(HOW) DO YOU KNOW WHAT I MEAN?

TRANSLATION & IMPLICATION

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Economists like to model uncertainty via state-spaces:

- A set W; each $w \in W$ represents a complete resolution of uncertainty
- Reasoning about states is hard — everything is included in a state
- Awareness complicates the picture

Introspectively, it seems we reason about state*ments* not states.

- This is dependent of language of the decision maker
- Under rationality assumptions, equivalent to the state-space model
- But, what if two agents speak different languages?

This paper:

- Is a theory of translation
- Posits how an agent’s language represents their understanding of uncertainty
- How this might be communicated between agents
- When there is a universal perspective that unifies the different agent’s views

Model: Languages

A **language** is a Boolean Algebra $\mathcal{L}$ representing sets of statements that can be true or false:

- closed under negation $\neg$
  - if λ is a statement in $\mathcal{L}$, then so is $\neg\lambda$
- closed under disjunction $\vee$
  - if λ and η are statements in $\mathcal{L}$, then so is $\lambda \vee \eta$
- We define $\lambda \wedge \eta = \neg(\neg\lambda \vee \neg\eta)$

A **truth assignment** for $\mathcal{L}$ is a function

$$w : \mathcal{L} \to \{0, 1\}$$

such that for all $\lambda, \eta \in L$:

- $w(\neg\lambda) = 1 - w(\lambda)$
- $w(\lambda \vee \eta) = \max\{w(\lambda), w(\eta)\}$
- Therefore, $w(\lambda \wedge \eta) = \min\{w(\lambda), w(\eta)\}$

Then $W(\mathcal{L})$ is the set of all truth assignments for $\mathcal{L}$

EXAMPLE

$\mathcal{L}$ is constructed from the primitive statements

- λ_{cat} =“*Martin* is a cat”
- λ_{dog} =“*Martin* is a dog”
- λ_{mam} =“*Martin* is a mammal”

under the axioms / presumption that:

- $\neg(\lambda_{cat} \wedge \lambda_{dog})$
  - It is not possible to be a dog and a cat
- $\lambda_{mam} \vee (\neg\lambda_{cat} \wedge \neg\lambda_{dog})$
  - Martin is either a mammal or he is not a dog and is not a cat

Example

w_1	w_2	w_3	w_4
λ_{cat}	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$
$\neg\lambda_{dog}$	λ_{dog}	$\neg\lambda_{dog}$	$\neg\lambda_{dog}$
λ_{mam}	λ_{mam}	λ_{mam}	$\neg\lambda_{mam}$

Implication

Say that λ **implies** η (in $\mathcal{L}$) if

$$w(\lambda) \leq w(\eta)$$

for all $w \in W(\mathcal{L})$. We then write:

$$\lambda \Rightarrow_{\mathcal{L}} \eta$$

- A **tautology** is a statement that gets mapped to 1 under every $w \in W(\mathcal{L})$
  - For example: $\lambda \vee \neg\lambda$
- A **contradiction** is a statement that gets mapped to 0 under every $w \in W(\mathcal{L})$
  - For example: $\lambda \wedge \neg\lambda$

$W(\mathcal{L})$ acts as a state-space for $\mathcal{L}$:

- The event corresponding to λ is $\mathbf{v}(\lambda) = \{w \in W \mid w(\lambda) = 1\}$
- Implication is containment: $\lambda \Rightarrow \eta$ if and only if $\mathbf{v}(\lambda) \subseteq \mathbf{v}(\eta)$
- Tautology maps to entire state-space, Contradiction to empty-set

Example

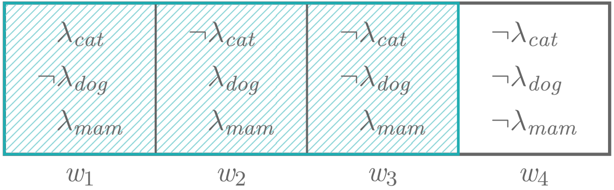
λ_{cat}	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$
$\neg\lambda_{dog}$	λ_{dog}	$\neg\lambda_{dog}$	$\neg\lambda_{dog}$
λ_{mam}	λ_{mam}	λ_{mam}	$\neg\lambda_{mam}$
w_1	w_2	w_3	w_4

Example

w_1	w_2	w_3	w_4
λ_{cat}	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$
$\neg\lambda_{dog}$	λ_{dog}	$\neg\lambda_{dog}$	$\neg\lambda_{dog}$
λ_{mam}	λ_{mam}	λ_{mam}	$\neg\lambda_{mam}$

