Modal Logic Notes

Evante Garza-Licudine

September 3, 2012

Administrative Stuff

Instructor: Alexandru Baltag

TA: Jort Bergfeld

Schedule:

Week 1: Basic Concepts (Ch. 1)
2, 3: Models (2)
4, 5: Finite model properties and van Benthem's characterization theorem (2)
6: Strong completeness (4)
7: Canonicity (4)
8: break (take home exam)
9: Completeness for PDL (4.8)
10, 11: Frame def. and correspondence theory (3.1, .2, .5, .6)
12, 13: Decidability and complexity (5, 6)
14: Advanced topics and applications
15: Take home exam

Modalities

- necessity/possibility
- knowledge/belief
- obligation/permission
- temporal modalities
- provability

Kripke Semantics (relational)

A model $M := \langle W, R \rangle$, where W is the universe of discourse and R is a set of relations on W. Modal logics are pairs (W, R) where R is a binary relation.

$\forall w \in W$:

$$w \models \Diamond\phi \Leftrightarrow \exists w' : R(w, w') \wedge w' \models \phi$$

$$w \models \Box\phi \Leftrightarrow \forall w' : R(w, w') \rightarrow w' \models \phi$$

Definition. *Connected relations are binary relations s.t. for all w, w' we have $R(w, w'), R(w', w')$ or $w = w'$.*

Definition. *A tree is a pair $\mathbf{T} := (T, S),\ S \subseteq T^2$, subject to:*

1. $\mathbf{T}$ *has a unique root; that is, there exists a unique $r \in T$ s.t. for any $t \neq r$ in T, $S^+(r, t)$. Here, S^+ iterates S one, two, ... times.*
2. *Every node (except the root) has a unique predecessor. For all $t \neq r$, there is a unique t' in T s.t. $S(t', t)$.*
3. $\mathbf{T}$ *is acyclic. For all t in T, $\neg S^+(t, t)$.*

Similarity type: $T = \langle O, \tau, \Phi \rangle$ where $O \neq \emptyset$ and $\tau : O \mapsto \mathbb{N}$.

$\tau(\Box) = \tau(\Diamond) = 1$. Φ is some set of propositional letters.

$\phi ::= p \mid \phi \vee \phi \mid \Delta(\phi_1, \ldots, \phi_n),\ \Delta \in O,\ \tau(\Delta) = n$ and $\nabla(\phi_1, \ldots, \phi_n) := \neg\Delta(\neg\phi_1, \ldots, \neg\phi_n)$