Maps and Sets

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1 Hashing

Hashing is a way that we can easily store and obtain a value associated with key in a table. We use a *hash function* to perform some operation on a key to get an index in a table. As a result, the times it takes to obtain a value given a key and retrieve it are just O(1). In Java, there is a hashCode() for each object.

2 Maps

A map is a data structure that can store paired data called "keys" and "values." If the key is known, its corresponding value can be found. In Java, Map$< K, V >$ is an interface, which has the following methods:

- boolean containsKey(Object key) returns true if the map contains the key
- V get(Object key) returns the value paired with the key
- Set$< K >$ keySet() returns the set of keys
- V put(K key, V value) associates the key with the value and returns the value. Maps can't have duplicated keys. If the key already exists, the existing pair is replaced with the current pair.

TreeMaps and HashMaps are the two concrete classes in Java that are commonly used. They have the similar except that HashMaps stores their keys based on the hash function and TreeMaps stores their keys based on the natural order. Thus, it's best to use TreeMaps when order matters, and HashMaps when time does.

3 Sets

A set is a data structure that contains a collection of distinct objects. In Java, Set is an interface containing the following methods:

- boolean add(Object o) which adds an object into the set if not already present and returns true is the set did not contain the object.

- boolean contains(Object o) returns true if object is present in set
- remove(Object o) removes the object and returns true if the set contains the object.

TreeSets and HashSets are the two concrete classes in Java. There are similar except a TreeSet stores objects in their natural order whereas a HashSet stores objects based on the hash function. HashSets have an O(1) for the add, remove, and contains methods whereas TreeSets have an O(log n) for all three methods.

4 Problems

1. To keep his cows intellectually stimulated, Farmer John has placed a large map of the USA on the wall of his barn. Since the cows spend many hours in the barn staring at this map, they start to notice several curious patterns. For example, the cities of Flint, MI and Miami, FL share a very special relationship: the first two letters of "Flint" give the state code ("FL") for Miami, and the first two letters of "Miami" give the state code ("MI") for Flint.
   Let us say that two cities are a "special pair" if they satisfy this property and come from different states. The cows are wondering how many special pairs of cities exist. Please help them solve this amusing geographical puzzle!
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2. For some reason, Farmer John's cows always seem to be running laser light shows. For their latest show, the cows have procured a large powerful laser – so large, in fact, that they cannot seem to move it easily from the location where it was delivered. They would like to somehow send the light from the laser to the barn on the other side of FJ's property. Both the laser and the barn can be considered to be located at points in the 2D plane on a map of FJ's farm. The cows plan to point the laser so that it sends a beam of light out either horizontally or vertically (i.e., aligned with the x or y axes). They will then bounce this beam off a number of mirrors to direct it to the barn.
   On the farm there are NN fence posts ($1 \leq N \leq 100,0001 \leq N \leq 100,000$) located at distinct 2D points (also distinct from the laser and the barn) at which the cows can mount mirrors. The cows can choose not to mount a mirror on a fence post, in which case the laser would simply pass straight over the top of the post without changing direction. If the cows do mount a mirror on a fence post, they align it diagonally like / or \so that it will re-direct a horizontal beam of light in a vertical direction or vice versa.

   Please compute the minimum possible number of mirrors the cows need to use in order to re-direct the laser to the barn.

   (USACO Gold 2017)