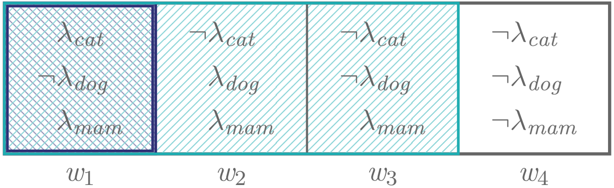
Event $\mathbf{v}(\lambda_{cat})$ = "Martin is a cat"

Example



Event $\mathbf{v}(\lambda_{mam})$ = "Martin is a Mammal"

Example



$\mathbf{v}(\lambda_{cat}) \subseteq \mathbf{v}(\lambda_{mam})$, cat implies mammal

Example

w_1	w_2	w_3	w_4
λ_{cat}	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$	$\neg\lambda_{cat}$
$\neg\lambda_{dog}$	λ_{dog}	$\neg\lambda_{dog}$	$\neg\lambda_{dog}$
λ_{mam}	λ_{mam}	λ_{mam}	$\neg\lambda_{mam}$

Event $\mathbf{v}(\neg(\lambda_{cat} \vee \lambda_{dog}))$ = “Martin is not a cat or a dog”

Model: Translation

There are two agents, 1 and 2, each endowed with a language $\mathcal{L}_1$ and $\mathcal{L}_2$

- let $\Rightarrow_i$ denote implication in i's language
- let t_i and f_i denote the tautology and contradiction

A **translation operator** (from i to j) is a function

$$\mathsf{T}_{i \to j} : \mathcal{L}_i^* \to \mathcal{L}_j^*$$

- $\mathsf{T}_{i \to j}(\lambda)$ is the translation of λ into the language j.
- $\mathcal{L}^* = \mathcal{L} \cup \{*\}$
- We allow $\mathsf{T}_{i \to j}(\lambda) = *$ to indicate that the translation of λ is undefined

We consider two kinds of translations

- Inner Translation operator $\mathrm{T}^{-}_{i \to j}$
  - Provides a *more specific* approximation
- Outer Translation operator $\mathrm{T}^{+}_{i \to j}$
  - Provides a *more general* approximation

Example

Consider a Spanish speaker, $\mathcal{L}_j$, who is never heard of 'mammals'

- η_{gat} ="*Martin* es un gato"
- η_{per} ="*Martin* es un perro"
- η_{ani} ="*Martin* es un animal"

η_{gat}	$\neg\eta_{gat}$	$\neg\eta_{gat}$	$\neg\eta_{gat}$
$\neg\eta_{per}$	η_{per}	$\neg\eta_{per}$	$\neg\eta_{per}$
η_{ani}	η_{ani}	η_{ani}	$\neg\eta_{ani}$
w'_1	w'_2	w'_3	w'_4

We could translation λ_{cat} = “Martin is a cat” from $i \to j$:

$$\mathsf{T}^{-}_{i \to j}(\lambda_{cat}) = \eta_{gat} = \mathsf{T}^{+}_{i \to j}(\lambda_{cat})$$

- There is no gap between $\mathsf{T}^{-}_{i \to j}$ and $\mathsf{T}^{+}_{i \to j}$
- This is a ‘perfect’ translation

How then should we translation λ_{mam} = "Martin is a mammal" from $i \to j$?

- j has never heard of a mammal, there is no statement in his language that captures it exactly
- All cats and dogs are mammals so $(\eta_{gat} \vee \eta_{per})$ is more specific
  - $\mathsf{T}^{-}_{i \to j}(\lambda_{mam}) = (\eta_{gat} \vee \eta_{per})$
- All mammals are animals so η_{ani} is a more general
  - $\mathsf{T}^{+}_{i \to j}(\lambda_{mam}) = \eta_{ani}$

Example

$\mathcal{L}_i$	$\mathcal{L}_j$
λ_{egg} =“*Tonya* lays eggs”	η_{huev} =“*Tonya* pone huevos”
λ_{mam} =“*Tonya* is a mammal”	η_{mam} =“*Tonya* es un mamífero”
λ_{plat} =“*Tonya* is a platypus”	

- i is aware of platypus, so $(\lambda_{egg} \wedge \lambda_{mam}) \Rightarrow_i \lambda_{plat}$
- j is not aware of platypus, so $(\eta_{huev} \wedge \eta_{mam}) \Rightarrow_j f_j$

Example

How should we translate λ_{mam} from $i \to j$?

