Designing, Building, and Testing Novel Metallophones

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Objective:

To design and build musical instruments with novel acoustic properties

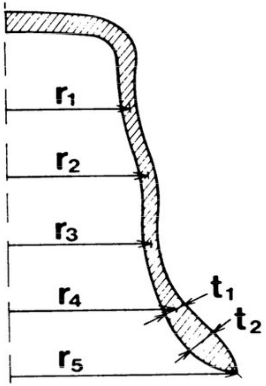
Overview

- Designed instrument using simulations
  - The **F**inite **E**lement **M**ethod **(FEM)**
- Built an apparatus to image its modes
  - **E**lectronic **S**peckle **P**attern **I**nterferometry **(ESPI)**

Comparison of methods

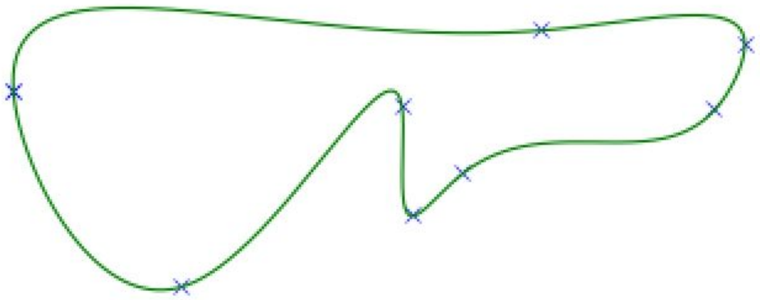
Schoofs et. al:

- Rotation, parametrized by thickness
- Modal frequencies from FEM
- Optimized using simplex method



Peairs:

- Flat, parametrized by splines
- Modal frequencies from FEM
- Best optimization method TBD



Schoofs, A., F. Van Asperen, P. Maas, and A. Lehr. “I. Computation of Bell Profiles Using Structural Optimization.” *Music Perception: An Interdisciplinary Journal* 4, no. 3 (April 1987): 245–54. doi:10.2307/40285368.

FEM for Modal Analysis

$$m\ddot{x} + kx = 0$$

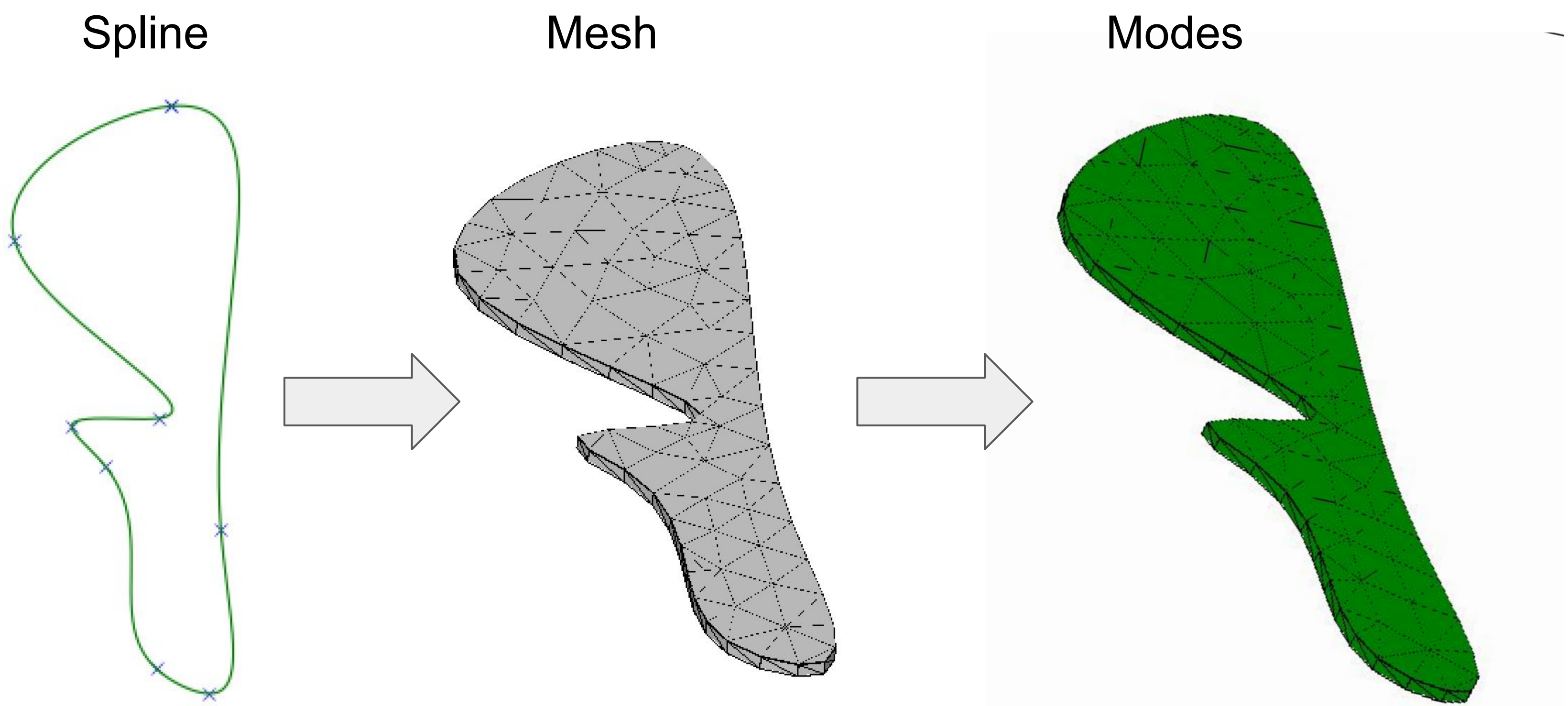
$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0$$

