The Transformer

NLP Week 8

Thanks to Dan Jurafsky for most of the slides this week!

Plan for today

- 1. Review of notation and matrix multiplication
- 2. (Multi-head) self-attention
- 3. Residual stream
- 4. Position embeddings
- 5. Putting it all together —> The Transformer
- 6. GPT and BERT
- 7. Group exercises

This semester

We will build language models adding to each layer of their complexity:

- 1. Bag of words models (basic statistical models of language)
- 2. **N-gram models** (+ sequential dependencies)
- 3. Hidden Markov models (+ latent categories)
- 4. **Recurrent neural networks** (+ distributed representations)
- 5. **LSTM language models** (+ long distance dependencies)
- 6. Transformer language models (+ attention-based dependency learning)

= Today's language models!

A note on notation

Quick recap on our notation and matrix-matrix and matrix-vector multiplication

- Let \mathbf{A} denote an $p \times d$ matrix
- Let X denote an $d \times n$ matrix
- Let **x** denote a $d \times 1$ vector (column vector)
- \mathbf{x}^{T} is a $1 \times d$ vector (row vector)
- Note that: $(\mathbf{A}\mathbf{X})^{\mathsf{T}} = \mathbf{X}^{\mathsf{T}}\mathbf{A}^{\mathsf{T}}$

We need to understand:

- y = Ax
- $\bullet \ \mathbf{y}^{\mathsf{T}} = \mathbf{x}^{\mathsf{T}} \mathbf{A}^{\mathsf{T}}$
- $\bullet Y = AX$
- $\bullet \ \mathbf{Y}^{\mathsf{T}} = \mathbf{X}^{\mathsf{T}} \mathbf{A}^{\mathsf{T}}$

will do this on the whiteboard ...

The paper that started it all

Transformer: a specific kind of network architecture, like a fancier feedforward network, but based on attention

Attention Is All You Need

Ashish Vaswani*

Google Brain avaswani@google.com Noam Shazeer*

Google Brain noam@google.com Niki Parmar*

Google Research nikip@google.com Jakob Uszkoreit*

Google Research usz@google.com

Llion Jones*

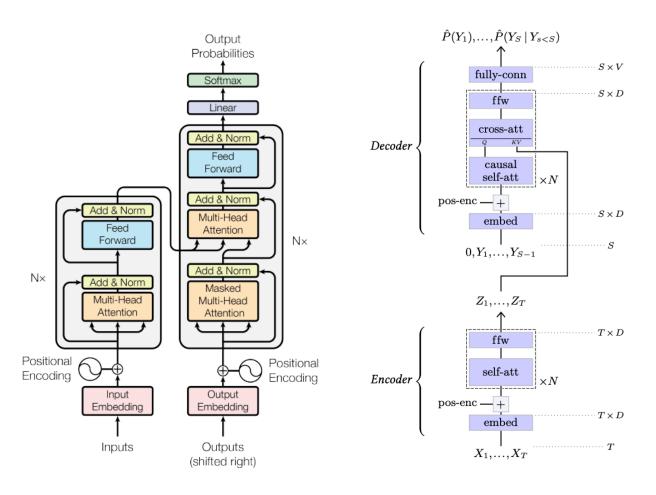
Google Research llion@google.com Aidan N. Gomez* †

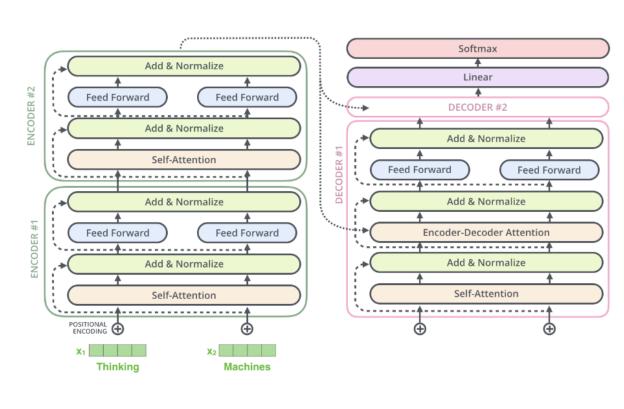
University of Toronto aidan@cs.toronto.edu Łukasz Kaiser*

Google Brain lukaszkaiser@google.com

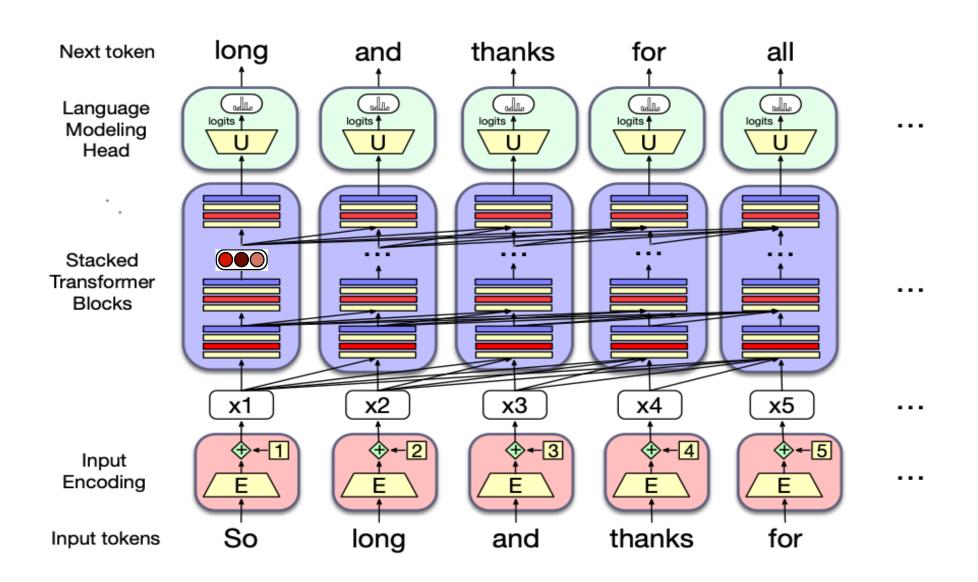
Illia Polosukhin* †
illia.polosukhin@gmail.com

What is a Transformer?

