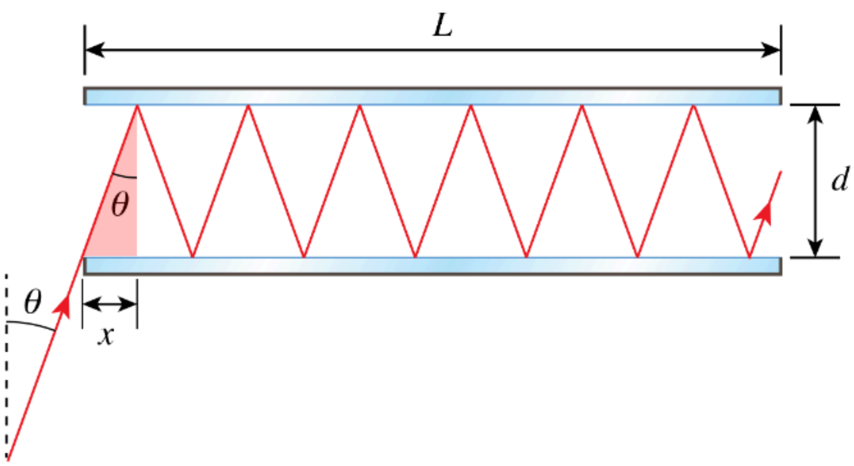
EXAMPLE

Two plane mirrors are horizontal and facing each other. The distance between the mirrors is $d = 10.0\,\text{cm}$ and their length is $L = 45.0\,\text{cm}$. A ray of light making an angle $\theta = 20.0°$ with vertical enters the space between the mirrors, as shown. (a) How far horizontally does a light ray travels as it moves from one reflection to the next? [HINT: Identify the appropriate right triangle for this calculation.] (b) How many times does the light ray reflect from the top mirror. (c) How many times does the light ray reflect from the bottom mirror. (d) Sketch the ray's path as it passes between the mirrors, as is consistent with your results from (b) and (c).



SOLUTION

(a) The diagram below shows the reflections. The normal to the mirrors is vertical, so for each reflection both the incident and reflected angles make an angle θ to vertical.



Consider the shaded right triangle that highlights the first reflection from the top mirror, where x is the length of the side opposite θ and d is the side adjacent. Since $\tan\theta = x/d$, we have

$$x = d\tan\theta = (10.0\,\mathrm{cm})\tan(20.0^\circ) = 3.6397\,\mathrm{cm}$$

(b) Note that the total number of reflections is

$$\frac{L}{x} = \frac{45.0\,\mathrm{cm}}{3.6397\,\mathrm{cm}} = 12.4 \to 12$$

Thus, there are 12 reflections. Note that the first reflection is from the top mirror, so *odd* reflections are off the top mirror.

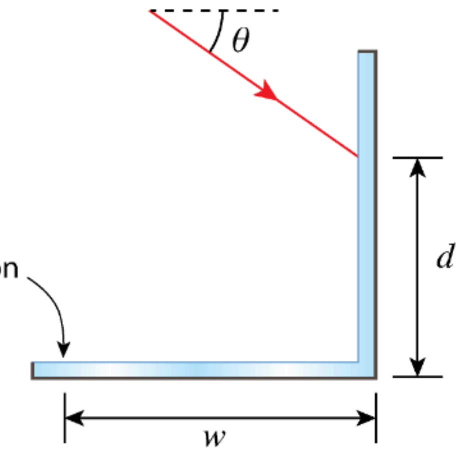
The odd reflections are 1, 3, 5, 7, 9, and 11, so there are six reflections from the top mirror.

(c) The even reflections are 2, 4, 6, 8, 10, and 12, so there are six reflections off the bottom mirror.

(d) See diagram above.