$$\mathbf{x} = \bar{\mathbf{x}}e^{i\omega t} \implies \ddot{\mathbf{x}} = -\omega^2\bar{\mathbf{x}}$$

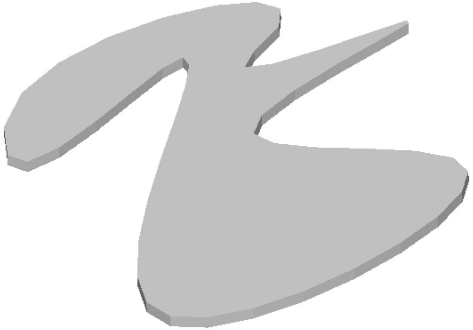
$$|-\omega^2\mathbf{M} + \mathbf{K}| = 0$$

Zienkiewicz, O. C., and Robert L. Taylor. *The Finite Element Method*. 5th ed. Oxford ; Boston: Butterworth-Heinemann, 2000.

FEM for Modal Analysis

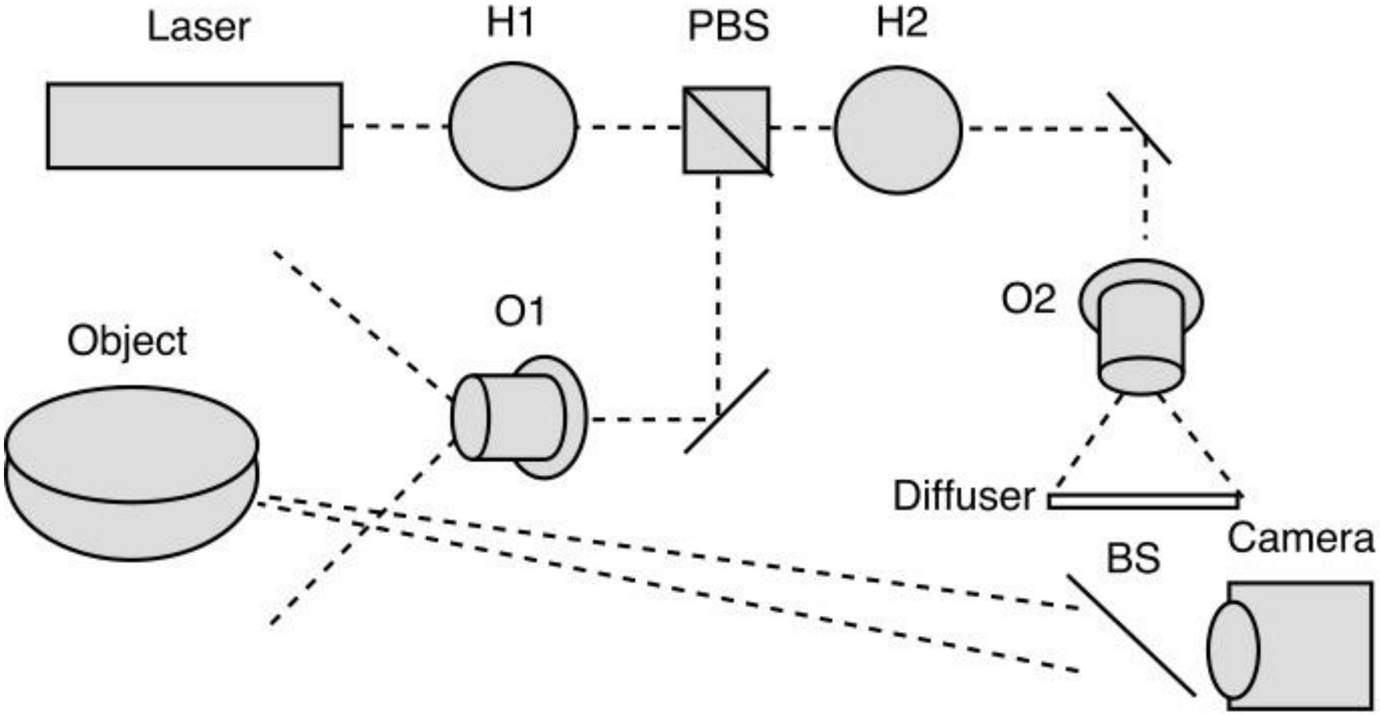


Results: Simplex Method for Bell (Computational)



Note	Ideal Freq (f/f_0)	Simulated Freq (f/f_0)	Percent Error
Hum	0.5	0.482	3.60
Fundamental	1.0	1.000	0
Minor Third	1.2	1.209	0.75
Fifth	1.5	1.504	0.20
Octave	2.0	2.001	0.05

ESPI

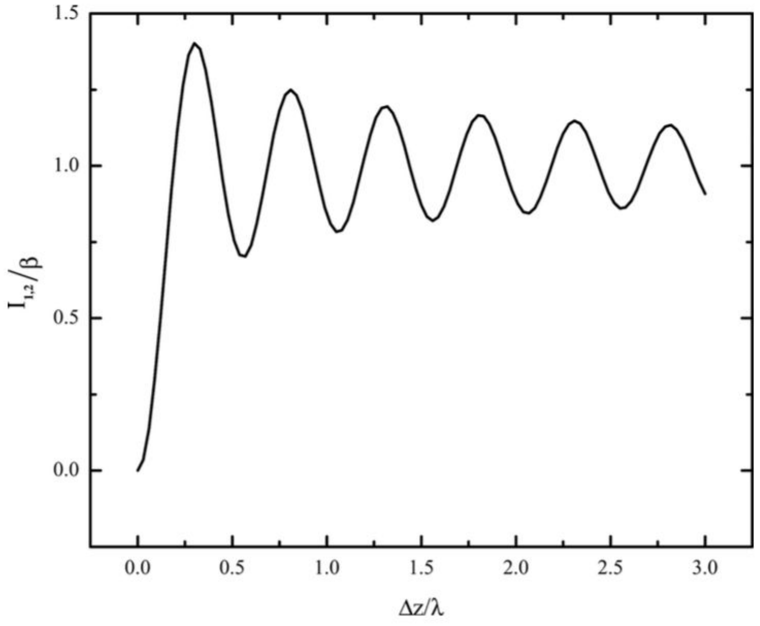


ESPI

$$\langle I \rangle = I_{\text{obj}} + I_{\text{ref}} + 2\sqrt{I_{\text{obj}} I_{\text{ref}}} \langle \cos(\phi_0 + \xi \sin \omega_0 t) \rangle$$
$$= I_{\text{obj}} + I_{\text{ref}} + 2\sqrt{I_{\text{obj}} I_{\text{ref}}} \cos(\phi_0) J_0(\xi)$$