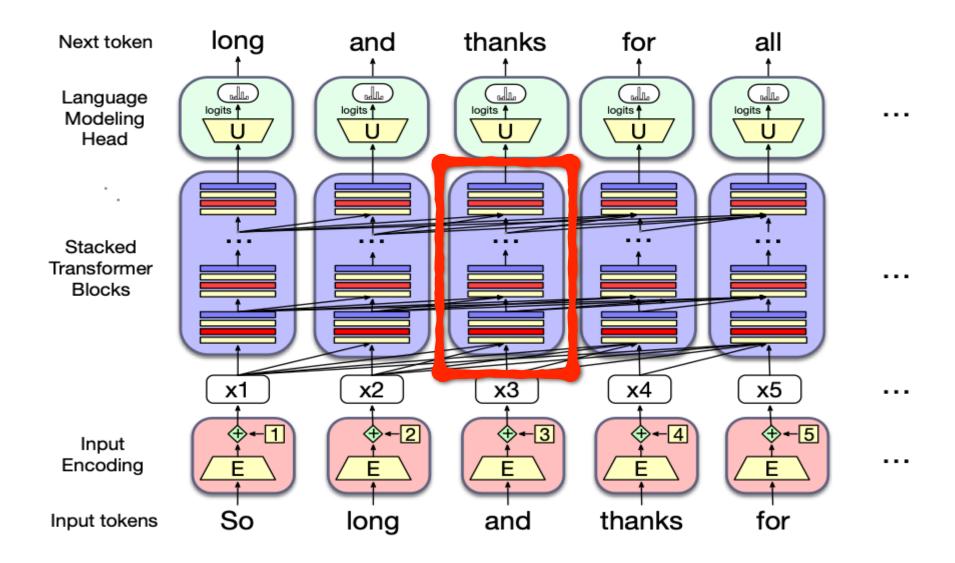




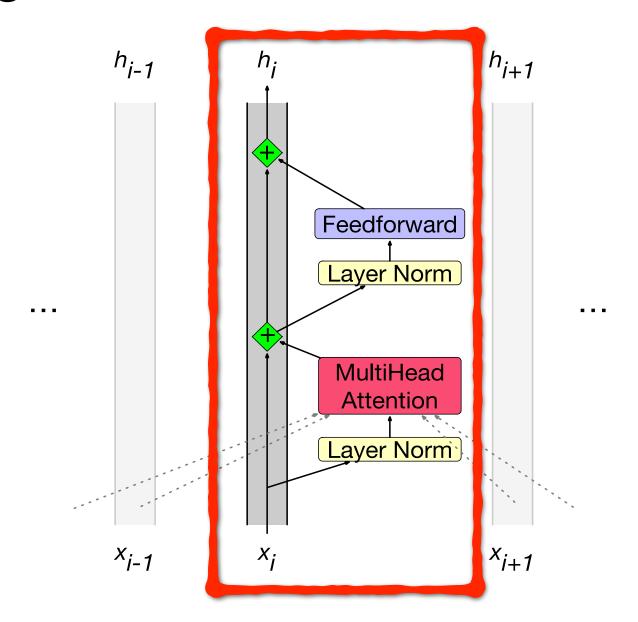
We will stick to the following illustration



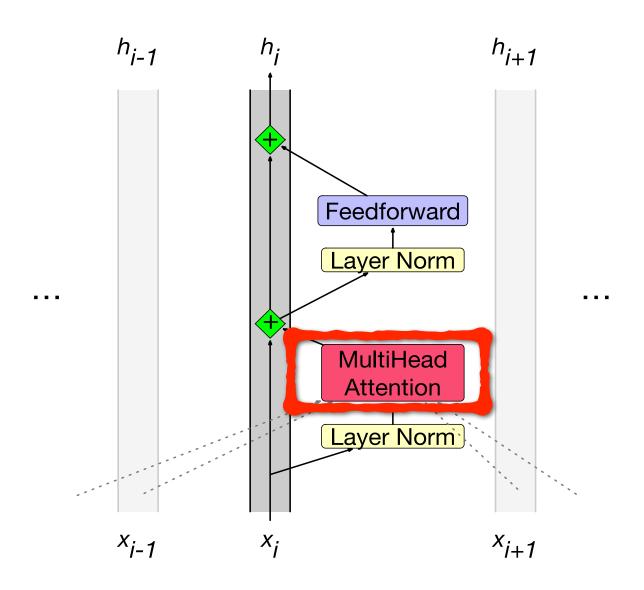
The Transformer



Zooming in



Zooming in

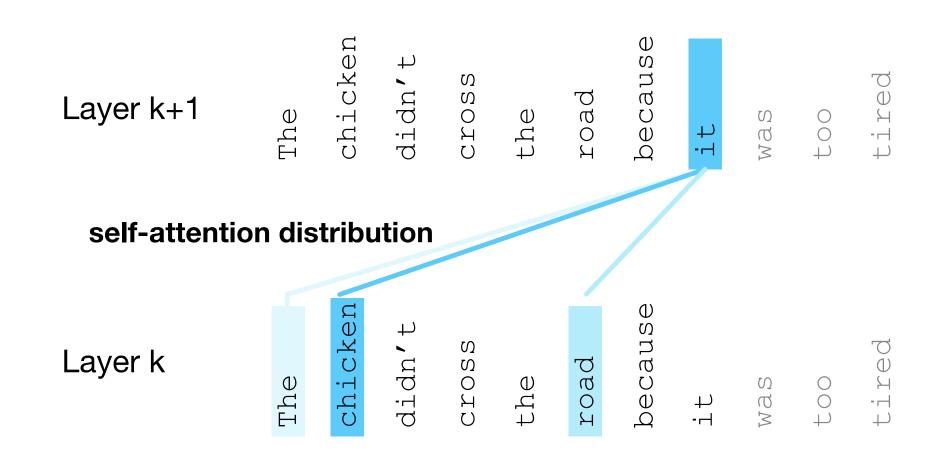


Intuition of attention

- Build up the representation of a word by selectively integrating information from all the neighbouring words
- We say that a word "attends to" some neighbouring words more than others

Intuition of attention

columns corresponding to input tokens



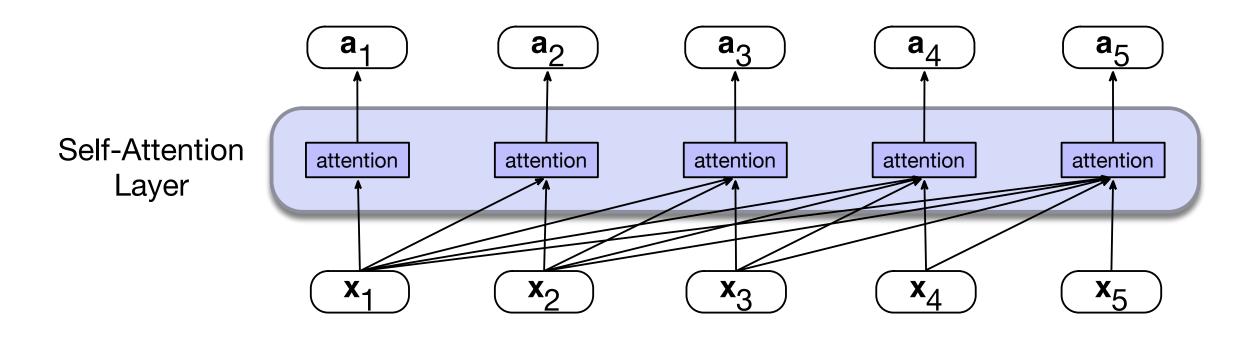
Attention definition

A mechanism for helping compute the embedding for a token by selectively attending to and integrating information from surrounding tokens (at the previous layer).

