# ITERATED REVELATION:

## HOW TO INCENTIVIZE EXPERTS TO REVEAL NOVEL ACTIONS

Evan Piermont
Royal Holloway, University of London

University College London --- May 13 2025

- Many economic models (cheap talk, persuasion, delegation, etc) find a decision maker seeking the council of an expert.
- Almost always: expert provides *statistical* info about the resolution of uncertainty

- Many economic models (cheap talk, persuasion, delegation, etc) find a decision maker seeking the council of an expert.

- Almost always: expert provides *statistical* info about the resolution of uncertainty

| Decision Maker | Expert | Information |
|---|---|---|
| investor | analyst | economic forecast |
| regulator | firm | cost structure |
| Ph.D student | supervisor | prob. of success |

- There is another reason to seek advice: to learn novel actions

- There is another reason to seek advice: to learn novel actions

| Decision Maker | Expert | Information | Novel Actions |
|---|---|---|---|
| investor | analyst | economic forecast | assets, firms, strat. |
| regulator | firm | cost structure | feasible mergers |
| Ph.D student | supervisor | prob. of success | research ideas |

- There is another reason to seek advice: to learn novel actions

| Decision Maker | Expert | Information | Novel Actions |
|---|---|---|---|
| investor | analyst | economic forecast | assets, firms, strat. |
| regulator | firm | cost structure | feasible mergers |
| Ph.D student | supervisor | prob. of success | research ideas |
| homeowner | architect | ??? | house design |

# Project Choice

Armstrong and Vickers (2010), Guo and Shmaya (2023) study project choice:

- A manager (decision maker) is uncertain about which projects are feasible
- A subordinate (expert) makes recommendations
- The manager commits to a selection rule
  - how to choose a project from the subordinate's recommendations

But what if ex-ante commitment to a selection rule is infeasible?

- Unawareness: a student unaware of state-of-the-art research ideas
- Inexpressibility: impractical to express every possible house design
- Enforceability: A regulator may be unable to make reasonable commitments

# Literature

- Delegation / Project Choice
  - Holmstrom (1980); Armstrong and Vickers (2010); Guo and Shmaya (2023)
- Incomplete Contracting / Unawareness in Contracting
  - Grossman and Hart (1986); Maskin and Tirole (1999); Tirole (2009); Hart (2017); Piermont (2017); Lei and Zhao (2021); Francetich and Schipper (2021)
- Strategic Information Transmission
  - Milgrom (1981), Crawford and Sobel (1982); Seidmann and Winter (1997); Aumann and Hart (2003); Chakraborty and Harbaugh (2010)
- Evidentiary disclosure
  - Dye, 1985; Green and Laffont, 1986; Grossman and Hart, 1986; Bull and Watson, 2007; Ben-Porath et al., 2019
- Robust Mechanism Design
  - Bergemann and Morris (2005); Jehiel et al., (2006); Carroll (2015, 2019).

# ❦ *Model* ❦

# Environment

The environment is described by

- $\mathcal{A}$ — a (compact) set of actions
- $(u_d, u_e)$ — (continuous) utility functions $\mathcal{A} \to \mathbb{R}$
- $\mathcal{R}$ — a collection of (compact) subsets of $\mathcal{A}$ such that
  - for $r \in \mathcal{R}$, there are a finite $r' \in \mathcal{R}$ with $r' \subseteq r$.

# Revelation Types

- A **revelation type** $r \in \mathcal{R}$ is a set of actions / projects that an agent can express
- Say that $r$ is **more expressive** than $r'$, if $r' \subseteq r$
- Fix types $r_d$ and $r_e$
  - common knowledge that $r_d \subseteq r_e$

# Key Assumptions

**Information Spillover**: Only $r \in \mathcal{R}$, such that $r \subseteq r_e$ can be revealed

**Voluntary Disclosure**: The expert can always masquerade as a type $r_d \subseteq r \subseteq r_e$

**Hard Evidence**: If the expert reveals $r$, then any $a \in r$ is 'real'

# Direct Mechanisms

An **selection function** is a function from types to actions:

$$\begin{array}{cccc} f: & r & \mapsto & a \\ & \pitchfork & & \pitchfork \\ & \mathcal{R} & \rightarrow & r \end{array}$$

- Outcome profiles are *direct mechanisms*
- These are the object of study in the project choice literature
- Inexpressibility precludes direct mechanisms / revelation principle

Call $f$ **monotone** if ex's payoff is monotone in her type

$$u_e(f(r')) \leq u_e(f(r)) \tag{1}$$

whenever $r' \subseteq r$, and **strongly monotone** if in addition (1) holds strictly whenever $f(r) \neq f(r')$.

- If direct mechanisms existed, monotonicity is necessary:
- Direct mechanisms don't exist: even with monotonicity, there need not be any 'strategic' way of enacting an selection function.