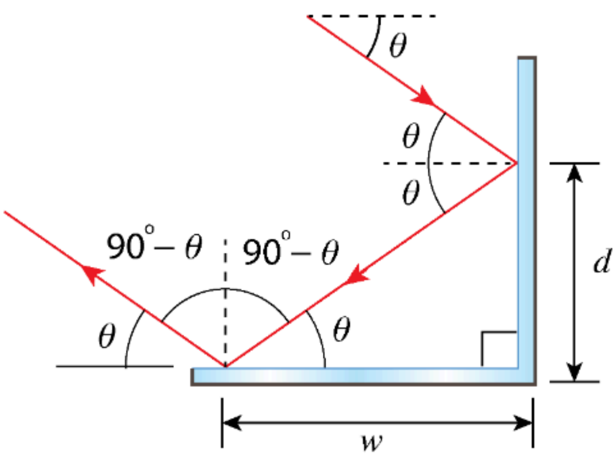
EXAMPLE

Two plane mirrors, one horizontal and the other vertical, are joined to make the L-shaped mirror in the diagram below. A light ray traveling in a direction $\theta = 30.0°$ below horizontal reflects from the mirror a distance $d = 45.0\,\text{cm}$ above the horizontal mirror, then reflects a second time a distance w from the vertical mirror. (a) Sketch the path of the ray as it reflects from the mirrors. (b) What is w? (c) In what direction is the light ray traveling after it reflects from the horizontal mirror?



SOLUTION

(a) The normal to the vertical mirror is horizontal and that of the horizontal mirror is vertical. The diagram below shows the reflections, drawn utilizing the law of reflection.



(b) From $\tan\theta = d/w$, we have

$$w = \frac{d}{\tan\theta} = \frac{45.0\text{ cm}}{\tan(30.0°)} = 77.9\text{ cm}$$

(c) After reflecting from the horizontal mirror, the ray is traveling in a direction θ above horizontal.

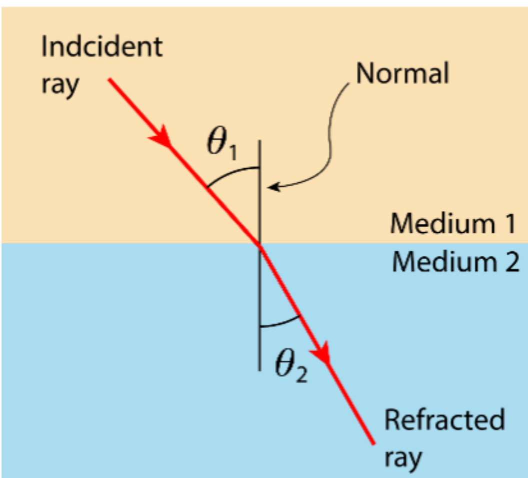
Refraction: Snell's Law

If light traveling in one medium crosses a boundary with another medium, then the wave speed changes. The following animation illustrates this situation, but with simulated water waves that are viewed from overhead and have approximately linear wave fronts. (Simulation footage courtesy of www.falstad.com.)

The waves cross from deeper water, where the waves travel fast, into shallow water, where the waves travel more slowly. Points on a particular wave front enter the shallow region at different times. The result is that the orientation of a particular wave front changes as it crosses the boundary. In other words, the propagation direction of the wave changes as it enters the slow medium. This phenomenon is called refraction. Note also the reflected wave, whose direction obeys the law of reflection.

[See animation: Refraction]

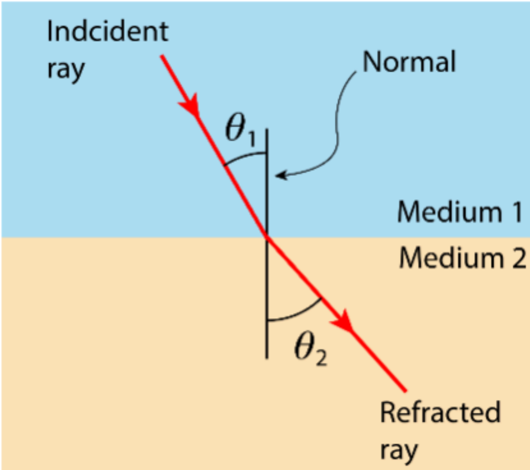
The following figure illustrates refraction of light. A light ray traveling in medium 1 and is incident on the boundary with medium 2. The normal is a line drawn perpendicular to the boundary of the mediums. The incident ray makes an angle θ_1 with the normal, while the refracted ray makes an angle θ_2 with the normal.