More formally: a method for doing a weighted sum of vectors.

$$\mathbf{v}^{k+1} = \sum_{i=1}^{n} \alpha_i \cdot \mathbf{v}_i^k$$

Attention can respect time (causal)



$$\mathbf{a}_{j} = \sum_{i=1}^{n} (\alpha_{i} \cdot \mathbb{M}(i, j)) \cdot \mathbf{x}_{i} \qquad \mathbb{M} = \begin{cases} 1 & \text{if } i \leq j \\ 0 & \text{else} \end{cases}$$

Simplified version of attention: a sum of prior words weighted by their similarity with the current word

Given a sequence of token embeddings:

$$\mathbf{x}_1$$
 \mathbf{x}_2 \mathbf{x}_3 \mathbf{x}_4 \mathbf{x}_5 \mathbf{x}_6 \mathbf{x}_7 \mathbf{x}_1

Produce: \mathbf{a}_i = a weighted sum of \mathbf{x}_1 through \mathbf{x}_7 (and \mathbf{x}_i)

Weighted by their similarity to \mathbf{x}_i

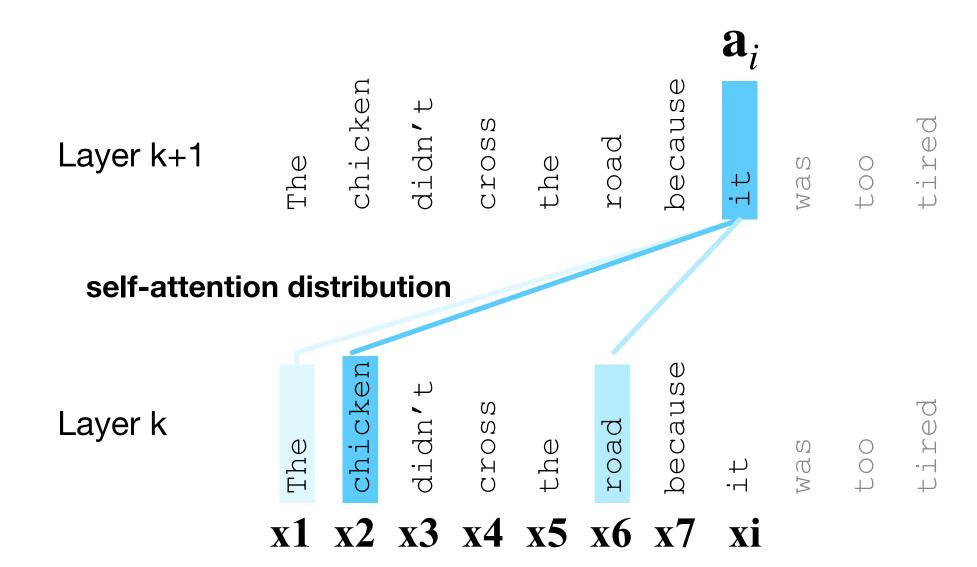
$$\mathbf{a}_i = \sum_{j < i} \alpha_j \cdot \mathbf{x}_j$$

$$score(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i \cdot \mathbf{x}_j$$

$$\alpha = \text{softmax}([\text{score}(\mathbf{x}_i, \mathbf{x}_j) \text{ for j in } 1...7, i])$$
 $\mathbf{a}_i = (\sum_{i=1}^{n} \alpha_i \cdot \mathbf{x}_j) + \alpha_i \cdot \mathbf{x}_i$

Intuition of attention

columns corresponding to input tokens



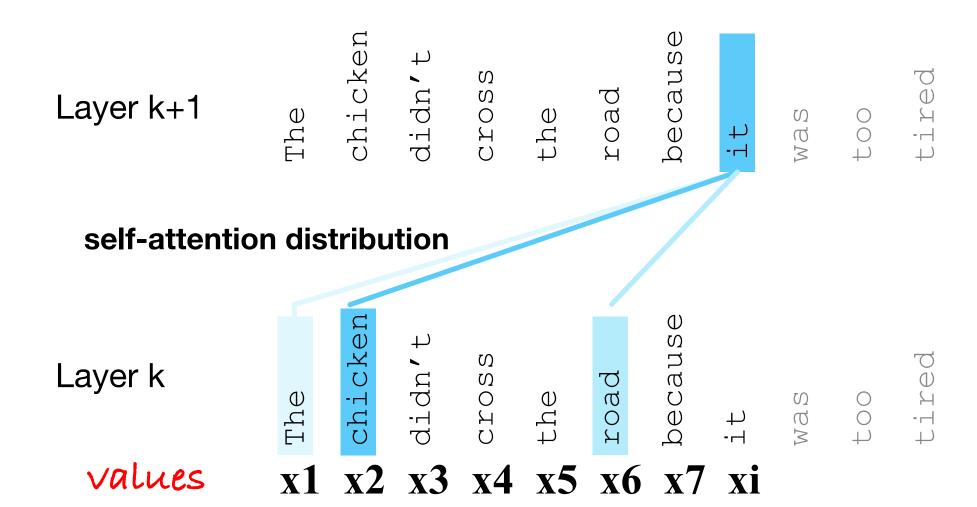
An Actual Attention Head is slightly more complicated

High-level idea: instead of using vectors (like x_i and x_4) directly, we'll represent 3 separate roles each vector x_i plays:

- query: As the current element being compared to the preceding inputs.
- key: as a preceding input that is being compared to the current element to determine a similarity
- value: a value of a preceding element that gets weighted and summed

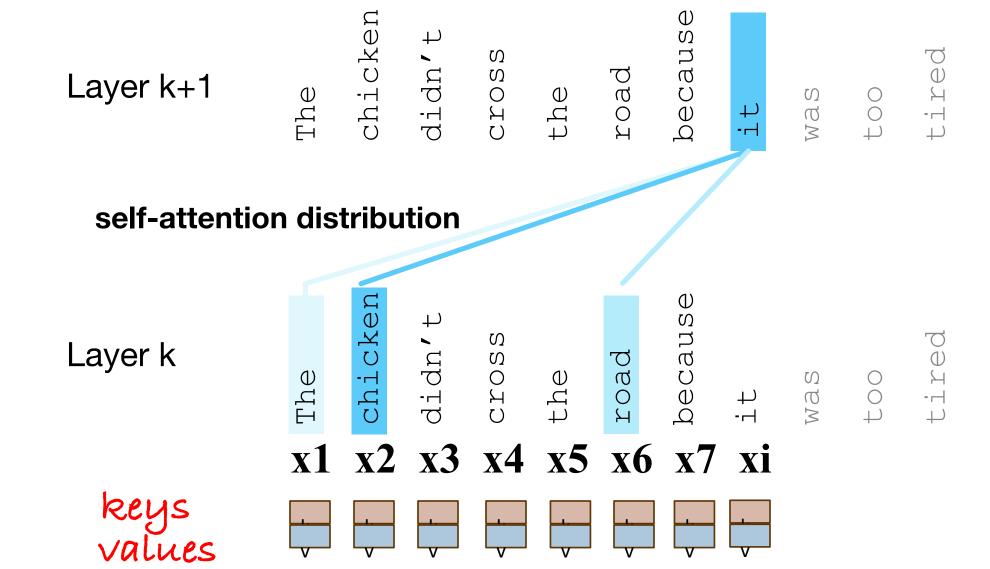
Intuition of attention





Intuition of attention





An Actual Attention Head is slightly more complicated

We'll use matrices to project each vector \mathbf{x}_i into a representation of its role as query, key, value:

- query: W^Q
- key: W^K
- value: W^v

$$\mathbf{q}_i = \mathbf{x}_i \mathbf{W}^Q \qquad \mathbf{k}_i = \mathbf{x}_i \mathbf{W}^K \qquad \mathbf{v}_i = \mathbf{x}_i \mathbf{W}^V$$

Note: \mathbf{x}_i , \mathbf{q}_i , \mathbf{k}_i , \mathbf{v}_i are row vectors here

An Actual Attention Head is slightly more complicated

Given these 3 representation of x_i

$$\mathbf{q}_i = \mathbf{x}_i \mathbf{W}^Q \qquad \mathbf{k}_i = \mathbf{x}_i \mathbf{W}^K \qquad \mathbf{v}_i = \mathbf{x}_i \mathbf{W}^V$$

To compute similarity of current element \mathbf{x}_i with some prior element \mathbf{x}_i

We'll use dot product between \mathbf{q}_i and \mathbf{k}_j .

And instead of summing up \mathbf{x}_i , we'll sum up \mathbf{v}_i

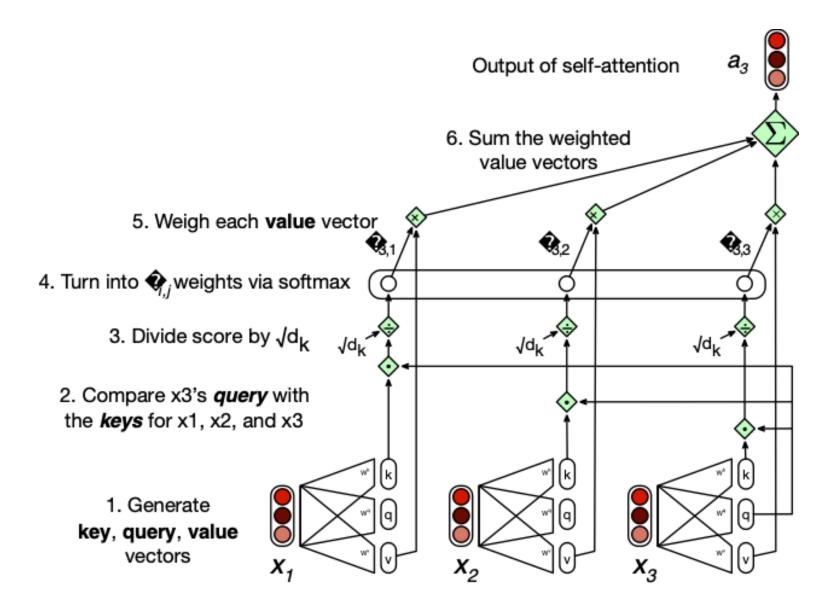
Final equations for one attention head

$$\mathbf{q}_i = \mathbf{x}_i \mathbf{W}^Q \qquad \mathbf{k}_i = \mathbf{x}_i \mathbf{W}^K \qquad \mathbf{v}_i = \mathbf{x}_i \mathbf{W}^V$$

$$score(\mathbf{x}_i, \mathbf{x}_j) = \frac{\mathbf{q}_i \mathbf{k}_j^{\mathsf{T}}}{\sqrt{d^k}} \qquad \alpha = softmax([score(\mathbf{x}_i, \mathbf{x}_j) \, \forall \, j \leq i])$$

$$\mathbf{a}_i = \sum_{j \le i} \alpha_j \cdot \mathbf{v}_j$$

Example: calculating the value of a3



An Actual Attention Head is slightly more complicated

- Instead of one attention head, we'll have lots of them!
- Intuition: each head might be attending to the context for different purposes
 - E.g., different linguistic relationships or patterns in the context

$$\mathbf{q}_i^c = \mathbf{x}_i \mathbf{W}^{Qc} \quad \mathbf{k}_i^c = \mathbf{x}_i \mathbf{W}^{Kc} \quad \mathbf{v}_i^c = \mathbf{x}_i \mathbf{W}^{Vc}$$

$$score^{c}(\mathbf{x}_{i}, \mathbf{x}_{j}) = \frac{\mathbf{q}_{i}^{c} \mathbf{k}_{j}^{c\top}}{\sqrt{d^{k}}} \qquad \alpha_{i}^{c} = softmax([score^{c}(\mathbf{x}_{i}, \mathbf{x}_{j}) \forall j \leq i])$$

$$\mathsf{head}_i^c = \sum_{j \le i} \alpha_{i,j}^c \cdot \mathbf{v}_j^c \qquad \qquad \mathbf{a}_i = (\mathsf{head}^1 \oplus \mathsf{head}^2 \dots \oplus \mathsf{head}^h) \mathbf{W}^O$$

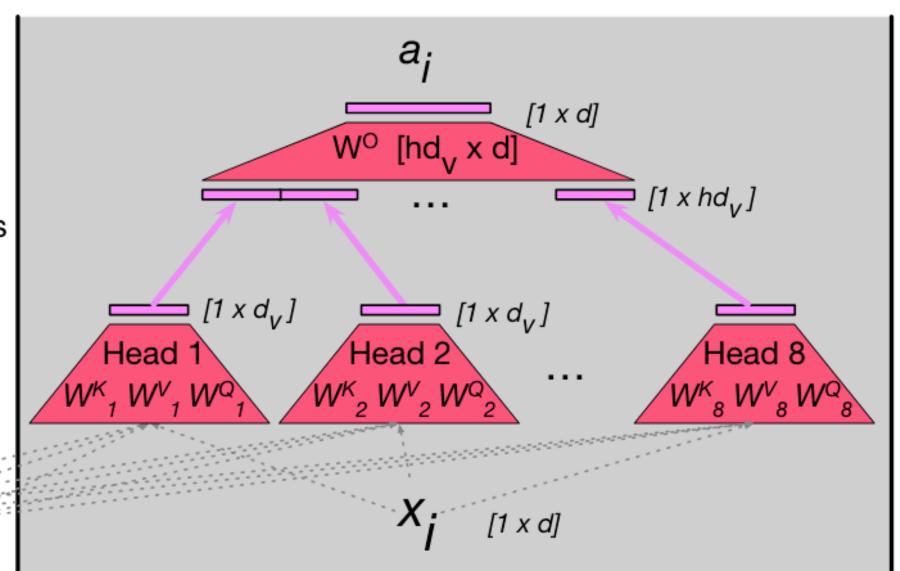
MultiHeadAttention(\mathbf{x}_i , [\mathbf{x}_1 , ..., \mathbf{x}_n]) = \mathbf{a}_i

Multi-head attention

Project down to d

Concatenate Outputs

Each head attends differently to context



Parallelizing computation using X

For attention/transformer block we've been computing a **single** output at a **single** time step *i* in a **single** residual stream.