- η_{mam} is more specific than λ_{mam}, so $\mathsf{T}^{-}_{i\to j}(\lambda_{mam}) = \eta_{mam}$ makes sense
- But there is *no* element in $\mathcal{L}_j$ more general
  - Nothing in j's language allows for platypus
- So, we can set $\mathsf{T}^{+}_{i\to j}(\lambda_{mam}) = *$

In the example, we use exogenously impose what is more specific / general:

- i.e., took for granted that gato $\Rightarrow$ mammal, etc
  - Where do they come from?
- What if we just observe the operators $\mathsf{T} = \langle \mathsf{T}^{-}_{i\rightarrow j}, \mathsf{T}^{+}_{i\rightarrow j}, \mathsf{T}^{-}_{j\rightarrow i}, \mathsf{T}^{+}_{j\rightarrow i} \rangle$
  - When does T behave like a translation?

❦ Abstract Translation ❦

We posit two axioms that T should satisfy

C1: Galois

For all $\lambda \in \mathcal{L}_i^*$ and $\eta \in \mathcal{L}_j^*$:

$$\eta \Rightarrow_j \mathsf{T}^{-}_{i\to j}(\lambda) \qquad \text{if and only if} \qquad \mathsf{T}^{+}_{j\to i}(\eta) \Rightarrow_i \lambda.$$

- If η is more specific than the $\mathsf{T}^{-}_{i\to j}(\lambda)$...
  - η_{gat} was more specific than $\mathsf{T}^{-}_{i\to j}(\lambda_{mam}) = (\eta_{gat} \vee \eta_{per})$
- Then λ is more general than $\mathsf{T}^{+}_{j\to i}(\eta)$.
  - λ_{mam} was more general than $\mathsf{T}^{+}_{j\to i}(\eta_{gat}) = (\lambda_{cat})$
- `C1` is abstract but provides a lot of structure

T1. T^- and T^+ *preserve contradiction*:

$$\mathsf{T}^-_{i\to j}(f_i) = \mathsf{T}^+_{i\to j}(f_i) = f_j$$

T2. T^- and T^+ *preserves implication*:

$$\lambda \Rightarrow_i \lambda' \text{ implies } \begin{cases} \mathsf{T}^-_{i\to j}(\lambda) \Rightarrow_j \mathsf{T}^-_{i\to j}(\lambda') \text{ and} \\ \mathsf{T}^+_{i\to j}(\lambda) \Rightarrow_j \mathsf{T}^+_{i\to j}(\lambda') \end{cases}$$

T3. $\mathsf{T}^-_{i\to j}$ *preserves conjunction*:

$$\mathsf{T}^-_{i\to j}(\lambda \wedge \lambda') = \mathsf{T}^-_{i\to j}(\lambda) \wedge \mathsf{T}^-_{i\to j}(\lambda')$$

T4. $\mathsf{T}^+_{i\to j}$ *preserves disjunction*:

$$\mathsf{T}^+_{i\to j}(\lambda \vee \lambda') = \mathsf{T}^+_{i\to j}(\lambda) \vee \mathsf{T}^+_{i\to j}(\lambda')$$

C2: Approximation

For all $\lambda_i \in \mathcal{L}_i^*$ we have $\mathsf{T}^-_{i\to j}(\lambda_i) \Rightarrow_j \mathsf{T}^+_{i\to j}(\lambda_i)$.

- The inner translation should be more specific than the outer translation

❦ Cross Language Implication ❦

Cross Language Implication

Consider a binary relation $\Rightarrow^{\star}$ over $\mathcal{L}_i \cup \mathcal{L}_j$

- represents when one statement implies another, *across* languages
- this is, in principle, able to arise naturally
- further is is observable to some outside modeler

Cross Language Implication

- j is trying to ascertain whether *perro* (dog) implies *mammal*.
    - j could point to various *perros*
    - i affirm that these are all also *mammals*
    - exhibits the implication holds
- now i is trying to ascertain whether *mammal* implies *perro*:
    - i could point to various *mammals*
    - when pointing at a *cat*, j can deny that it is a *mammal*
    - refutes the implication holds

We posit four axioms that $\Rightarrow^\star$ should satisfy

I1: Within Language Consistency

For all $\lambda_i, \lambda_i' \in \mathcal{L}_i$:

$$\lambda_i \Rightarrow^\star \lambda_i' \qquad \text{if and only if} \qquad \lambda_i \Rightarrow_i \lambda_i'.$$

