ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/43500603

The EM Algorithm and Extensions

Article in Biometrics · March 1998

DOI: 10.2307/2534032 · Source: OAI

CITATIONS

READS

3,263

3,411

2 authors, including:



G. J. Mclachlan

University of Queensland

308 PUBLICATIONS 19,747 CITATIONS

SEE PROFILE

Some of the authors of this publication are also working on these related projects:



Clustering View project

All content following this page was uploaded by G. J. Mclachlan on 07 March 2017.

The user has requested enhancement of the downloaded file.

The EM Algorithm and Extensions

GEOFFREY J. McLACHLAN
THRIYAMBAKAM KRISHNAN



Contents

Preface			xiii	
1.	General Introduction			1
	1.1	Introd	luction, 1	
	1.2	Maxir	num Likelihood Estimation, 3	
	1.3	1.3 Newton-Type Methods, 5		
		1.3.1	•	
		1.3.2	·	
		1.3.3	Quasi-Newton Methods, 6	
			Modified Newton Methods, 7	
	1.4	Introd	luctory Examples, 9	
		1.4.1	Introduction, 9	
		1.4.2	Example 1.1: A Multinomial Example, 9	
		1.4.3	Example 1.2: Estimation of Mixing Proportions, 16	
	1.5 Formulation of the EM Algorithm, 21		ulation of the EM Algorithm, 21	
		1.5.1	EM Algorithm, 21	
		1.5.2		
			Survival Times, 23	
		1.5.3	E- and M-Steps for Regular Exponential Family, 26	
		1.5.4	Example 1.4: Censored Exponentially Distributed Survival	
			Times (Example 1.3 Continued), 27	
		1.5.5	5 ,	
		1.5.6	• • • • • • • • • • • • • • • • • • • •	
		1.5.7	<i>y</i>	
		1.5.8	11 6,	
	1.6		lgorithm for Maximum a Posteriori and Maximum Penalized	
	Likelihood Estimation, 30		hood Estimation, 30	
		1.6.1	Maximum a Posteriori Estimation, 30	
		1.6.2	Example 1.5: A Multinomial Example (Example 1.1,	
			Continued), 31	

viii Contents

	1.6.3	Maximum Penalized Likelihood Estimation, 32			
1.7	Brief S	summary of the Properties of the EM Algorithm, 32			
1.8	History	y of the EM Algorithm, 34			
	1.8.1 1.8.2	Work Before Dempster, Laird, and Rubin (1977), 34 EM Examples and Applications Since Dempster, Laird, and Rubin (1977), 36			
	1.8.3 1.8.4	Two Interpretations of EM, 37 Developments in EM Theory, Methodology, and Applications, 38			
1.9	Overv	iew of the Book, 41			
1.10	Nota	tion, 42			
Exan	nples of	f the EM Algorithm	45		
2.1	Introdu	action, 45			
2.2	Multiv	ariate Data with Missing Values, 45			
		Example 2.1: Bivariate Normal Data with Missing Values, 45 Numerical Illustration, 49 Multivariate Data: Buck's Method, 50			
2.3	Least Squares with Missing Data, 51				
		Healy-Westmacott Procedure, 51			
		Example 2.2: Linear Regression with Missing Dependent Values, 52			
	2.3.3 2.3.4	Example 2.3: Missing Values in a Latin Square Design, 54 Healy-Westmacott Procedure as an EM Algorithm, 55			
2.4	Examp	le 2.4: Multinomial with Complex Cell Structure, 57			
2.5	Examp	le 2.5: Analysis of PET and SPECT Data, 60			
2.6	Examp Freedo	le 2.6: Multivariate <i>t</i> -Distribution with Known Degrees of m, 63			
	2.6.1 2.6.2	ML Estimation of Multivariate <i>t</i> -Distribution, 63 Numerical Example: Stack Loss Data, 67			
2.7	Finite 1	Normal Mixtures, 68			
	2.7.1 2.7.2 2.7.3	Example 2.7: Univariate Component Densities, 68 Example 2.8: Multivariate Component Densities, -71 Numerical Example: Red Blood Cell Volume Data, 72			
2.8	Example 2.9: Grouped and Truncated Data, 74				
	2.8.1 2.8.2 2.8.3	Introduction, 74 Specification of Complete Data, 74 E-Step, 77			
	2.8.4 2.8.5	M-Step, 78 Confirmation of Incomplete-Data Score Statistic, 78			

2.