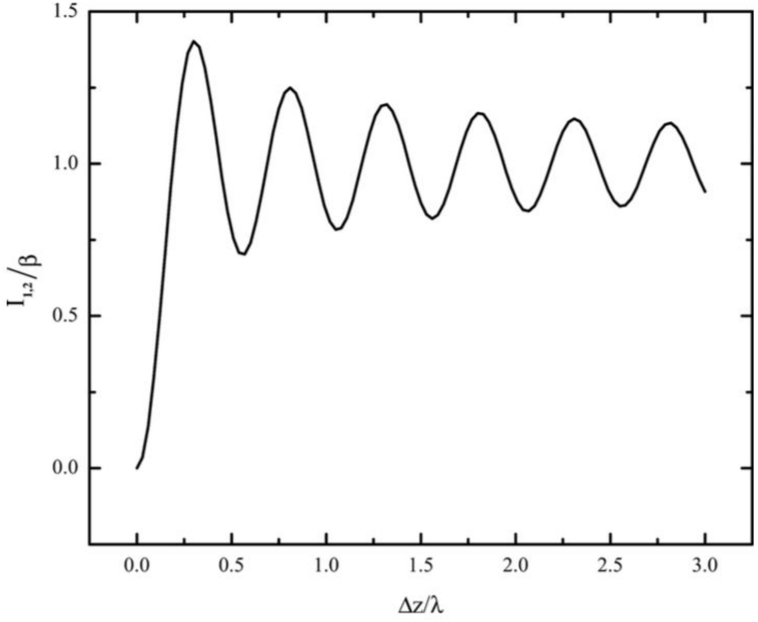
$$\Delta I(\xi) \equiv |\langle I \rangle(0) - \langle I \rangle(\xi)|$$
$$= 2\sqrt{I_{\text{obj}} I_{\text{ref}}} \cos(\phi_0) |1 - J_0(\xi)|$$
$$= \beta |1 - J_0(\xi)|$$