But we can pack the N tokens of the input sequence into a single matrix \mathbf{X} of size $[N \times d]$.

Each row of X is the embedding of one token of the input.

X can have 1K-32K rows, each of the dimensionality of the embedding *d* (the **model dimension**)

$$\mathbf{Q} = \mathbf{X}\mathbf{W}^Q \qquad \mathbf{K} = \mathbf{X}\mathbf{W}^K \qquad \mathbf{V} = \mathbf{X}\mathbf{W}^V$$

QKT

Now can do a single matrix multiply to combine Q and K^T

N

$$score(\mathbf{x}_i, \mathbf{x}_j) = \frac{\mathbf{q}_i \mathbf{k}_j^{\mathsf{T}}}{\sqrt{d^k}}$$

$$\mathbf{S} = \frac{\mathbf{Q}\mathbf{K}^{\top}}{\sqrt{d^k}}$$

q1•k1	q1•k2	q1•k3	q1•k4
q2•k1	q2•k2	q2•k3	q2•k4
q3•k1	q3•k2	q3•k3	q3•k4
q4•k1	q4•k2	q4•k3	q4•k4

N

Parallelizing attention

- Scale the scores, take the softmax, and then multiply the result by V resulting in a matrix of shape N × d
 - An attention vector for each input token

$$\mathbf{A} = \operatorname{softmax} \left(\mathbb{M} \left(\frac{\mathbf{Q} \mathbf{K}^{\top}}{\sqrt{d^k}} \right) \right) \mathbf{V}$$

Masking out the future

- What is this mask function?
 QK^T has a score for each query dot every key, including those that follow the query.
- Guessing the next word is pretty simple if you already know it!

$$\mathbf{A} = \operatorname{softmax} \left(\mathbb{M} \left(\frac{\mathbf{Q} \mathbf{K}^{\top}}{\sqrt{d^k}} \right) \right) \mathbf{V}$$

Masking out the future

Add -∞ to cells in upper triangle The softmax will turn it to 0

$$\mathbf{A} = \operatorname{softmax} \left(\mathbb{M} \left(\frac{\mathbf{Q} \mathbf{K}^{\mathsf{T}}}{\sqrt{d^k}} \right) \right) \mathbf{V}$$
 N

q1•k1	-8	-8	-8
q2•k1	q2•k2	-8	-8
q3·k1	q3·k2	q3•k3	-8
q4•k1	q4•k2	q4•k3	q4·k4

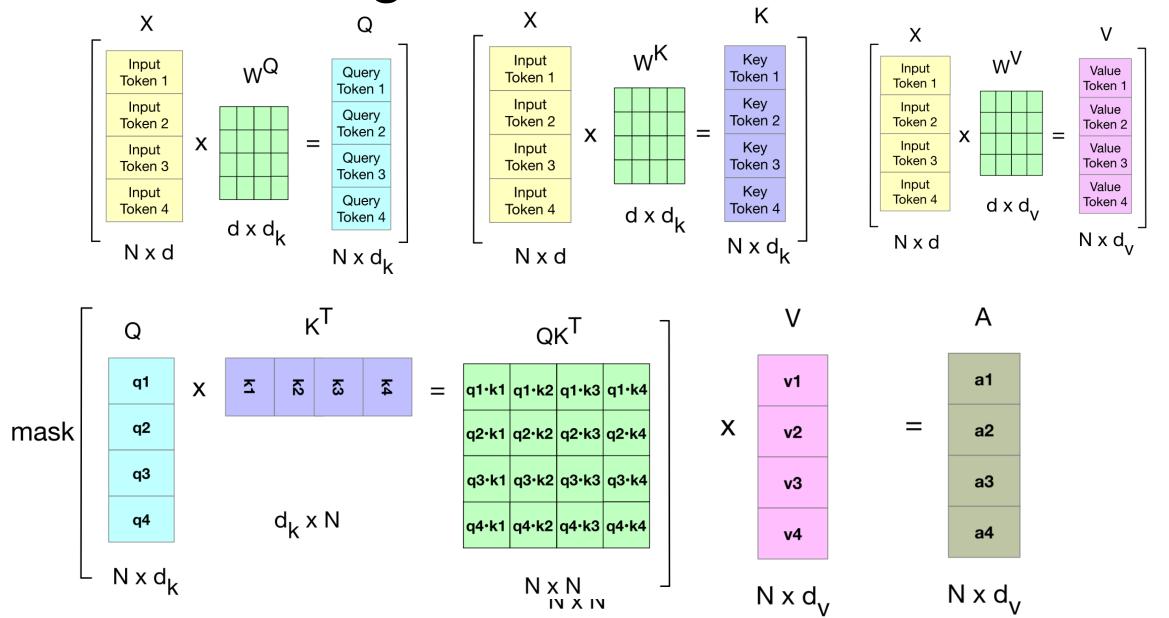
Another point: Attention is quadratic in length

$$\mathbf{A} = \operatorname{softmax} \left(\mathbb{M} \left(\frac{\mathbf{Q} \mathbf{K}^{\top}}{\sqrt{d^k}} \right) \right) \mathbf{V} \qquad \mathbb{N}$$