Call $f$ **efficient** if outcome is Pareto undominated:

for all $r \supseteq r_d$, is no $a \in r$ such that

$$u_d(a) \geq u_d(f(r)) \qquad \text{and} \qquad u_e(a) \geq u_e(f(r)), \tag{2}$$

with at least one inequality holding strictly.

So, what can we do without ex-ante commitment to an selection function?

# Example

- A regulator (the decision maker) is evaluating mergers (i.e., projects) :
  - can only choose a merger structure it is aware of
- A firm (the expert) may be aware of novel ways of structuring a merger
- choice of merger structure determines payoffs for both players
  - the firm cares about producer welfare
  - the regulator cares about consumer welfare foremost, but
  - also cares about efficiency

# Example A

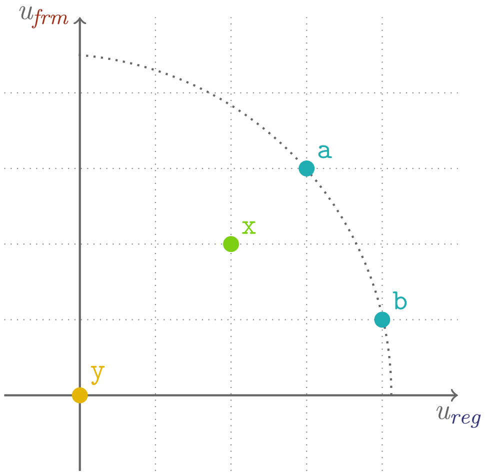
- Each merger yields $(x_{reg}, x_{frm})$:
  - $x_{reg}$ is regulator's payoff (consumer welfare)
  - $x_{frm}$ is firms's (producer welfare)
- The regulator is initially aware of two ways to structure the merger:

$$\mathtt{x} = (2, 2) \qquad \mathtt{y} = (0, 0)$$

- The firm is also aware of:

$$\mathtt{a} = (3, 3) \qquad \mathtt{b} = (4, 1)$$

# Example A



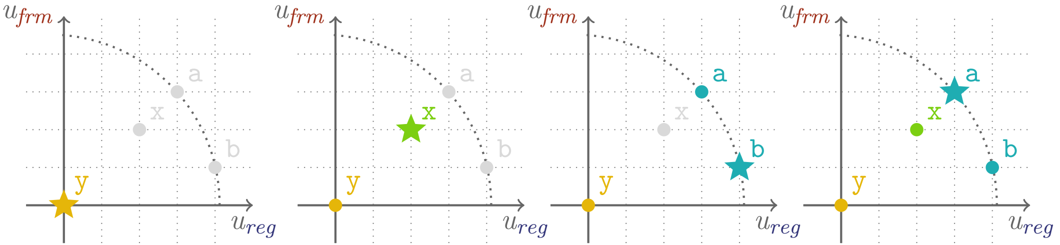
# Example A

- If the firm had full control over what to reveal: simply reveal $\mathtt{a} = (3, 3)$
- However, not all mergers can be independently revealed:
  - Revealing one merger in a 'class' reveals the existence of the whole class, etc
- What if $\mathtt{a}$ and $\mathtt{b}$ must be revealed together?

$$\mathcal{R} = \{\{\mathtt{y}\}, \{\mathtt{y}, \mathtt{x}\}, \{\mathtt{y}, \mathtt{a}, \mathtt{b}\}, \{\mathtt{y}, \mathtt{x}, \mathtt{a}, \mathtt{b}\}\}$$

# Example A

A monotone and efficient selection function:

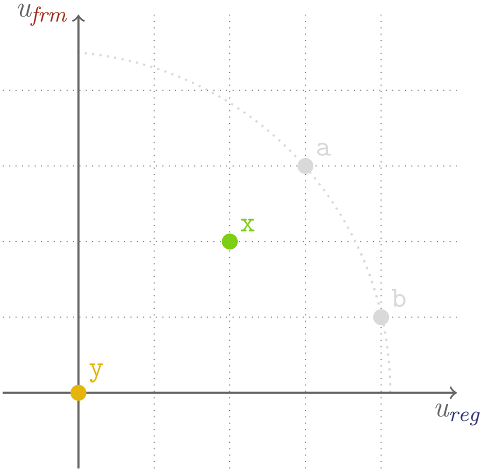



$f(\{\mathtt{y}\}) = \mathtt{y}$ $\quad$ $f(\{\mathtt{y}, \mathtt{x}\}) = \mathtt{x}$ $\quad$ $f(\{\mathtt{y}, \mathtt{a}, \mathtt{b}\}) = \mathtt{b}$ $\quad$ $f(\{\mathtt{y}, \mathtt{x}, \mathtt{a}, \mathtt{b}\}) = \mathtt{a}$