Let n_1 and n_2 be the indices of refraction of medium 1 and 2, respectively. The angles and indices of refraction are related by Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The preceding figure illustrations the situation when $n_2 > n_1$, in which case $\theta_2 < \theta_1$. In this case, we say the light ray bends toward the normal. If $n_2 < n_1$, on the other hand, then $\theta_2 > \theta_1$. In this case we say the light ray bends away from the normal, as illustrated in the following figure.



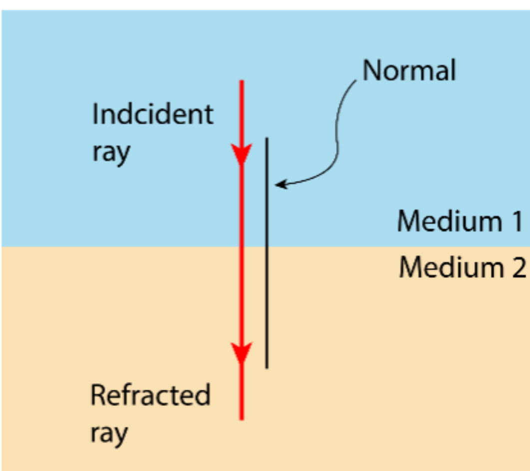
Summarizing elements of the preceding discussion, there are two qualitatively different ways in which the light ray changes direction:

- When a light ray traveling in one medium enters a medium of higher refractive index ($n_2 > n_1$), then the light bends toward the normal.
- When a light ray traveling in one medium enters a medium of lower refractive index ($n_2 < n_1$), then the light bends away from the normal.

There a case for which the light ray does not change direction at all:

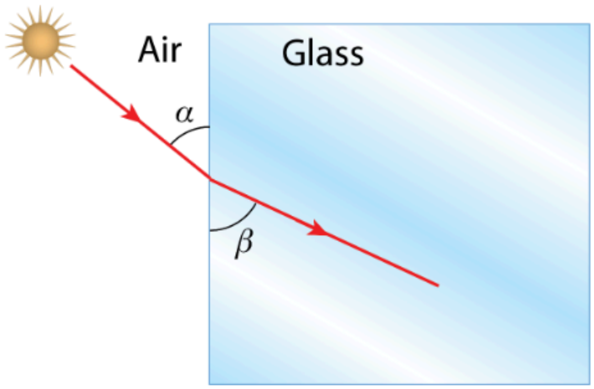
- If $\theta_1 = 0$ then $\theta_2 = 0$. In other words, if the incident ray is parallel to the normal, then the refracted ray will be parallel to the normal.

This point is illustrated in the following figure.



EXAMPLE

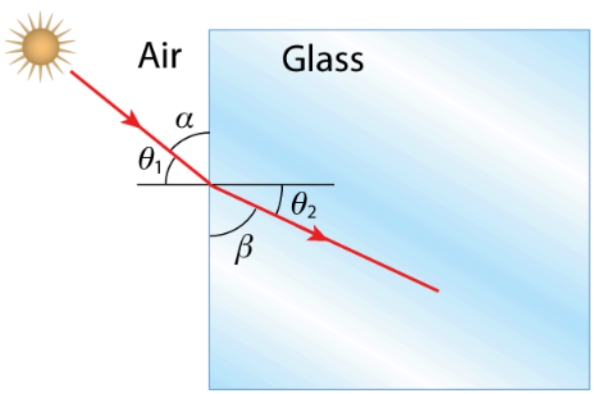
Rectangular slab of glass, whose index of refraction is 1.480, stands vertically, and light rays enter a vertical face at an angle of $\alpha = 53.0°$ to the vertical face, as shown. After passing into the glass, the light ray makes an angle β to the vertical face, as shown.



What is β?

SOLUTION

The normal to the vertical face is horizontal, and the following diagram illustrates the relationship between α, β, and the angles of incidence and refraction. Note that $\theta_1 = 90° - \alpha$ and $\theta_2 = 90° - \beta$.