Contents ix

	2.8.6 2.8.7		tep for Grouped Normal Data, 79 nerical Example: Grouped Log Normal Data, 80	
3.	Basi	sic Theory of the EM Algorithm 8		
	3.1	Introdu	action, 82	
	3.2	Monot	onicity of the EM Algorithm, 82	
	3.3	Monot	onicity of a Generalized EM Algorithm, 84	
	3.4	Conve	rgence of an EM Sequence to a Stationary Value, 84	
		3.4.1	Introduction, 84	
		3.4.2	Regularity Conditions of Wu, 86	
		3.4.3 3.4.4	Main Convergence Theorem for a Generalized EM Sequence, A Convergence Theorem for an EM Sequence, 88	87
	3.5.	Conve	ergence of an EM Sequence of Iterates, 89	
		3.5.1	Introduction, 89	
		3.5.2	Two Convergence Theorems of Wu, 89	
		3.5.3	Convergence of an EM Sequence to a Unique Maximum Likelihood Estimate, 91	
	3.6	Examp	oles of Nontypical Behavior of an EM (GEM) Sequence, 91	
		3.6.1	Example 3.1: Convergence to a Saddle Point, 91	
			Example 3.2: Convergence to a Local Minimum, 94	
		3.6.3	Example 3.3: Nonconvergence of a Generalized EM Sequence, 97	
	3.7	Score S	Statistic, 99	
	3.8	8 Missing Information, 100		
		3.8.1 3.8.2	Missing Information Principle, 100 Example 3.4 (Example 1.3 Continued), 102	
	3.9	Rate of	f Convergence of the EM Algorithm, 105	
		3.9.1	Rate Matrix for Linear Convergence, 105	
		3.9.2	Measuring the Linear Rate of Convergence, 106	
		3.9.3	Rate Matrix in Terms of Information Matrices, 106	
		3.9.4	Rate Matrix for Maximum a Posteriori Estimation, 108	100
		3.9.5	Derivation of Rate Matrix in Terms of Information Matrices,	108
		3.9.6	Example 3.5 (Example 3.4 Continued), 109	
١.	Stan	dard E	rrors and Speeding Up Convergence	110
	4.1		action, 110	
	4.2	Observ	red Information Matrix, 111	
		4.2.1	Direct Evaluation, 111	
		4.2.2	Extraction of Observed Information Matrix in Terms of the Complete-Data Log Likelihood, 111	
		4.2.3	Regular Case, 113	

Contents X

5.1

Introduction, 166

5.

	4.2.4	Evaluation of the Conditional Expected Complete-Data	
		Information Matrix, 114	
	4.2.5	Examples, 115	
4.3	Appro	ximations to the Observed Information Matrix: i.i.d. Case, 12	0
4.4	Observ	ed Information Matrix for Grouped Data, 123	
	4.4.1	Approximation Based on Empirical Information, 123	
	4.4.2	Example 4.3: Grouped Data from an Exponential	
		Distribution, 125	
4.5		mented EM Algorithm, 128	
	4.5.1	Definition, 128	
	4.5.2 4.5.3	• ,	
	4.5.4		
	4.5.5		
	4.5.6		36
4.6	Bootsti	rap Approach to Standard Error Approximation, 139	
4.7	Accele	ration of the EM Algorithm via Aitken's Method, 141	
	4.7.1	Aitken's Acceleration Method, 141	
	4.7.2	•	
	4.7.3	•	
	4.7.4 4.7.5	Example 4.7: Geometric Mixture, 143 Grouped and Truncated Data (Example 2.8 Continued), 147	
4.8		ken Acceleration-Based Stopping Criterion, 148	
4 .8	•••		
٦.۶	4.9.1	Conjugate Gradient Method, 149	
	4.9.2	A Generalized Conjugate Gradient Algorithm, 150	
	4.9.3	Accelerating the EM Algorithm, 151	
4.10	Hybri	d Methods for Finding Maximum Likelihood Estimate, 152	
	4.10.1	Introduction, 152	
	4.10.2	Combined EM and Modified Newton-Raphson Algorithm,	152
4.11		neralized EM Algorithm Based on One	
		on-Raphson Step, 154	
	4.11.1	Derivation of a Condition to be a Generalized EM	
	4.11.2	Sequence, 154 Simulation Experiment, 155	
4.12		tradient Algorithm, 156	
4.13		asi-Newton Acceleration of the EM Algorithm, 158	
7.13	4.13.1	The Method, 158	
	4.13.1	Example 4.8: Dirichlet Distribution, 161	
		1	
Extensions of the EM Algorithm 166			