$$\beta |1 - J_0(\xi)|$$

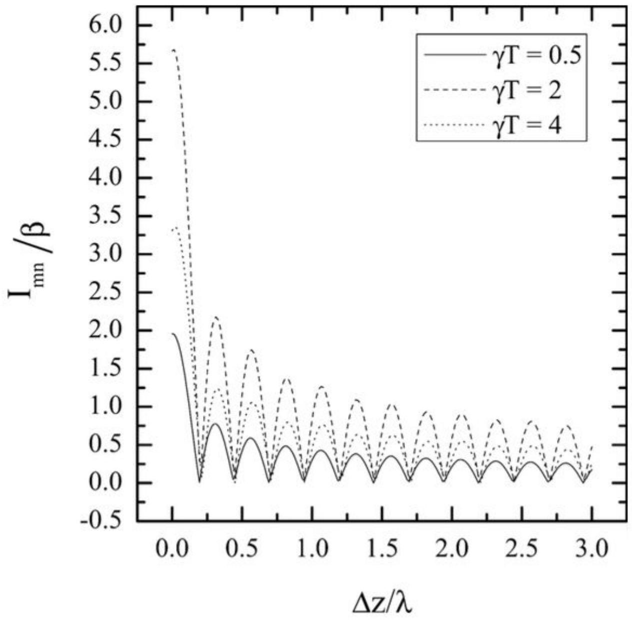


ESPI

$$\beta|1 - J_0(\xi)|$$



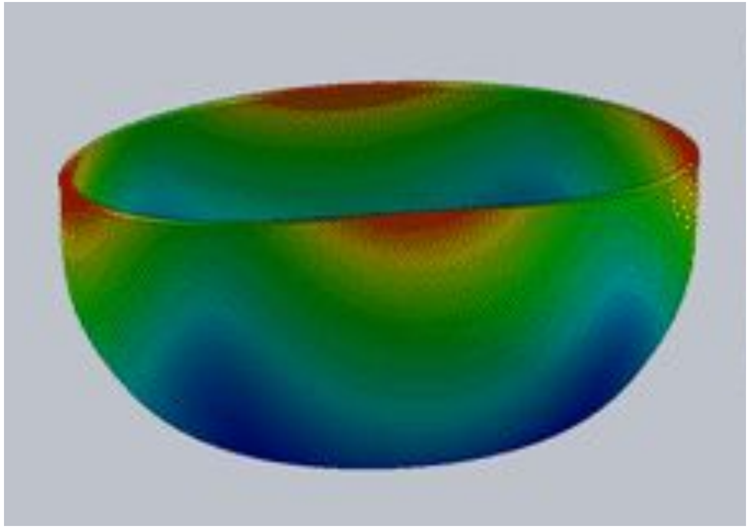
$$\beta \int_0^{2\pi} \left| \frac{1}{T} \int_0^1 \cos[\phi_m + \gamma T\tau + \xi \sin\{\omega_0 T(\tau)\}]\mathrm{d}\tau - \frac{1}{T}\int_0^1 \cos[\phi_m + \gamma T(1+\tau) + \xi \sin\{\omega_0 T(1+\tau)\}]\mathrm{d}\tau \right| \mathrm{d}\phi_m.$$



Moore, Thomas R., and Jacob J. Skubal. “Time-Averaged Electronic Speckle Pattern Interferometry in the Presence of Ambient Motion. Part I. Theory and Experiments.” *Applied Optics* 47, no. 25 (2008): 4640–48.