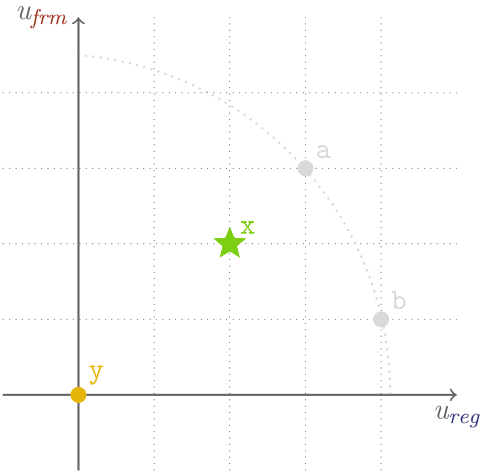
But, the regulator cannot express `a` and `b`:

- Cannot commit to these mergers until they are revealed
- Cannot express the selection function above

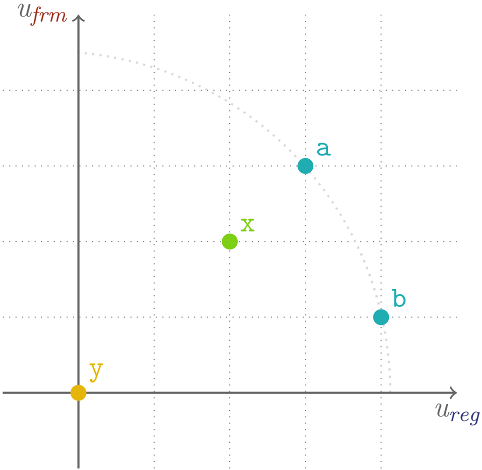
# Example A



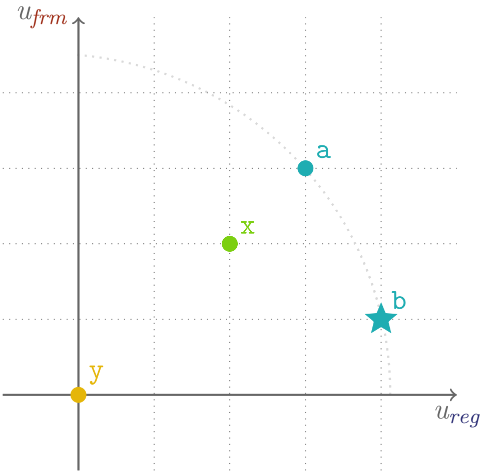
# Example A



# Example A



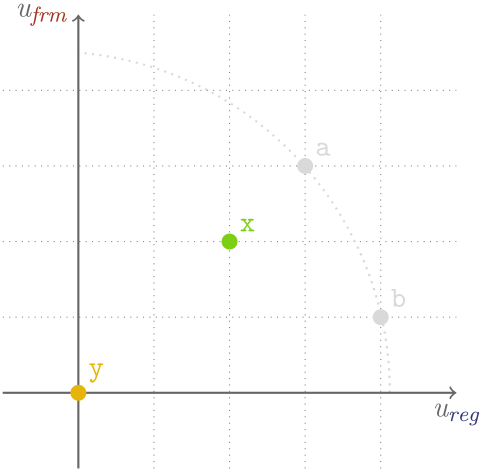
# Example A



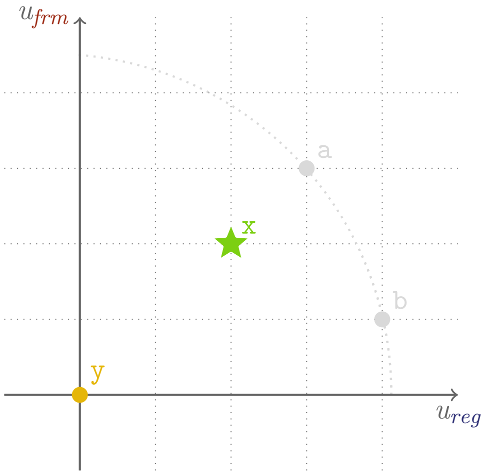
# Example A

- Since the firm prefers `x` to `b`, she would choose not to reveal.
- This is Pareto Inefficient: `a` dominates `x`
- What if regulator and firm can create the following contract (before revelation):
  - shortlist an ‘outside option’ (that the regulator is aware of)
  - this can be re-negotiated after revelation
  - the regulator can propose a new merger, but the firm can veto (therefore implement outside option)