Let n_a be the index of refraction of air and n_g be the index of refraction of the glass. Applying Snell's law,

$$n_\mathrm{a}\sin(90° - \alpha) = n_\mathrm{g}\sin(90° - \beta)$$

Solving for $90° - \beta$ and calculating,

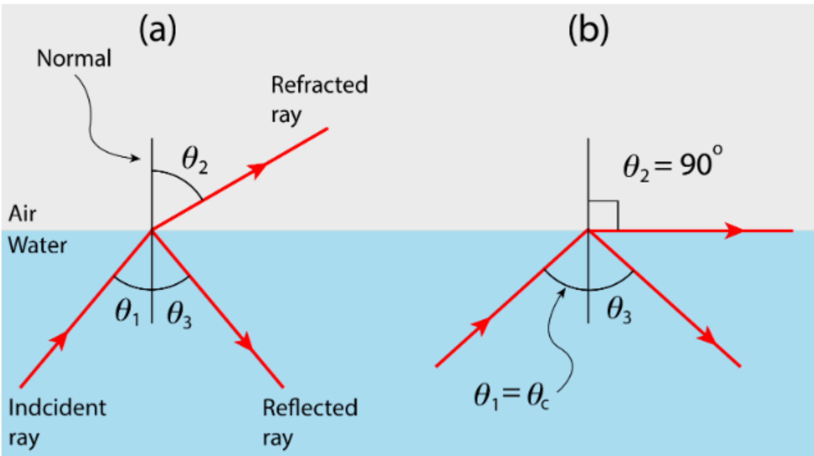
$$90° - \beta = \sin^{-1}\left(\frac{n_\mathrm{a}\sin(90° - \alpha)}{n_\mathrm{g}}\right) = \sin^{-1}\left(\frac{(1.000)\sin(90° - 53.0)}{1.480}\right) = 23.990°$$

Thus,

$$\beta = 90° - 23.990° = 66.0°$$

Total Internal Reflection

The following figure shows a light ray traveling in water ($n_1 = 1.333$), up towards the boundary with air ($n_2 = 1.000$). In part (a), the incident ray makes an angle θ_1 with the normal to the water-air boundary. Some light is reflected, and the reflected ray makes an angle θ_3 with the normal. According to the law of reflection, $\theta_1 = \theta_3$. Some light is transmitted into the air, and the refracted ray makes an angle θ_2 with the normal. Recall that when a light ray traveling in one medium enters a medium of lower refractive index, then the light bends away from the normal. This is the case here, so $\theta_2 > \theta_1$.



Now imagine increasing θ_1 gradually. At some point θ_2 will become 90° while θ_1 is less than 90°, as illustrated in part (b) of the figure. At this point no light is transmitted in to the air. The incident angle θ_c at which this happens is called the critical angle for total internal reflection. We can derive a formula for θ_c by applying Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

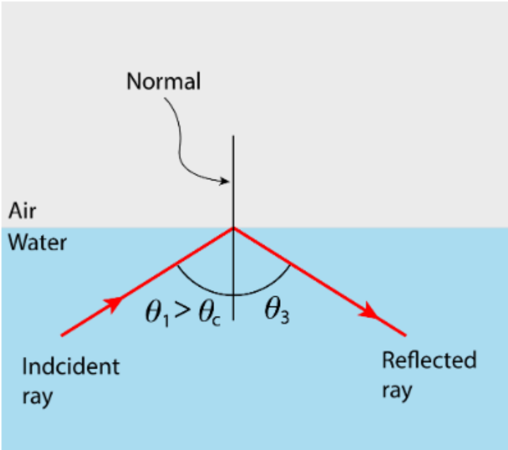
With $\theta_2 = 90°$ and $\theta_1 = \theta_c$, we have

$$n_1 \sin\theta_c = n_2 \sin(90°)$$

But $\sin(90°) = 1$, so we can rearrange the previous equation as follows:

$$\sin\theta_c = \frac{n_2}{n_1}$$

This equation allows you to calculate the critical angle for any boundary between two transparent mediums for which $n_1 > n_2$. The following figure illustrates the situation for which the incident angle is greater than the critical angle for total internal reflection.



