12

Computability

Exercise 12.1. Is the Launches-Missiles Problem described below computable? Provide a convincing argument supporting your answer.

Launches-Missiles

Input: A specification of a procedure.

Output: If an application of the procedure would lead to the missiles being launched, outputs True. Otherwise, outputs False.

You may assume that the only thing that causes the missiles to be launched is an application of the *launchMissiles* procedure.

Solution.

Exercise 12.2. Is the Same-Result Problem described below computable? Provide a convincing argument supporting your answer.

Same-Result

Input: Specifications of two procedures, P and Q.

Output: If an application of P terminates and produces the same value as applying Q, outputs True. If an application of P does not terminate, and an application of Q also does not terminate, outputs True. Otherwise, outputs False.

Solution.

Exercise 12.3. Is the Check-Proof Problem described below computable? Provide a convincing argument supporting your answer.

Check-Proof

Input: A specification of an axiomatic system, a statement (the theorem), and a proof (a sequence of steps, each identifying the axiom that is applied).

Output: Outputs True if the proof is a valid proof of the theorem in the system, or False if it is not a valid proof.

Solution.

Exercise 12.4. Is the Find-Finite-Proof Problem described below computable? Provide a convincing argument supporting your answer.

Find-Finite-Proof

Input: A specification of an axiomatic system, a statement (the theorem), and a maximum number of steps (max-steps).

Output: If there is a proof in the axiomatic system of the theorem that uses max-steps or fewer steps, outputs True. Otherwise, outputs False.

Solution.

Exercise 12.5. [⋆] Is the Find-Proof Problem described below computable? Provide a convincing argument why it is or why it is not computable.

Find-Proof

Input: A specification of an axiomatic system, and a statement (the theorem).

Output: If there is a proof in the axiomatic system of the theorem, outputs True. Otherwise, outputs False.

Solution.

Exercise 12.6. Confirm that the machine showing in Figure 12.3 runs for 6 steps before halting.

Solution.

Exercise 12.7. Prove the Beaver Bound problem described below is also noncomputable:

Beaver-Bound

Input: A positive integer, n.

Output: A number that is greater than the maximum number of steps a Turing Machine with n states and a two-symbol tape alphabet can run starting on an empty tape before halting.

A valid solution to the Beaver-Bound problem can produce any result for n as long as it is greater than the Busy Beaver value for n.

Solution.

Exercise 12.8. [⋆⋆⋆] Find a 5-state Turing Machine that runs for more than 47,176,870 steps, or prove that no such machine exists.

Solution.