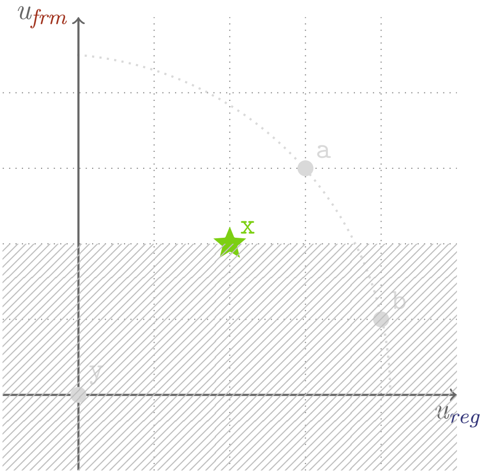
# Example A



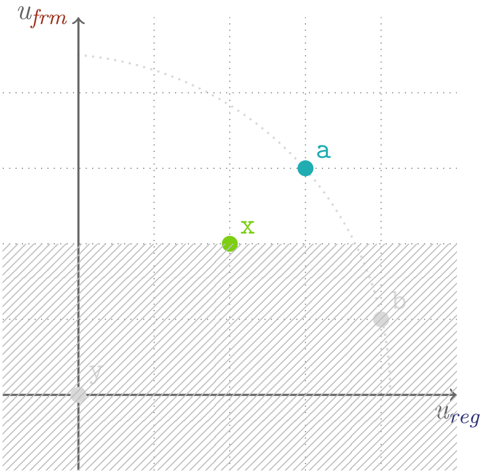
# Example A



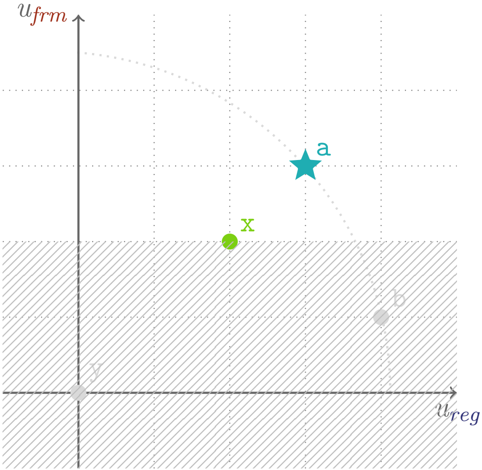
# Example A



# Example A



# Example A



So a two stage game with commitment to not revoke the prior proposal resulted in

- full revelation
- an efficient contract

So a two stage game with commitment to not revoke the prior proposal resulted in

- full revelation
- an efficient contract

Does this always work?

So a two stage game with commitment to not revoke the prior proposal resulted in

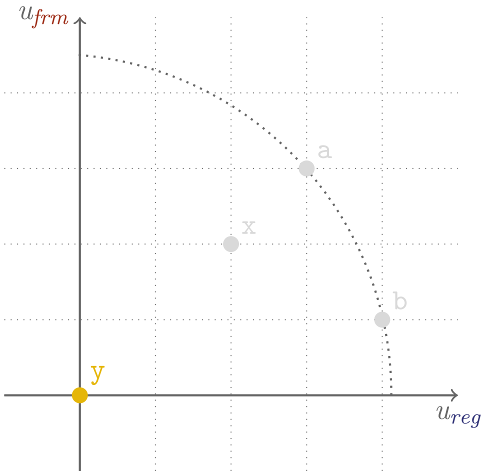
- full revelation
- an efficient contract

Does this always work? No

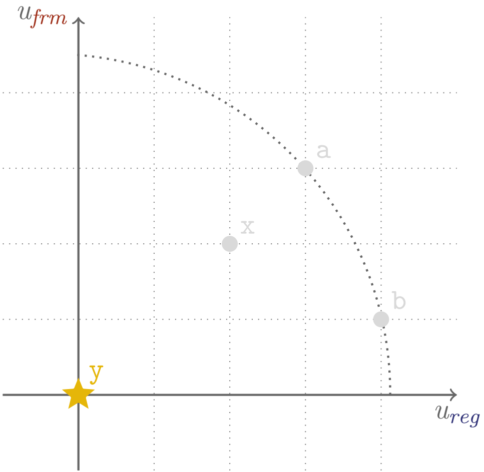
# Example B

- What if the regulator was also initially unaware of $\mathtt{x}$
- $\mathtt{x}$ and $\{\mathtt{a}, \mathtt{b}\}$ can be revealed independently