What is the critical angle for total internal reflection for a light ray traveling in water toward the boundary with air? Using $n_1 = 1.333$ and $n_2 = 1.000$, we have

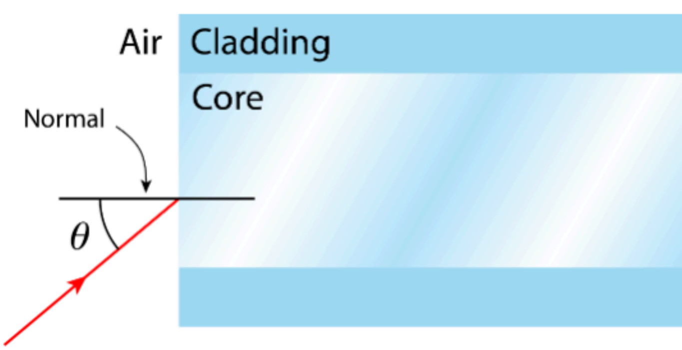
$$\theta_c = \sin^{-1}\left(\frac{1.000}{1.333}\right) = 48.61^\circ$$

Total internal reflection can occur only when light traveling in one medium is incident upon a medium of lower index of refraction. In this case the following points apply:

- If $\theta_1 < \theta_c$ then some light is transmitted into the medium of lower index of refraction, with the direction of the refracted ray governed by Snell's law.
- If $\theta_1 > \theta_c$ then all of the light is reflected back into the medium of higher index of refraction, with the direction of the reflected ray governed by the law of reflection.

EXAMPLE

A fiber-optic cable consists of a fine glass wire (the *core*) sheathed by another material (the *cladding*) of lower refractive index. The diagram shows a ray of light incident upon the glass core of a horizontal fiber-optic cable. The ray enters the glass with an angle of incidence of $\theta = 45.0^\circ$, then travels in the core, where it is then incident upon the cladding. Will there be total internal reflection when the ray of light first encounters the cladding? Assume the air-core boundary is vertical and the cladding-core boundary is horizontal. The indices of refraction of air, core, and cladding are 1.00, 1.55, and 1.30, respectively.

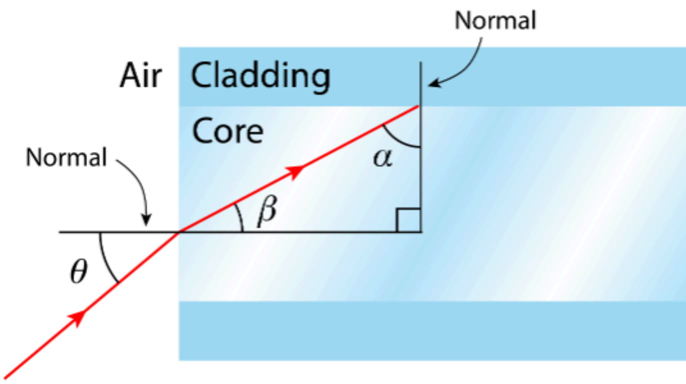


SOLUTION

When light traveling in a medium with index of refraction n_1 encounters the boundary with a medium of index of refraction n_2, the angle of incidence θ_1 and angle of refraction θ_2 are related by Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

The diagram below illustrates the incident and refracted rays for the light ray entering the core.



Here $n_1 = 1.00$, $n_2 = 1.55$, $\theta_1 = \theta$, and $\theta_2 = \beta$. Solving for β in $n_1 \sin\theta = n_2 \sin\beta$, and calculating, we have

$$\beta = \sin^{-1}\left(\frac{n_1 \sin\theta}{n_2}\right) = \sin^{-1}\left(\frac{(1.00)\sin(45.0^\circ)}{1.55}\right) = 27.142^\circ$$

The diagram also illustrates the ray incident upon the cladding at incident angle α. The normal here is vertical, whereas the normal to the air-core boundary is horizontal, so $\beta + \alpha = 90^\circ$, and

$$\alpha = 90^\circ - \beta = 90^\circ - 27.142^\circ = 62.858^\circ$$

When light traveling in one medium, with index of refraction n_1, is incident upon the boundary with a medium of lower index of refraction n_2, there will be total internal reflection if the angle of incidence is greater or equal to the *critical angle for total internal reflection*, θ_c, where