Contents xi

- 5.2 ECM Algorithm, 167
 - 5.2.1 Motivation, 167
 - 5.2.2 Formal Definition, 167
 - 5.2.3 Convergence Properties, 169
 - 5.2.4 Speed of Convergence, 170
 - 5.2.5 Discussion, 171
- 5.3 Multicycle ECM Algorithm, 171
- 5.4 Example 5.1: Normal Mixtures with Equal Correlations, 173
 - 5.4.1 Normal Components with Equal Correlations, 173
 - 5.4.2 Application of ECM Algorithm, 173
 - 5.4.3 Fisher's Iris Data, 175
- 5.5 Example 5.2: Mixture Models for Survival Data, 175
 - 5.5.1 Competing Risks in Survival Analysis, 175
 - 5.5.2 A Two-Component Mixture Regression Model, 176
 - 5.5.3 Observed Data, 177
 - 5.5.4 Application of EM Algorithm, 178
 - 5.5.5 M-Step for Gompertz Components, 179
 - 5.5.6 Application of a Multicycle ECM Algorithm, 180
- 5.6 Example 5.3: Contingency Tables with Incomplete Data, 181
- 5.7 ECME Algorithm, 183
- 5.8 Example 5.4: Maximum Likelihood Estimation of *t*-Distribution with Unknown Degrees of Freedom, 184
 - 5.8.1 Application of EM Algorithm, 184,
 - 5.8.2 M-Step, 185
 - 5.8.3 Application of the ECM Algorithm, 185
 - 5.8.4 Application of ECME Algorithm, 186
 - 5.8.5 Some Standard Results, 187
 - 5.8.6 Missing Data, 188
 - 5.8.7 Numerical Examples, 190
 - 5.8.8 Theoretical Results on the Rate of Convergence, 191
- 5.9 Example 5.5: Variance Components, 191
 - 5.9.1 A Variance Components Model, 191
 - 5.9.2 E-Step, 192,
 - 5.9.3 M-Step, 194
 - 5.9.4 Application of Two Versions of ECME Algorithm, 195
 - 5.9.5 Numerical Example, 195
- 5.10 Example 5.6: Factor Analysis, 197
 - 5.10.1 ECM Algorithm for Factor Analysis, 197
 - 5.10.2 Numerical Example, 199
- 5.11 Efficient Data Augmentation, 200
 - 5.11.1 Motivation, 200
 - 5.11.2 Maximum Likelihood Estimation of t-Distribution, 201
 - 5.11.3 Variance Components Model, 206

xii **CONTENTS**

Subject Index

	5.12	Alternating ECM Algorithm, 206	
	5.13	EMS Algorithm, 208	
	5.14	One-Step-Late Algorithm, 209	
	5.15	Variance Estimation for Penalized EM and OSL Algorithms, 210	
		5.15.1 Penalized EM Algorithm, 210	
		5.15.2 OSL Algorithm, 211	
		5.15.3 Example 5.7 (Example 4.1 Continued), 211	
	5.16	Linear Inverse Problems, 213	
6.	Misc	ellaneous Topics 2	214
	6.1 Iterative Simulation Algorithms, 214		
	6.2	Monte Carlo E-Step, 214	
6.3 Stochastic EM Algorithm, 216		Stochastic EM Algorithm, 216	
6.4 Data Augmentation Algorithm, 218			
	6.5 Multiple Imputation, 220		
6.6 Sampling-Importance Resampling, 221		Sampling-Importance Resampling, 221	
	6.7	Gibbs Sampler, 222	
		6.7.1 The Algorithm, 222	
		6.7.2 Example 6.1: Gibbs Sampler for the Mixture Problem, 224	
		6.7.3 Gibbs Sampler Analogs of ECM and ECME Algorithms, 225	
		6.7.4 Some Other Analogs of the Gibbs Sampler, 226	
	6.8	Missing-Data Mechanism and Ignorability, 227	
6.9 Competing Methods and Some Comparisons w Algorithm, 228			
		6.9.1 Introduction, 228	
		6.9.2 Simulated Annealing, 229	
		6.9.3 Comparison of SA and EM Algorithm for Normal Mixtures, 230	
	6.10	The Delta Algorithm, 230	
	6.11	Image Space Reconstruction Algorithm, 232	
	6.12	Further Applications of the EM Algorithm, 232	
		6.12.1 Introduction, 232	
		6.12.2 Hidden Markov Models, 233	
		6.12.3 AIDS Epidemiology, 2376.12.4 Neural Networks, 239	
		0.12.4 Inculal Networks, 239	
Ref	erenc	es 2	43
Aut	Author Index		

265