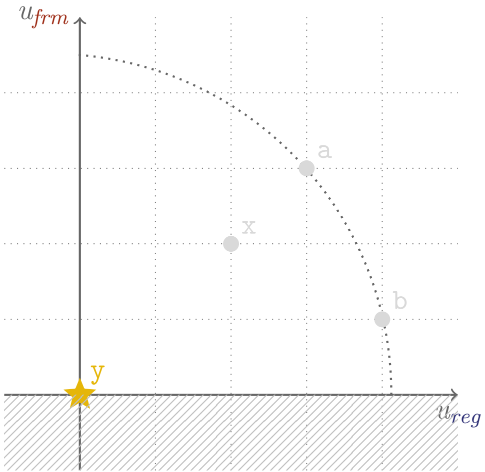
# Example B



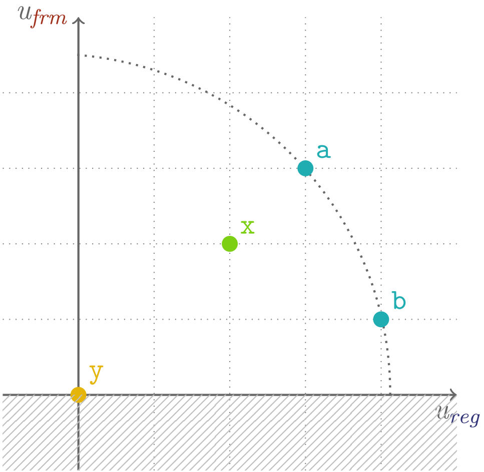
# Example B



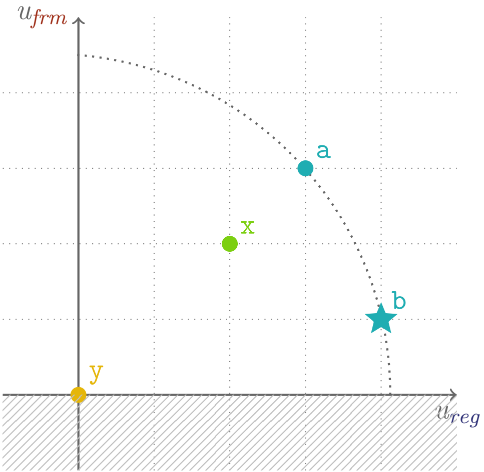
# Example B



# Example B

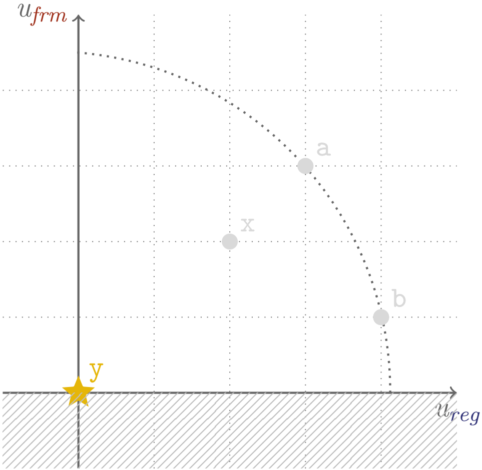


# Example B

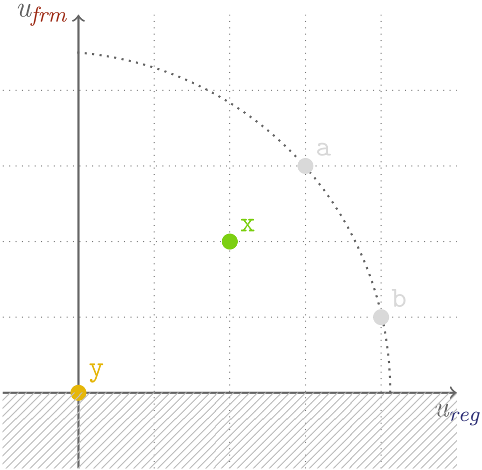


But the firm does not have to reveal everything! Instead, reveal only `x`

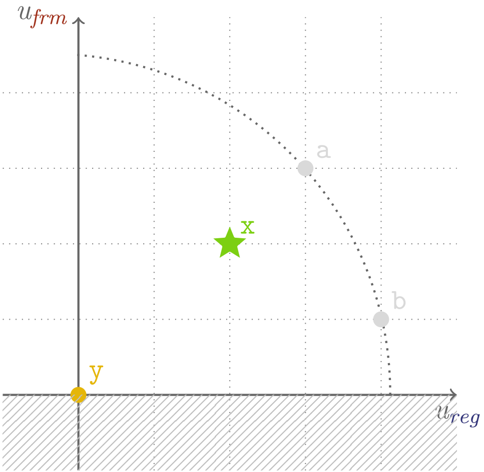
# Example B



# Example B



# Example B



The firm prefers `x` to `b`:

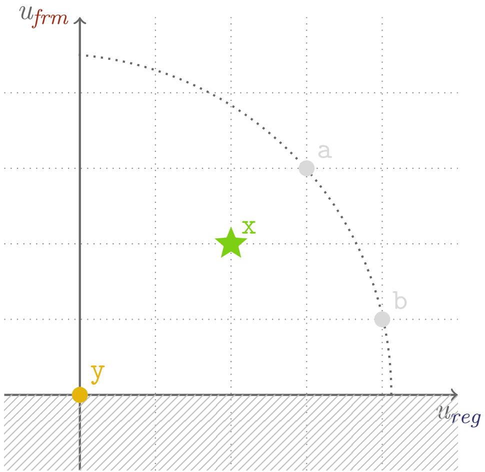
- it will only partially reveal
- again, this is inefficient: `a` dominates `x`

The firm prefers `x` to `b`:

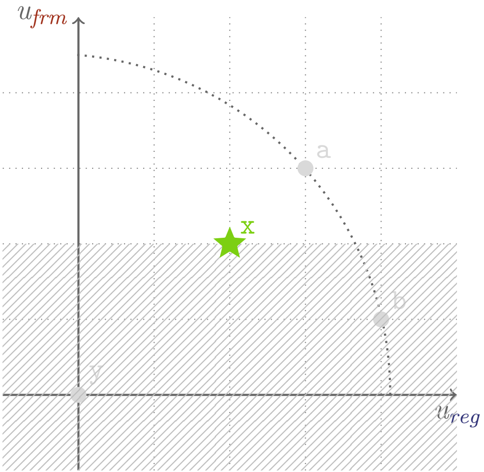
- it will only partially reveal
- again, this is inefficient: `a` dominates `x`

But, we can repeat!