I2: Transitivity

$\Rightarrow^\star$ is transitive

I3: Connective Consistency

Let $\eta_j, \eta'_j \in \mathcal{L}_j$ and $\lambda_i \in \mathcal{L}_j$. Then:

(i) $\lambda_i \Rightarrow^\star \eta_j$ and $\lambda_i \Rightarrow^\star \eta'_j$ implies $\lambda_i \Rightarrow^\star (\eta_j \wedge \eta'_j)$

(ii) $\eta_j \Rightarrow^\star \lambda_i$ and $\eta'_j \Rightarrow^\star \lambda_i$ implies $(\eta_j \wedge \eta'_j) \Rightarrow^\star \lambda_i$

I4: Principle of Explosion

For all $\lambda_j \in \mathcal{L}_j$,

$$f_i \Rightarrow^{\star} \lambda_j$$

Theorem

The following are equivalent:

(1) T satisfies C1 and C2

(2) There exists some $\Rightarrow^\star$ satisfying I1—I4 such that

$$\mathsf{T}^{-}_{i\to j}(\lambda_i) = \bigvee\{\eta_j \in \mathcal{L}_j \mid \eta_j \Rightarrow^\star \lambda_i\}, \quad \text{and}$$
$$\mathsf{T}^{+}_{i\to j}(\lambda_i) = \bigwedge\{\eta_j \in \mathcal{L}_j \mid \lambda_i \Rightarrow^\star \eta_j\},$$

- We define $\bigwedge \varnothing = *$

- $W(\mathcal{L}_i)$ and $W(\mathcal{L}_j)$ act as ‘local’ state-spaces
- If $W^\star$ is some ‘global’ state-space that nests both:
  - Then translation operators appear as inner and outer approximation
  - As in measure theory, etc