$$\sin\theta_c = \frac{n_2}{n_1}$$

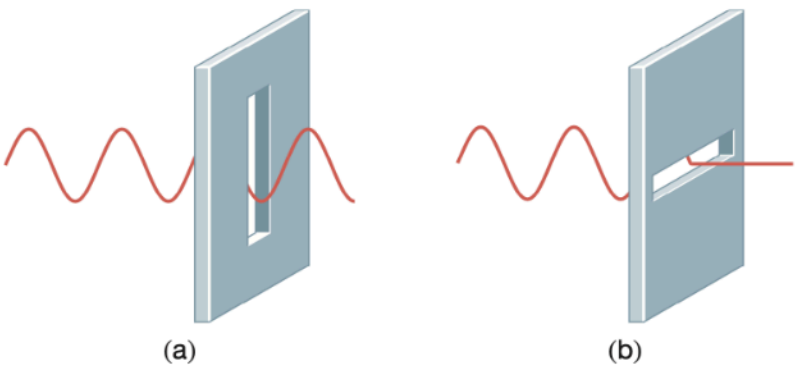
Calculating θ_c for the core-cladding boundary,

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_2}\right) = \sin^{-1}\left(\frac{1.30}{1.55}\right) = 57.0^\circ$$

Because $\alpha > \theta_c$ there will be total internal reflection.

Polarization and Polarizers

Polarization is a phenomenon that is unique to transverse waves—that is, waves in which the vibrations of the medium are perpendicular to wave velocity. To understand why, consider the transverse wave on a string pictured in the following figure. The vibrations of the string are vertical, so this wave is vertically polarized. In part (a), the string passes through a vertical slit, which does not impede the motion of the string and the wave continues through. In part (b), the string encounters a horizontal slit, which prevents vertical motion, so the wave is stopped. A longitudinal wave—one in which the vibrations are parallel to the direction of the wave velocity—would be unaffected in this manner. The wave is linearly polarized because the vibrations of the wave are along only one direction.

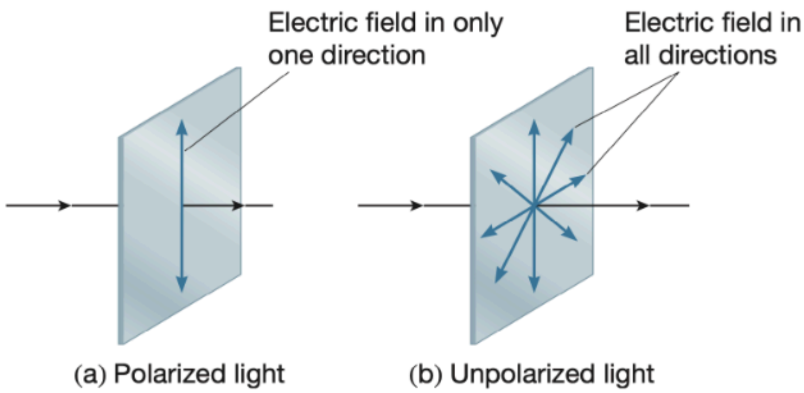


The following animation illustrates what polarization means for electromagnetic waves. The direction of the polarization of an electromagnetic wave is defined as the direction parallel to the electric field, so we show only the electric part of the wave. In part (a) of the following animation, the electric field is directed either vertically upward or downward, so this wave is vertically polarized. In part (b), the electric field is horizontal, so this wave is horizontally polarized. The polarization in part (c) is 20° from vertical. Run the animation to see the wave motion.

[See animation: PolarizedLight]

Polarized electromagnetic waves can be produced by a dipole antenna. Electrons oscillating along the antenna create the wave and the polarization direction is parallel to the antenna. Most light sources, however, create unpolarized light—that is, electromagnetic waves with randomly directed electric fields. In light sources such as the Sun or the hot filament of an incandescent light bulb, the light is created by collisions between charges or the oscillation of electrons within an atom. These processes produce very short bursts of electromagnetic radiation. Due to the large number of atoms and the random nature of the collisions and oscillations, these bursts have randomly oriented electric fields, so the light is completely unpolarized.