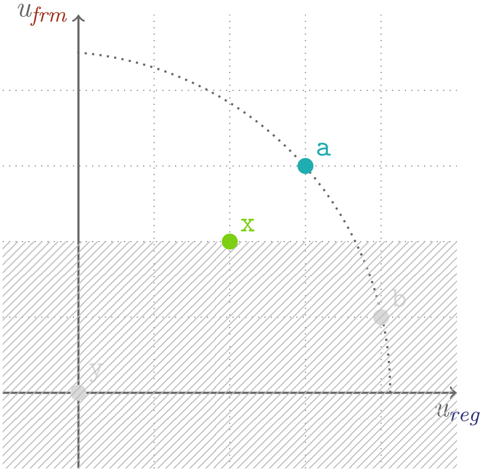
# Example B



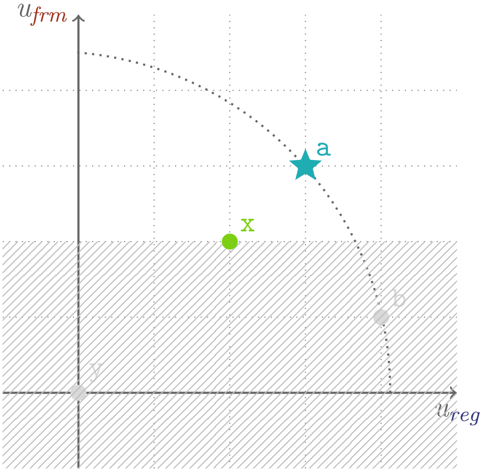
# Example B



# Example B



# Example B



# Iterated Revelation Protocol

# Iterated Revelation Protocol

INITIAL STEP — The decision maker announces $r_0 \in \mathcal{R}$, and shortlists $a_0 \in r_0$.

ITERATIVE STEP — Given $(r_0, \ldots, r_{n-1})$ distinct prior revelations, the expert reveals $r_n \in \mathcal{R}$.

- If $r_{n-1} \subsetneq r_n$, the dm shortlists $a_n \in r_n$, and the ITERATIVE STEP is repeated
- Otherwise, the protocol moves to the FINAL STEP

FINAL STEP — Given $(r_0, \ldots, r_n)$ distinct revelations, the expert chooses an action $a \in \{a_0, \ldots, a_n\}$.

Importantly:

- This protocol can be explained / contracted to without having to express any specific actions/outcomes
- Specifically, the only contractual obligations in an IRP are actions that *have already been* revealed.

# Strategies

Given the IRP, a **strategy**

- for the dm is a function from *sequences of revelations* to actions:

$$s : (r_0 \ldots r_n) \mapsto a_n \in r_n$$

- for the ex is a function from *sequences of shortlisted actions* to revelations:

$$\sigma : (a_0 \ldots a_{n-1}) \mapsto r_n \in \mathcal{R}$$

(and a choice out of the final shortlist)

# Implementation

Let $a(s,\sigma)$ denote the action enacted by playing strategies $s$ and $\sigma$.

Say that $s$ **implements** the selection function $f$ if for all $r \in \mathcal{R}$

$$f(r) = a(s,\sigma) \quad \text{for some best response for type } r$$

and **fully implements** $f$ if

$$f(r) = a(s,\sigma) \quad \text{for every best response for type } r$$

**Theorem**

The following are equivalent for a selection function $f$

(1) $f$ is monotone (resp. strongly monotone)

(2) there exists some $s$ that implements $f$, (resp. fully implements)

# Greedy Strategies & Efficiency

Each shortlist proposal in an IRP specifies:

(1) The outcome should the game end
- ◇ dm wants to maximize his own payoff

(2) The implicit incentive constraint should the game continue
- ◇ dm wants to minimize ex's payoff

In the examples, IRPs solved (1) ignoring (2)

### Definition

Call a strategy $s$ (for the decision maker) **mostly greedy** if for all $(r_0 \dots r_n)$, there is no $a \in r_n$ such that $V_e(s(r_0 \dots r_{n-1})) \leq V_e(a)$ and

$$u_d(a) > u_d(s(r_0 \dots r_n))$$

or such that

$$u_d(a) = u_d(s(r_0 \dots r_n)) \text{ and } u_e(a) > u_e(s(r_0 \dots r_n))$$

A mostly greedy strategy:

- maximizes the dm's payoff myopically (subject to IC constraint)
- does not account for effect on future incentive constraints
- breaks ties in favor of the expert (hence only *mostly* greedy)

### Theorem

Let $s$ be mostly greedy. Then $s$ implements the decision maker's preferred efficient selection function, $f^\star$.