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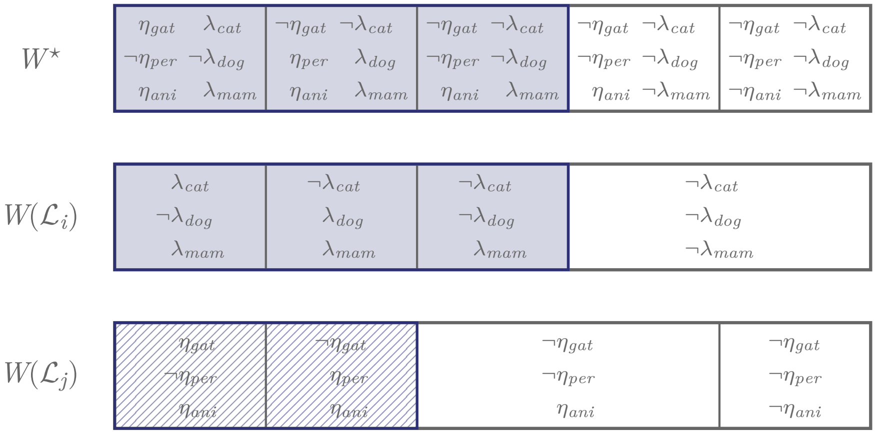
$W(\mathcal{L}_j)$	η_{gat} $\neg\eta_{per}$ η_{ani}	$\neg\eta_{gat}$ η_{per} η_{ani}	$\neg\eta_{gat}$ $\neg\eta_{per}$ η_{ani}	$\neg\eta_{gat}$ $\neg\eta_{per}$ $\neg\eta_{ani}$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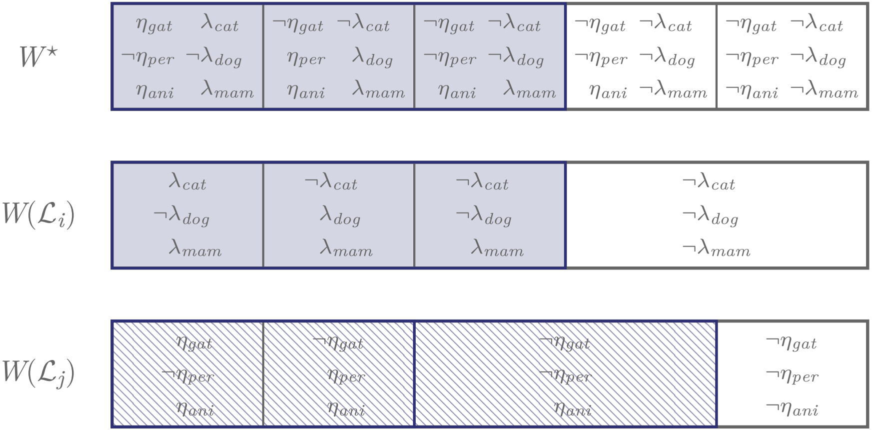
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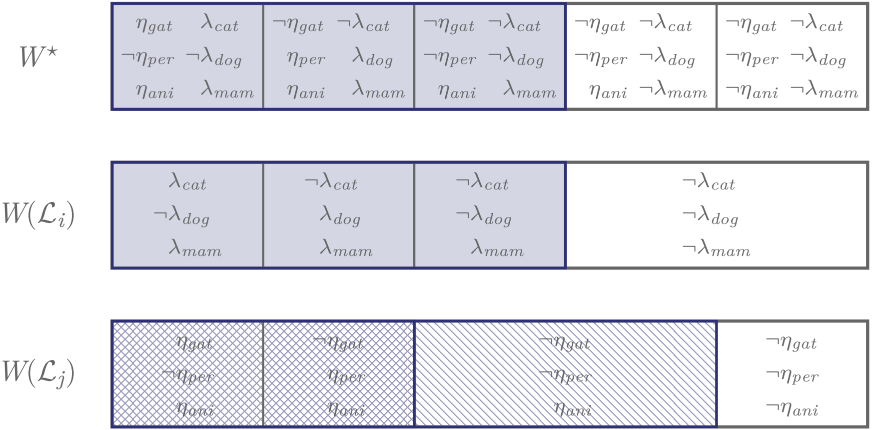
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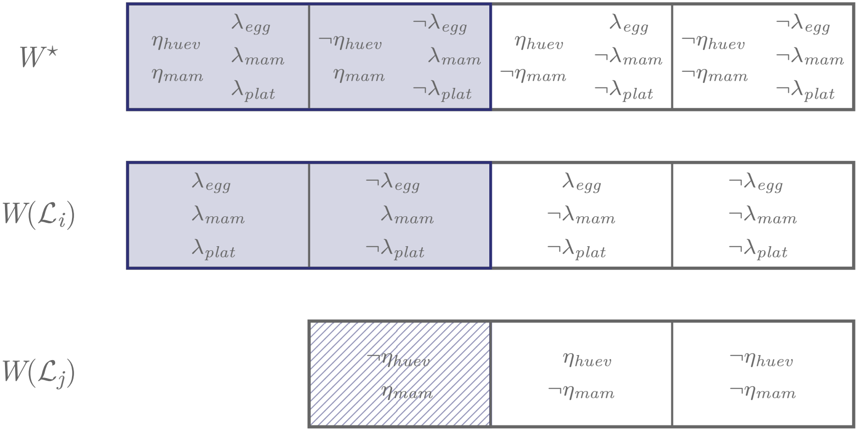
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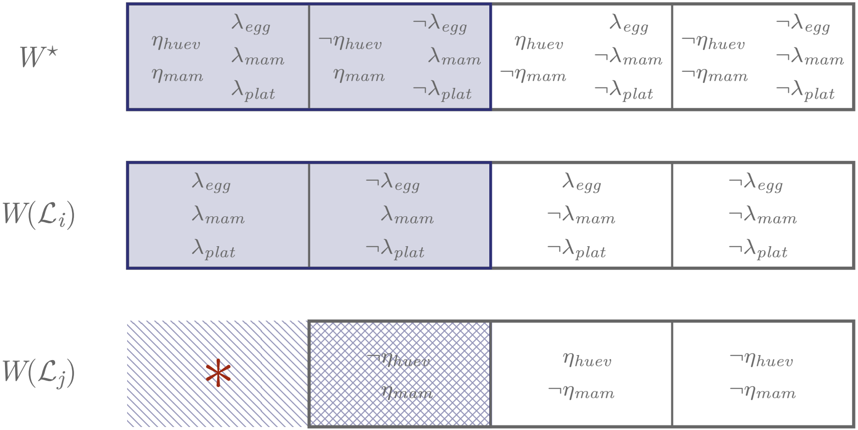
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$\neg\eta_{huev}$ $\neg\eta_{mam}$ $\neg\lambda_{egg}$ $\neg\lambda_{mam}$ $\neg\lambda_{plat}$
$W(\mathcal{L}_i)$
λ_{egg} λ_{mam} λ_{plat}
$\neg\lambda_{egg}$ λ_{mam} $\neg\lambda_{plat}$
λ_{egg} $\neg\lambda_{mam}$ $\neg\lambda_{plat}$
$\neg\lambda_{egg}$ $\neg\lambda_{mam}$ $\neg\lambda_{plat}$
$W(\mathcal{L}_j)$
$\neg\eta_{huev}$ η_{mam}
η_{huev} $\neg\eta_{mam}$
$\neg\eta_{huev}$ $\neg\eta_{mam}$

Does such a state-space $W^\star$ always exist?