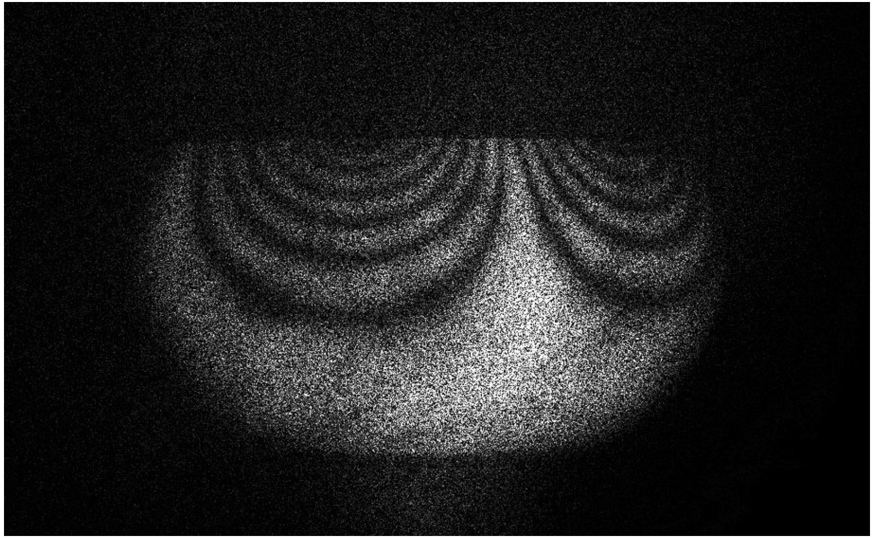
Comparing Results

FEM



506 Hz < freq < 580 Hz

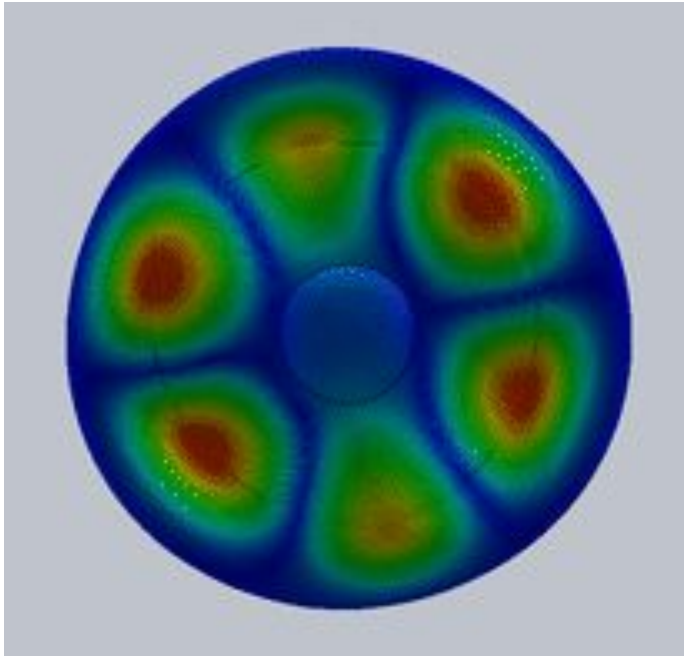
ESPI



freq = 514 Hz

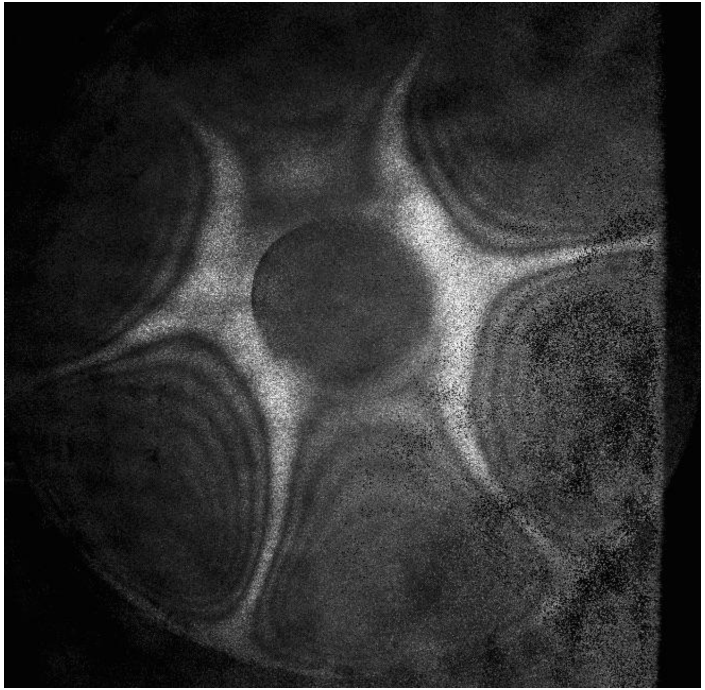
Comparing Results

FEM



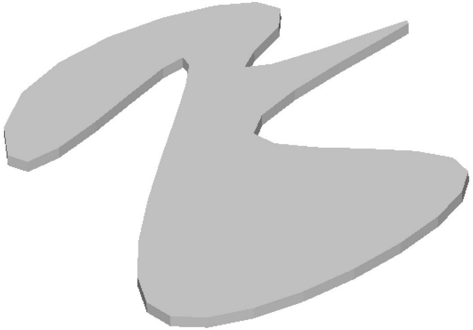
800 Hz < freq. < 850Hz

ESPI

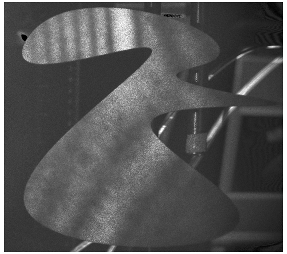


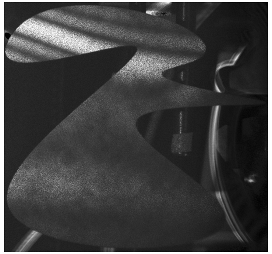
freq = 643.2

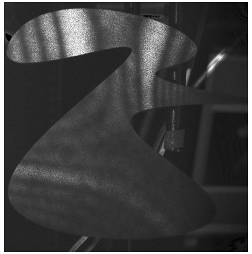
Results: Simplex Method for Bell (Experimental)

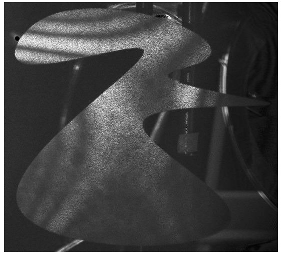


Note	Ideal Freq (f/f_0)	Measured Freq (f/f_0)	Percent Error
Hum	0.5	0.464	7.6
Fundamental	1.0	1.000	0.0
Minor Third	1.2	1.195	0.4
Fifth	1.5	1.432	4.7
Octave	2.0	2.050	2.4









Results: Comparing Simulation and Experiment

Remaining work

- make bells with other sounds
- try other optimization algorithms
- include relative loudness
- pick starting boundary

Acknowledgements

- John Essick
- Rand Worland (UPS)
- Jay