q1•k1	-8	-8	-&
q2•k1	q2•k2	-8	-&
q3·k1	q3·k2	q3·k3	-∞
q4•k1	q4•k2	q4•k3	q4•k4

Λ

Attention again



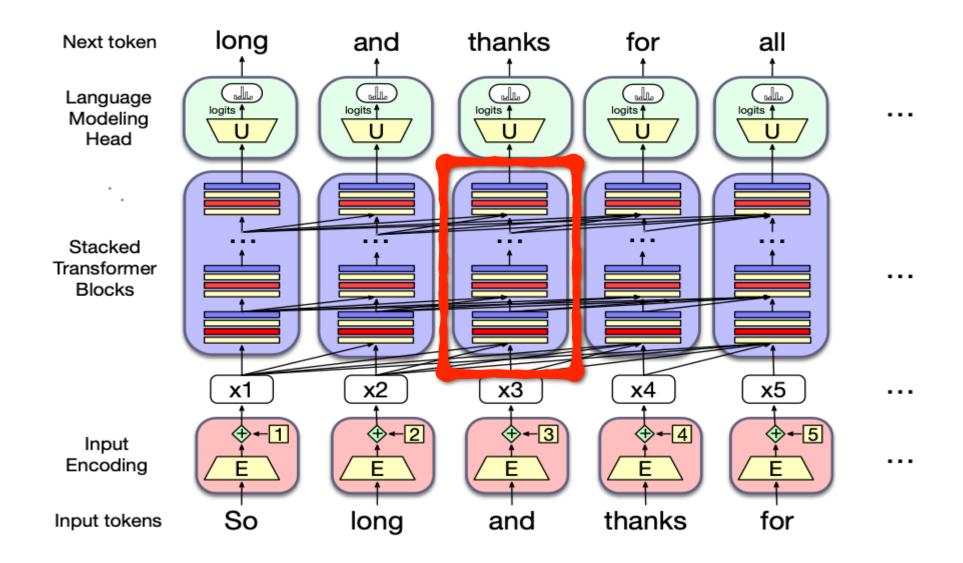
Parallelizing Multi-head Attention

$$\begin{aligned} \mathbf{Q^i} &= \mathbf{X} \mathbf{W^{Qi}} \; ; \; \; \mathbf{K^i} \; = \; \mathbf{X} \mathbf{W^{Ki}} \; ; \; \; \mathbf{V^i} = \mathbf{X} \mathbf{W^{Vi}} \\ \mathbf{head}_i &= \mathrm{SelfAttention}(\mathbf{Q^i}, \mathbf{K^i}, \mathbf{V^i}) \; = \; \mathrm{softmax} \left(\frac{\mathbf{Q^i} \mathbf{K^{iT}}}{\sqrt{d_k}} \right) \mathbf{V^i} \\ \mathrm{MultiHeadAttention}(\mathbf{X}) \; &= \; (\mathbf{head}_1 \oplus \mathbf{head}_2 ... \oplus \mathbf{head}_A) \mathbf{W^O} \end{aligned}$$

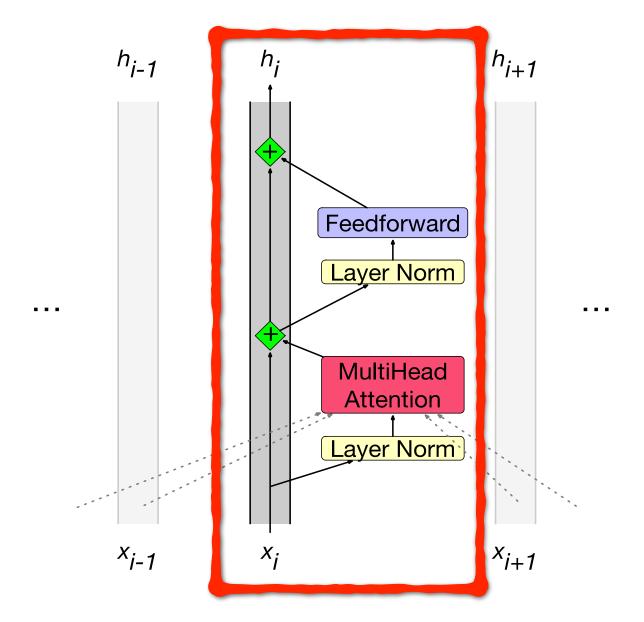


This is equivalent to running the attention heads in parallel and adding their results back to the residual stream (Whiteboard)

Reminder: transformer architecture

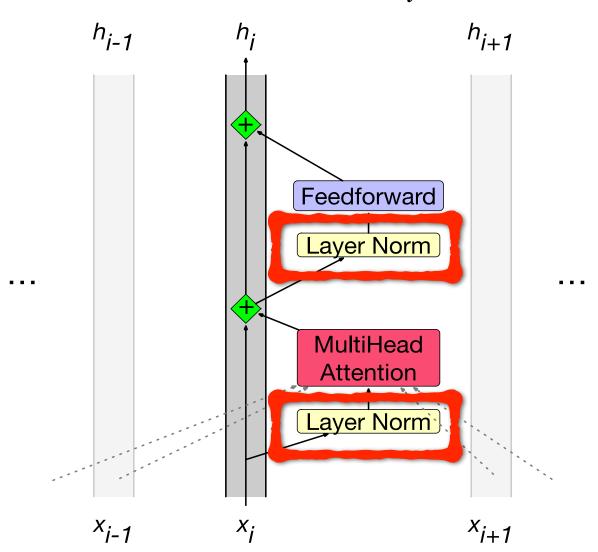


A single transformer block



Sublayers of the transformer block: Layer Norm

LayerNorm(\mathbf{x}_i) = ...



Layer Norm

Layer norm is a variation of the z-score from statistics, applied to a single vector in a hidden layer

$$\mu = \frac{1}{d} \sum_{i=1}^{d} x_i$$

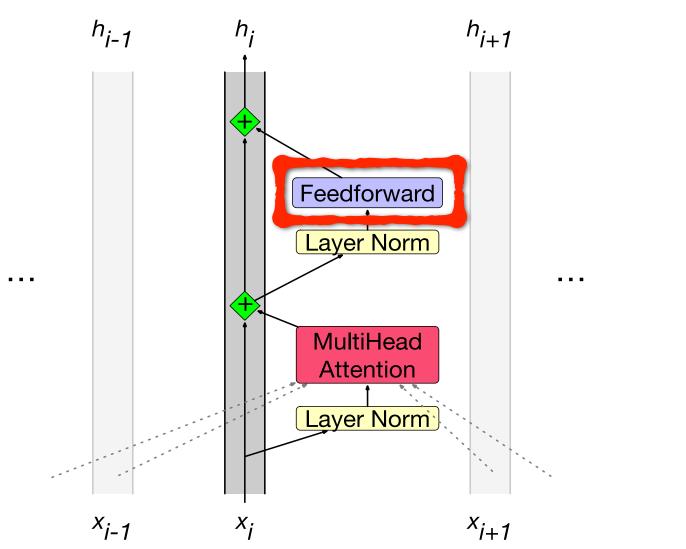
$$\sigma = \sqrt{\frac{1}{d} \sum_{i=1}^{d} (x_i - \mu)^2}$$

$$\hat{\mathbf{x}} = \frac{(\mathbf{x} - \mu)}{\sigma}$$

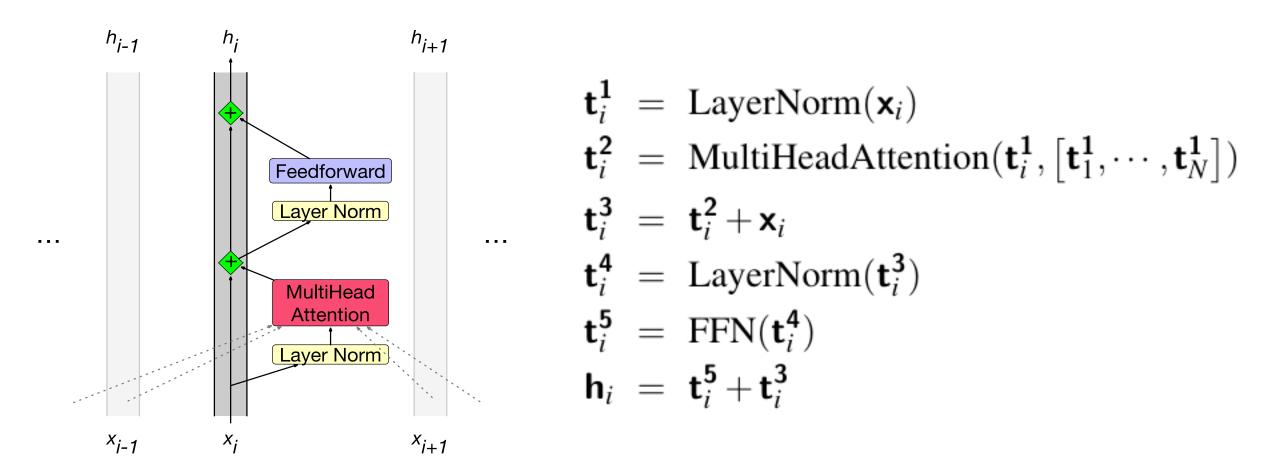
LayerNorm(
$$\mathbf{x}$$
) = $\gamma \frac{(\mathbf{x} - \mu)}{\sigma} + \beta$