Example

$\mathcal{L}_i$	$\mathcal{L}_j$
$\lambda =$ "God exists"	$\eta =$ "Dios es bueno"
$\neg\lambda =$ "God does not exist"	$\neg\eta =$ "Dios es malvado"

- i defines God as benevolent, but is unsure God exists
  - i cannot conceive of an evil God
- j defines God as what is in the universe, but is unsure of the moral character God
  - j cannot conceive God that does not exist

Example

$\mathcal{L}_i$	$\mathcal{L}_j$
$\lambda =$ "God exists"	$\eta =$ "Dios es bueno"
$\neg\lambda =$ "God does not exist"	$\neg\eta =$ "Dios es malvado"

$$\mathsf{T}^{-}_{i\to j}(f_i) = f_j \qquad \mathsf{T}^{+}_{i\to j}(f_i) = f_j$$

$$\mathsf{T}^{-}_{i\to j}(\lambda) = \eta \qquad \mathsf{T}^{+}_{i\to j}(\lambda) = \eta$$

$$\mathsf{T}^{-}_{i\to j}(\neg\lambda) = f_j \qquad \mathsf{T}^{+}_{i\to j}(\neg\lambda) = t_j$$

$$\mathsf{T}^{-}_{i\to j}(t_i) = t_j \qquad \mathsf{T}^{+}_{i\to j}(t_i) = t_j$$

This satisfies our axioms but admits no state-space representation:

- That $\mathsf{T}^{-}_{i\to j}(t_i) = t_j = \mathsf{T}^{+}_{i\to j}(t_i)$ requires that t_i and t_j map to the same event
- That $\mathsf{T}^{-}_{i\to j}(\lambda) = \eta = \mathsf{T}^{+}_{i\to j}(\lambda)$ requires that λ and η map to the same event
- Then, preservation of $\neg$ as complementarities requires that $\neg\lambda$ and $\neg\eta$ map to the same event
- But this last requirement does not hold!

If $\mathsf{T}^{-}_{i\to j}(t_i) = t_j = \mathsf{T}^{+}_{i\to j}(t_i)$:

- i.e., aware of the same states
- then a state-space exists iff and only if

C3: Duality

$$\mathsf{T}^{-}_{i\to j}(\neg\lambda_i) = \neg\mathsf{T}^{+}_{i\to j}(\lambda_i)$$

- In the example: $\mathsf{T}^{-}_{i\to j}(\neg\lambda) = f_j \neq \neg\eta = \neg\mathsf{T}^{+}_{i\to j}(\lambda)$

For $\lambda_i \in \mathcal{L}_i$ and $\eta_j \in \mathcal{L}_j$ such that $\lambda_i \Rightarrow^\star t_j$ (or $\lambda_i \Rightarrow_i \mathsf{T}^-_{j\to i}(t_j)$ or $\mathsf{T}^+_{i\to j}(\lambda_i) \neq *$):

C3: Strong (Inner) Consistency

$$\eta_j \Rightarrow_j \mathsf{T}^-_{i\to j}(\neg\lambda_i) \qquad \text{implies} \qquad \lambda_i \Rightarrow_i \mathsf{T}^-_{j\to i}(\neg\eta_j).$$

C3: Restricted Duality

$$\mathsf{T}^-_{i\to j}(\neg\lambda_i) = \neg\mathsf{T}^+_{i\to j}(\lambda_i) \wedge \mathsf{T}^-_{i\to j}(t_i).$$

C3: Negation Consistency

$$\eta_j \Rightarrow^\star \neg\lambda_i \qquad \text{implies} \qquad \lambda_i \Rightarrow^\star \neg\eta_j.$$

Theorem

A translation T satisfies C1 and C2 any (hence all) version of C3 if and only if there exists a joint state-space that represents both $\mathcal{L}_i$ and $\mathcal{L}_j$.

- there exists $\mathbf{v}_i : \mathcal{L}_i \to 2^W$ and $\mathbf{v}_j : \mathcal{L}_j \to 2^W$ are ‘locally’ Boolean

❦ Thank You ❦