- If $f$ is any other monotone and efficient selection function, then for all $r \supseteq r_d$

$$u_d(f^\star(r)) \geq u_d(f(r))$$

# Comparative Statics: Expressibility

Let $f^r$ denote the efficient, monotone selection function induced by the mostly greedy strategy for type $r$. Then:

- If the decision maker can express more actions ex-ante, he does better
  - $r_d^\dagger \subseteq r_d^\star \subseteq r_e$ implies $u_d(f^{r_d^\dagger}(r_e)) \leq u_d(f^{r_d^\star}(r_e))$
- If the expert can express more actions ex-ante, the decision maker's payoff is unsigned
  - $r_d^\dagger \subseteq r_e \subseteq r_e'$ implies $u_d(f^{r_d^\dagger}(r_e)) \ ?? \ u_d(f^{r_d^\dagger}(r_e'))$

# Comparative Statics: Information Spillover

- Let $\mathcal{R}$ and $\mathcal{Q}$ be two different type spaces over the same set of actions:

$$\mathcal{R} \subseteq \mathcal{Q} \subseteq 2^{\mathcal{A}}$$

- Let $r_d, r_e \in \mathcal{R}$ and $q_d, q_e \in \mathcal{Q}$ be such that

$$r_d = q_d \subseteq r_e = q_e$$

- Then $u_d(f^{q_d}(q_e)) \leq u_d(f^{r_d}(r_e))$

# Comparative Statics: Information Spillover

- In the limit $\mathcal{R} = \{\mathcal{A}\}$ (all actions reveal all other actions)
  - As if dm maximizes subject to individual rationality constraint
- In the limit $\mathcal{R} = 2^{\mathcal{A}}$ (all actions can be revealed independently)
  - As if expert maximizes subject to individual rationality constraint
  - This coincides with the expert preferred efficient selection function
  - Corollary: efficient selection function is unique

# General Strategic Analysis

### Definition

Call a strategy $s$ **greedy** if for all $(r_0 \dots r_n)$, there is no $a \in r_n$ such that

$$V_e(s(r_0 \dots r_{n-1})) \le V_e(a) \quad \text{and} \quad V_d(s(r_0 \dots r_n)) < V_d(a)$$

- There is no way to for the dm to increase his own payoff
- Generalization of mostly greedy strategy

### Theorem

An selection function $f$ is implemented by a greedy $s$

if and only if

for all $r \in \mathcal{R}$, there is no other monotone selection function $f'$ such that

$$\inf_{r' \supseteq r} V_d(f(r')) < \inf_{r' \supseteq r} V_d(f'(r'))$$

## Definition

Call a strategy $s$ **locally rational** if for all $(r_0 \ldots r_n)$, there is no $a \in r_n$ such that

$$V_e(s(r_0 \ldots r_{n-1})) \leq V_e(a) < V_e(s(r_0 \ldots r_n)) \quad \text{and} \quad V_d(s(r_0 \ldots r_n)) < V_d(a)$$

- There is no way to simultaneously for the dm to
  - increase his own payoff
  - decrease the expert's payoff

## Theorem

An selection function $f$ is implemented by a locally rational $s$

if and only if

for all $r \in \mathcal{R}$, there is no other monotone selection function $f'$ such that

$$V_d(f(r')) \leq V_d(f'(r')) \qquad \text{for all } r' \supseteq r,$$
$$V_d(f(r')) < V_d(f'(r')) \qquad \text{for some } r' \supseteq r$$

- ‘if’ direction requires a richness condition on $\mathcal{R}$

# ❦ Payoff Uncertainty ❦

- The implementation above presupposes dm can anticipate ex’s acceptance / rejection
- What happens with private information:
  - Actions are state-dependent $u : \mathcal{A} \times \Omega \to \mathbb{R}$
  - assume ex knows the state, $\omega \in \Omega$
  - dm does not

# Example C

- $\Omega = \{\omega_L, \omega_R\}$, ex knows the state, dm believes equally likely
  - Each action is therefore given by $(\langle x_{d,L}, x_{d,R}\rangle, \langle x_{e,L}, x_{e,R}\rangle)$.
- The dm is initially aware of one action:

$$x = (\langle 0, 0\rangle, \langle 0, 0\rangle)$$

- The ex is also aware of:

$$a_L = (\langle 3, -1\rangle, \langle 3, -1\rangle) \qquad a_R = (\langle -1, 3\rangle, \langle -1, 3\rangle) \qquad b = (\langle 2, 2\rangle, \langle 2, 2\rangle)$$

- The only revelation type is $\{a_L, a_R, b\}$.

