B Supplemental Material

Recall the expected utility theory described in Example 1: $(\Delta, \Theta^{\Delta}, \Omega^{\Delta}, \mu)$. We will now recast structural invariance in a domain specific manner, illuminating that the concrete notion of invariance that is relevant to the expected utility model is *translation invariance*.

Consider the experiment $(A, \{P_1, \ldots, P_n\})$ and some other A'; notice that the linearity of each u requires that $u \in W_{A,P_i}$ if and only $u \in W_{\alpha A+(1-\alpha)A', \alpha P_i+(1-\alpha)A'}$. Indeed, there must be some $p \in P_i$ that maximizes u over A and $q \in B$ that maximizes u over B, so that $\alpha p + (1-\alpha)q \in \alpha P_i + (1-\alpha)B$ maximizes u over the mixture. Hence, translating an experiment by mixing both the decision problem and the observability partition with some common A' does not alter which preference sets can be identified. This particular form of invariance is captured by the following axiom.[13]

(B1) *Translation Invariance*. For $A, B \in \mathbb{D}$, we have

$$(A, \{P_1, \ldots, P_n\}) \sim (\alpha A + (1-\alpha)B, \{Q_1, \ldots, Q_n\}),$$

whenever $Q_i \subseteq (\alpha P_i + (1-\alpha)B)$ for all $i \leq n$.

Recall that A3 also implies that the value of identification should not depend on zero probability perturbations. The following axiom reflects this implication.

(B2) *Belief Consistency*. Fix $c \geq 0$, $A \in \mathbb{D}$, and let $\{P_1, P_2, \ldots, P_n\}$ be a partition of A such that $\mu(W_{A,P_1}) = 0$. Then:

$$(A, \{P_1, P_2, \ldots, P_n\}, c) \sim (A, \{P_1 \cup P_2, \ldots, P_n\}, c).$$

These two axioms capture invariance directly in the Expected Utility specialization of our framework:

Theorem 4. *Let $\succcurlyeq$ be defined over Δ. Then $\succcurlyeq$ satisfies* B1 *and* B2 *iff it satisfies* A3

Proof of Theorem 4. Let $(A, \{P_1, \ldots, P_n\})$ and $(B, \{Q_1, \ldots, Q_m\})$ be such that $\{W_{A,P_i}\}_{i \leq n}$ is μ-equivalent to $\{W_{B,Q_i}\}_{i \leq m}$. Furthermore, from Lemma 4, we can assume there are $1 \leq k \leq n$ elements of each partition with positive μ-probability and for each $i \leq k$, $\mu(W_{A,P_i}) = \mu(W_{A,P_i} \cap W_{B,Q_i}) = \mu(W_{B,Q_i})$.

Consider the problem $C = \frac{1}{2}A + \frac{1}{2}B$. For each $i \leq n$, define $R_i \subseteq C$ as $R_i = \{\frac{1}{2}P_i + \frac{1}{2}B\} \cap \text{ext}(C)$. Clearly, we have for each $i \leq n$, $R_i \subseteq \frac{1}{2}P_i + \frac{1}{2}B$; it follows from B1 that $(A, \{P_1, \ldots, P_n\}) \sim (C, \{R_1, \ldots, R_n\})$.

Now for each $i \leq k$, let $R_i' = R_i \cap (\frac{1}{2}P_i + \frac{1}{2}Q_i) \cap \text{ext}(C) = (\frac{1}{2}P_i + \frac{1}{2}Q_i) \cap \text{ext}(C)$. The final equality arises from the fact that each extreme point of C has a unique decomposition as elements of A and B (so that any $x \in (\frac{1}{2}P_i + \frac{1}{2}Q_i) \cap \text{ext}(C)$ was not in R_j for $j < i$).

[13] The reason there is a subset, rather than set equality, in the axiom is that it is possible that $r \in \alpha A + (1-\alpha)A'$ is not a unique mixture of two elements. That is, $r = \alpha p + (1-\alpha)q = \alpha p' + (1-\alpha)q'$ for some $p, p' \in A$ and $q, q' \in A'$. For these elements, the cell of the partition in which they reside is not determined, but, it turns out not to matter. See the appendix for the formal argument.

We claim that $\mu(W_{C,R_i\setminus R'_i}) = 0$. Indeed,

$$\begin{aligned} W_{C,R_i\setminus R'_i} &\subseteq W_{A,P_i} \cap \bigcup_{j\neq i} W_{B,Q_j} \\ &= \bigcup_{j\neq i} (W_{A,P_i} \cap W_{B,Q_j}) \\ &\subseteq \bigcup_{j\neq i} \big((W_{A,Q_i} \cap W_{B,Q_j}) \cup (W_{A,P_i} \setminus W_{B,Q_i})\big) \end{aligned}$$

The claim then follows from the fact that for all $i \neq j$, $W_{A,Q_i} \cap W_{B,Q_j} = \varnothing$ and $\mu(W_{A,P_i} \setminus W_{B,Q_i}) = 0$ (from μ-equivalence).

By repeatedly appealing to B2, we can see that

$$(C, \{R_1, \ldots, R_n\}) \sim (C, \{R'_1, \ldots, R'_k, R^\dagger\}),$$

where $R^\dagger = C \setminus \bigcup_{i\leq k} R'_i$. We make use the fact for $i > k$, $\mu(W_{C,R_i}) = 0$ on account of the fact that $W_{C,R_i} \subseteq W_{A,P_i}$. Thus we have

$$(A, \{P_1, \ldots, P_n\}) \sim (C, \{R_1, \ldots, R_n\}) \sim (C, \{R'_1, \ldots, R'_k, R^\dagger\})$$

A symmetric argument ensures that also $(B, \{Q_1, \ldots, Q_m\}) \sim C, \{R'_1, \ldots, R'_k, R^\dagger\})$, and so the two experiments are indifferent, as is required. ■