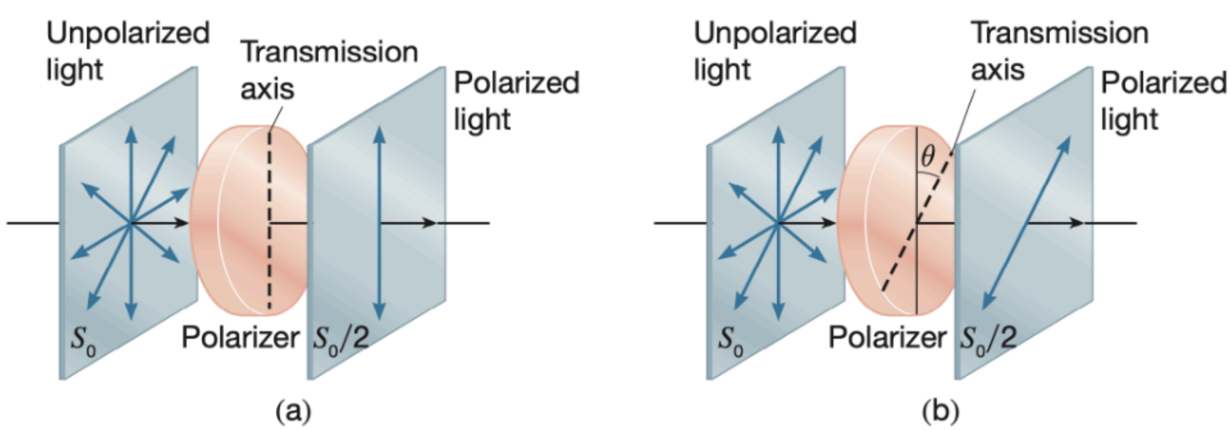
The following figure illustrates a way to represent polarized and unpolarized light schematically, without drawing the full electric wave. In part (a), we use a double-ended arrow to indicate the polarization direction. Part (b) represents unpolarized light by using multiple double-ended arrows. Keep in mind that light is a transverse wave, so in both cases the electric field is perpendicular to the wave velocity.



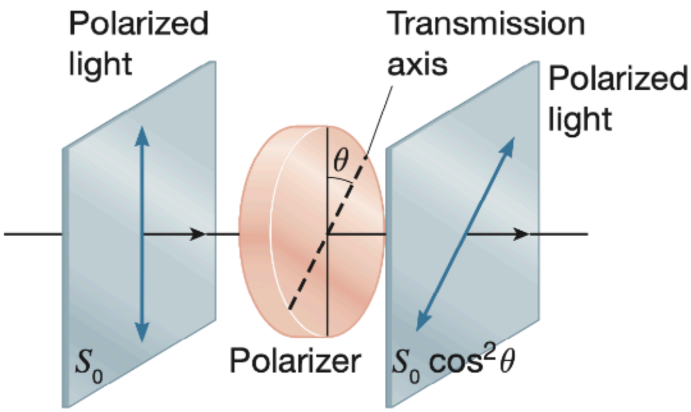
A polarizer is a thin sheet of transparent material in which there are many microscopic, electrically conducting, needle-like crystals. These crystals are forced to align during the manufacturing process, resulting in a material that is electrically conducting in the direction parallel to the crystals and insulating in the direction perpendicular to the crystals. This kind of polarizer tends to absorb light that is polarized parallel to the crystals and transmit light that is polarized perpendicular to the crystals. We define the transmission axis of such a polarizer as the imaginary line perpendicular to the crystals. Thus, when a light wave passes through a polarizer, only the component of the electric field wave that is parallel to the transmission axis passes through, while the perpendicular component of the electric field wave is absorbed.

Part (a) of the following figure illustrates what happens when unpolarized light of intensity S_0 passes through a polarizer whose transmission axis is vertical. The light undergoes two important changes as it passes through the polarizer. First, the transmitted intensity is one-half of the incident intensity. (In the context of wave propagation, the word incident describes the waves before they interact with an optical device, such as a polarizer, mirror, or transparent surface.) Second, the light has become polarized, with the direction of polarization the same as the direction of the transmission axis of the polarizer.

In part (b), the transmission axis has been rotated so that it makes an angle θ with the vertical. Once again, the transmitted intensity is one-half the incident intensity, but now the polarization of the transmitted light makes an angle θ with the vertical. The figure shows how we can create polarized light from unpolarized light and control the direction of polarization. The human eye cannot detect polarization, so the transmitted light in each case would look identical to the naked eye.



A polarizer can be used to make polarized light from unpolarized light, but what happens when polarized light passes through a polarizer? In the following figure, vertically polarized light with intensity S_0 passes through a polarizer whose transmission axis makes an angle of θ with the vertical.