# Example C

$$*x = \langle 0, 0\rangle, \langle 0, 0\rangle *$$

# Example C

$$*x = \langle \mathbf{0}, 0\rangle, \langle \mathbf{0}, 0\rangle*$$

$$*x = \langle 0, \mathbf{0}\rangle, \langle 0, \mathbf{0}\rangle*$$

# Example C

$$*x = \langle \mathbf{0}, 0\rangle, \langle \mathbf{0}, 0\rangle *$$

$$*x = \langle 0, \mathbf{0}\rangle, \langle 0, \mathbf{0}\rangle *$$

$$\begin{aligned} x &= (\langle \mathbf{0}, 0\rangle, \langle \mathbf{0}, 0\rangle) \\ b &= (\langle \mathbf{2}, 2\rangle, \langle \mathbf{2}, 2\rangle) \\ a_L &= (\langle \mathbf{3}, -1\rangle, \langle \mathbf{3}, -1\rangle) \\ a_R &= (\langle -\mathbf{1}, 3\rangle, \langle -\mathbf{1}, 3\rangle) \end{aligned}$$

$$\begin{aligned} x &= (\langle 0, \mathbf{0}\rangle, \langle 0, \mathbf{0}\rangle) \\ b &= (\langle 2, \mathbf{2}\rangle, \langle 2, \mathbf{2}\rangle) \\ a_L &= (\langle 3, -\mathbf{1}\rangle, \langle 3, -\mathbf{1}\rangle) \\ a_R &= (\langle -1, \mathbf{3}\rangle, \langle -1, \mathbf{3}\rangle) \end{aligned}$$

# Example C

$$*x = \langle \mathbf{0}, 0\rangle, \langle \mathbf{0}, 0\rangle*$$

$$\downarrow$$

$$\begin{aligned} x &= (\langle \mathbf{0}, 0\rangle, \langle \mathbf{0}, 0\rangle) \\ b &= (\langle \mathbf{2}, 2\rangle, \langle \mathbf{2}, 2\rangle) \\ a_L &= (\langle \mathbf{3}, -1\rangle, \langle \mathbf{3}, -1\rangle) \\ a_R &= (\langle -\mathbf{1}, 3\rangle, \langle -\mathbf{1}, 3\rangle) \end{aligned}$$

$$*x = \langle 0, \mathbf{0}\rangle, \langle 0, \mathbf{0}\rangle*$$

$$\downarrow$$

$$\begin{aligned} x &= (\langle 0, \mathbf{0}\rangle, \langle 0, \mathbf{0}\rangle) \\ b &= (\langle 2, \mathbf{2}\rangle, \langle 2, \mathbf{2}\rangle) \\ a_L &= (\langle 3, -\mathbf{1}\rangle, \langle 3, -\mathbf{1}\rangle) \\ a_R &= (\langle -1, \mathbf{3}\rangle, \langle -1, \mathbf{3}\rangle) \end{aligned}$$

# Example C

$$*x = \langle \mathbf{0}, 0\rangle, \langle \mathbf{0}, 0\rangle*$$

$$\downarrow$$

$$x = (\langle \mathbf{0}, 0\rangle, \langle \mathbf{0}, 0\rangle)$$
$$*\ b = (\langle \mathbf{2}, 2\rangle, \langle \mathbf{2}, 2\rangle)\ *$$
$$a_L = (\langle \mathbf{3}, -1\rangle, \langle \mathbf{3}, -1\rangle)$$
$$a_R = (\langle \mathbf{-1}, 3\rangle, \langle \mathbf{-1}, 3\rangle)$$

$$*x = \langle 0, \mathbf{0}\rangle, \langle 0, \mathbf{0}\rangle*$$

$$\downarrow$$

$$x = (\langle 0, \mathbf{0}\rangle, \langle 0, \mathbf{0}\rangle)$$
$$*\ b = (\langle 2, \mathbf{2}\rangle, \langle 2, \mathbf{2}\rangle)\ *$$
$$a_L = (\langle 3, \mathbf{-1}\rangle, \langle 3, \mathbf{-1}\rangle)$$
$$a_R = (\langle -1, \mathbf{3}\rangle, \langle -1, \mathbf{3}\rangle)$$

# Example C

- Preferences are completely aligned, but IRP does not allow delegation
- the protocol cannot use ex's private info.
  - this *creates* inefficiency
- Instead, dm chooses a **set of actions** $p_1 \subseteq r$. After revelation, propose

$$p_1 = \{a_L, a_R\}$$

and let the ex choose.

- A **generalized IRP** allows the dm to choose a set of actions at each step:
  - At each $(r_0 \ldots r_n)$, $s(r_0 \ldots r_n) \subseteq r_n$

- A **generalized selection function** is a function $f : \Omega \times \mathcal{R} \to \mathcal{A}$
  - For each $r \in \mathcal{R}$, $w \in \Omega$, we have $f(\omega, t) \in t$

# Full Revelation

**Theorem**

The following are equivalent for a gen. selection function $f$

(1) $f$ can be implemented by a gen. IRP

(2) $f$ is monotone: for all $\omega, r \in \Omega \times \mathcal{R}$

$$u_e(f(\omega', r'), \omega) \leq u_e(f(\omega, r), \omega)$$

for any other $\omega' \in \Omega$ and $r' \subseteq r$.

❦ Thank You ❦