Binary Search

ICT Officers

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1 Introduction

Binary Search is a technique where you search by taking advantage of some in-order property of the data. At each instance, you can determine if the target is higher or lower than the specified position, so you can discard half of the data. This makes binary search logarithmic - O(log n).

2 Basic Algorithm

Low represents the 0th index and high represents the largest index. Mid is computed by taking the floor of the sum of low and high (integer division). After logarithmic computations, mid will be the position of target. Notice the low<high-1 is intentional: that is to prevent it form getting trapped in an infinite loop.

Algorithm 1 Binary Search for Integers

```
high = N
low = 0
mid
while low<high do
    mid=(high+low)/2
    if a[mid]<target then
        low=mid+1
    else
        high=mid
```

3 Uses of Binary Search

Binary searching can be applied in many non-obvious ways. Consider the following: There are N (N≤10,000) cows going to perform on a stage numbered 1...N (they must appear in this order) and each cow takes time t to dance. Given max time T, at most a million, find the minimum K such that K cow dance.

The solution is to binary search K. The bounds are 1 and N. We test a K is valid by simulating it. If it is under T, we update the lower bounds and if over, update the upper bounds.

We can simulate it using a greedy approach: We loop through all N cows and find the minimum time of all cows on the stage. Once we reached K cows, we remove the cow with the minimum time and add that time to a total time elapsed. We do this until there are no more cows left to add or until it exceeds time. This should yield a C*NlogT solution, where C is a constant factor (due to low cost of simulation operations).

4 Practice

(UVa, Problem 11935: Through the Desert)

You are an explorer crossing a desert. You use a jeep with a 'large enough' fuel tank - initially full. You encounter a series of events such as drive (consumes fuel), experience gas leak (reduce amount of fuel by n), encounter gas station (refuels to original capacity of fuel tank), or reach goal state (done). Given an integer n, a series of events $(e_1, e_2, e_3, ...)$, and that the maximum possible fuel tank size is 10M, determine the smallest possible fuel tank capacity to reach the goal.

Solution: There is one key observation to solving the problem. Say that the correct answer is X. Any number between (0-X) will not be sufficient, whereas anything from (X, 10M) will be sufficient fuel. We can thus conduct a binary search over (0, 10M) for the optimal solution. This becomes more tricky because we are dealing with floating-point numbers. We can define some value EPS, such that if abs(high-low) in the binary search is less than EPS, then terminate.

Angry Cows (USACO 2016 January Contest, Silver)

In this problem, we have several hay bales on a number line and a few exploding cows (like Angry Birds!). If a cow is launched with power R landing at position x, this will causes a blast of "radius R", destroying all hay bales within the range x−R. . . x+R. There are N hay bales located at distinct integer positions x1,x2,. . . ,xN on the number line.

A total of K cows are available to shoot, each with the same power R. Please determine the minimum integer value of R such that it is possible to use the K cows to detonate every single hay bale in the scene.

Solution: Notice that if there is some radius r that makes all hay bales to explode, then any radius R>r also works. Therefore, we can binary search for the minimum radius r.

To check if a given radius r is feasible, consider the leftmost hay bale at location x. In order to make as many other hay bales as possible explode, it makes sense to move the cow to the right as much as possible while including the bale at x. Therefore, we would have the hay bale explode at location x+r and all hay bales in between x and x+2r explode. We then repeat this process for the leftmost hay bale that is at location greater than x+2r, then count the number of cows that we placed. If the number of cows used is less than or equal to K, then r is feasible. Otherwise, the answer must be at least r+1.