The light emerges from the polarizer with an intensity S given by

$$S = S_0 \cos^2 \theta$$

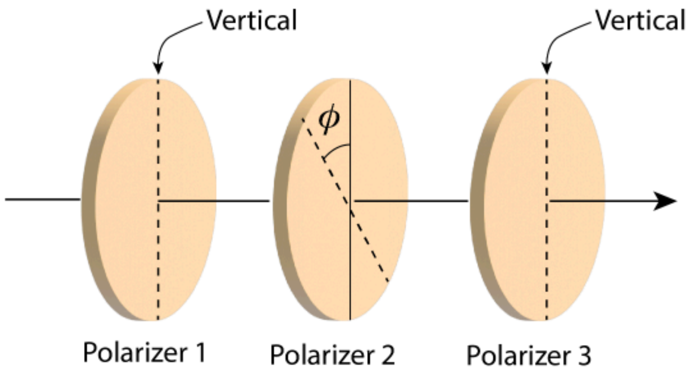
where $\cos^2 \theta$ is notation for $(\cos \theta)^2$. This equation is called Malus' law, because it was discovered by French engineer Etienne Louis Malus (1775–1812). The angle θ is defined as the angle between the polarization of the incident light and the transmission axis of the polarizer. An interesting consequence of Malus' law is that the transmitted intensity S is zero when $\theta = 90°$. This should be expected, however, because the polarizer only transmits electric field components parallel to the transmission axis.

Here is a concise summary of the preceding discussion:

- INCIDENT LIGHT UNPOLARIZED: When unpolarized light passes through a polarizer, the intensity of the transmitted light is 1/2 that of the incident light. Moreover, the transmitted light is polarized with its polarization parallel to the transmission axis of the polarizer.
- INCIDENT LIGHT POLARIZED: When polarized light passes through a polarizer, the intensity of the transmitted light is $\cos^2 \theta$ times that of the incident light, where θ is the angle between the polarization of the incident light and the transmission axis of the polarizer. The transmitted light is polarized with its polarization parallel to the transmission axis of the polarizer.

EXAMPLE

Three polarizers are arranged in series as shown. The transmission axis of the first polarizer is vertical, the transmission axis of the second makes an angle $\phi = 25.0°$ to the vertical, and the transmission axis of the third is vertical. Unpolarized light with intensity $50.0\ \mathrm{W/m^2}$ is incident on this combination, moving left to right. (a) What is the intensity of the light when it is between polarizers 1 nd 2? (b) What is direction of the polarization of the light when it is between polarizers 1 and 2? (c) What is the intensity of the light when it is between polarizers 2 nd 3? (d) What is the intensity of the light after it has passed through all three polarizers? (e) What is the direction of the polarization of the light after is has passed through all three polarizers?



SOLUTION

(a) When unpolarized light passes through a polarizer, the intensity of the transmitted light is one-half the intensity of the incident light, so

$$\frac{1}{2}(50.0\,\mathrm{W/m^2}) = 25.000\,\mathrm{W/m^2}$$

(b) When unpolarized light passes through a polaizer, the light becomes polarized and the polarization of the transmitted light is parallel to the transmission axis. Thus, the light is vertically polarized after passing through polarizer 1.

(c) When polarized light passes through a polarizer, the intensity of the transmitted light is equal to $\cos^2\theta$ times the intensity of the incident light, where θ is the angle between the polarization of the incident light and the transmission axis of the polarizer, so here $\theta = \phi$, and

$$(25.000\,\mathrm{W/m^2})\cos^2(25.0^\circ) = 20.535\,\mathrm{W/m^2}$$

(d) When polarized light passes through a polaizer, the polarization direction changes so that the polarization of the transmitted light is parallel to the transmission axis. Thus, the light between polarizers 2 and 3 is polarized at an angle ϕ to vertical. The transmission axis of polarizer 3 is vertical, so once again $\theta = \phi$, and

$$(20.535\,\mathrm{W/m^2})\cos^2(25.0^\circ) = 16.9\,\mathrm{W/m^2}$$

(e) When polarized light passes through a polaizer, the polarization direction changes so that the polarization of the transmitted light is parallel to the transmission axis. Thus, the light is polarized vertically after passing through all three polarizers.