Sublayers of the transformer block: FFN

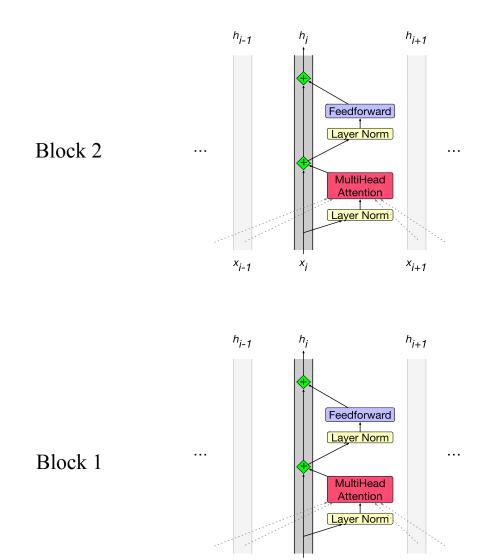
$$FFN(\mathbf{x}_i) = ReLU(\mathbf{x}_i\mathbf{W}_1 + \mathbf{b}_1)\mathbf{W}_2 + \mathbf{b}_2$$



Putting together a single transformer block

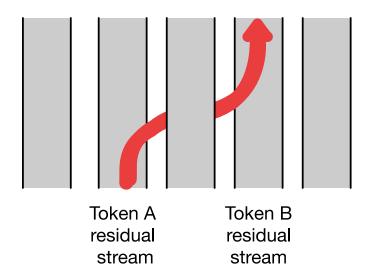


A transformer is a stack of these blocks so all the vectors are of the same dimensionality d

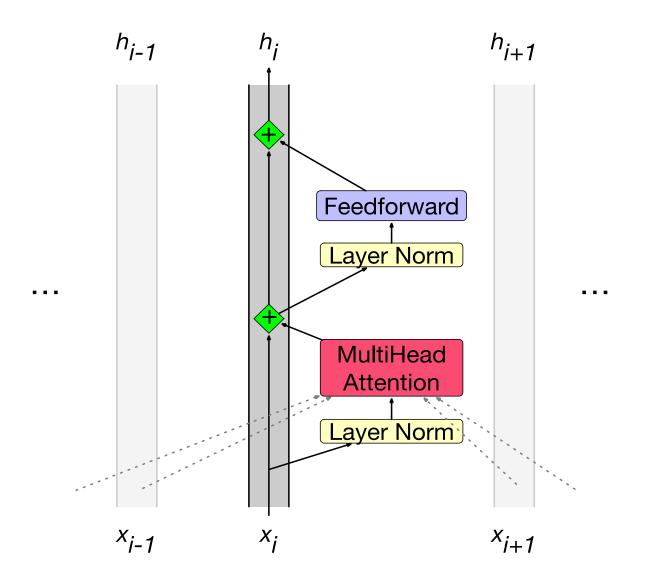


Residual streams and attention

- Notice that all parts of the transformer block apply to 1 residual stream except attention, which takes information from other tokens
- Elhage et al. (2021) show that we can view attention heads as literally moving information from the residual stream of a neighboring token into the current stream



Residual stream view



- FFN and attention layers read from and write to the residual stream
- FFN layers have access to "one lane" only. Same computation applied on every "lane"
- Attention layers can read from from other "lanes" too
- \mathbf{x}_i is transformed into \mathbf{h}_i^L through a sequence of non-linear transformations

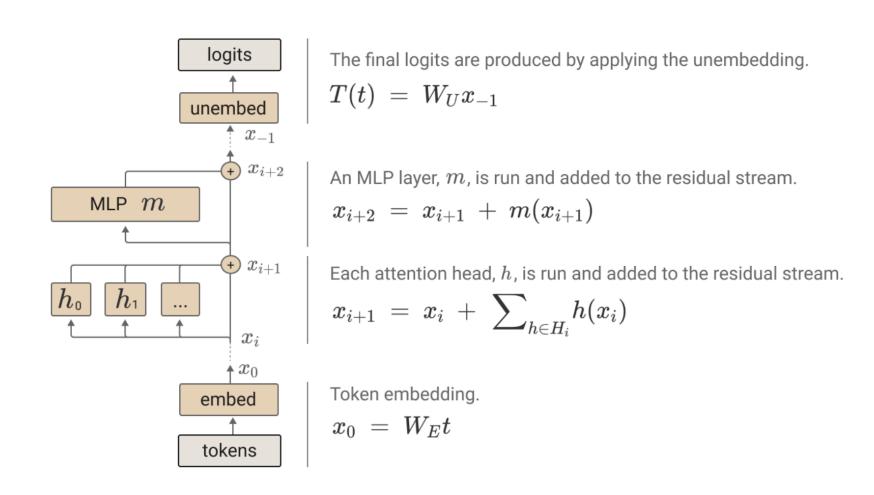
Putting together a single transformer block

Single vector:

Matrix of inputs:

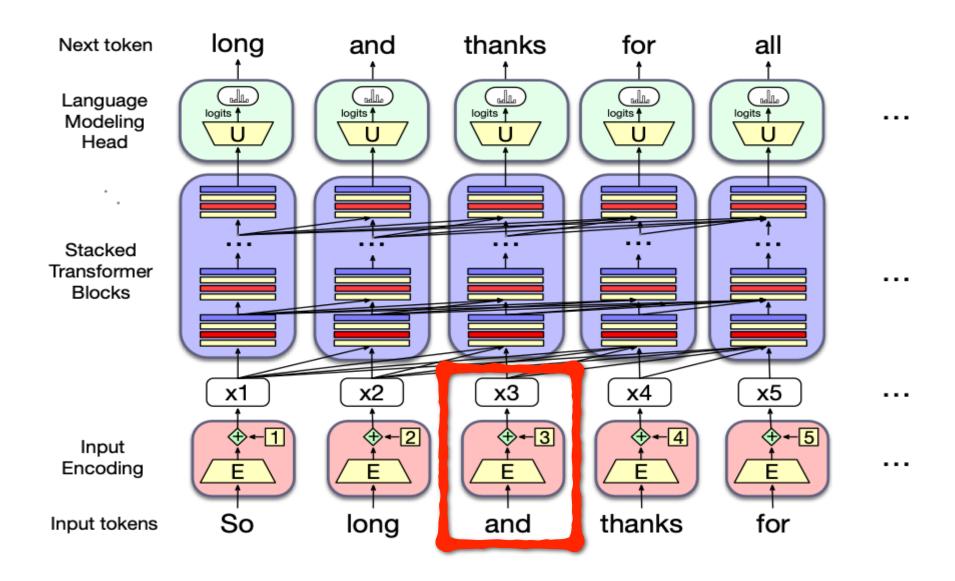
$$\mathbf{t}_{i}^{1} = \operatorname{LayerNorm}(\mathbf{x}_{i})$$
 $\mathbf{T}^{1} = \operatorname{LayerNorm}(\mathbf{X})$
 $\mathbf{t}_{i}^{2} = \operatorname{MultiHeadAttention}(\mathbf{t}_{i}^{1}, \left[\mathbf{t}_{1}^{1}, \cdots, \mathbf{t}_{N}^{1}\right])$ $\mathbf{T}^{2} = \operatorname{MultiHeadAttention}(\mathbf{T}^{1})$
 $\mathbf{t}_{i}^{3} = \mathbf{t}_{i}^{2} + \mathbf{x}_{i}$ $\mathbf{T}^{3} = \mathbf{T}^{2} + \mathbf{X}$
 $\mathbf{t}_{i}^{4} = \operatorname{LayerNorm}(\mathbf{t}_{i}^{3})$ $\mathbf{T}^{4} = \operatorname{LayerNorm}(\mathbf{T}^{3})$
 $\mathbf{t}_{i}^{5} = \operatorname{FFN}(\mathbf{t}_{i}^{4})$ $\mathbf{T}^{5} = \operatorname{FFN}(\mathbf{T}^{4})$
 $\mathbf{h}_{i} = \mathbf{t}_{i}^{5} + \mathbf{t}_{i}^{3}$ $\mathbf{H} = \mathbf{T}^{5} + \mathbf{T}^{3}$

Residual stream view



Elhage et al. (2021) - A Mathematical Framework for Transformer Circuits

Reminder: transformer architecture

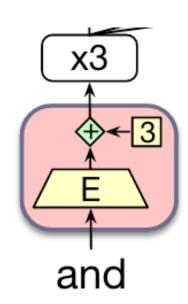


Token and Position Embeddings

The matrix X (of shape $N \times d$) has an embedding for each word in the context.

This embedding is created by adding two distinct embedding for each input

- token embedding
- positional embedding



Token Embeddings

Embedding matrix **E** has shape $|\mathcal{V}| \times d$.

- One row for each of the $|\mathcal{V}|$ tokens in the vocabulary.
- Each word is a row vector of d dimensions

Given: string "Thanks for all the"

1. Tokenize with BPE and convert into vocab indices

input_ids = [5,4000,10532,2224]

- 2. Select the corresponding rows from ${f E}$, each row an embedding
- (row 5, row 4000, row 10532, row 2224).

Position Embeddings

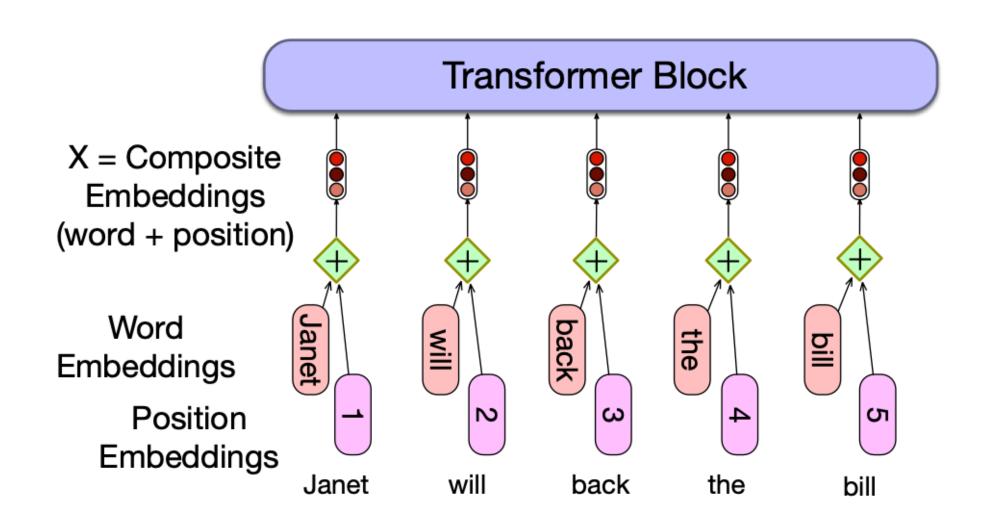
There are many methods, but we'll just describe the simplest: absolute position.

Goal: learn a position embedding matrix $\mathbf{E}_{\mathsf{pos}}$ of shape $N \times d$.

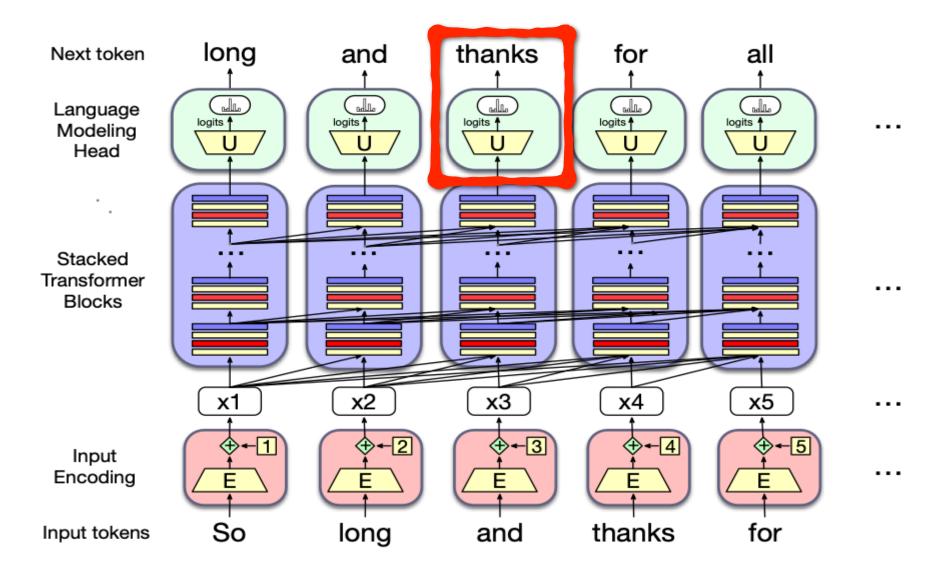
Start with randomly initialized embeddings

- one for each integer up to some maximum length.
- i.e., just as we have an embedding for token *fish*, we'll have an embedding for position 3 and position 17.
- As with word embeddings, these position embeddings are learned along with other parameters during training.

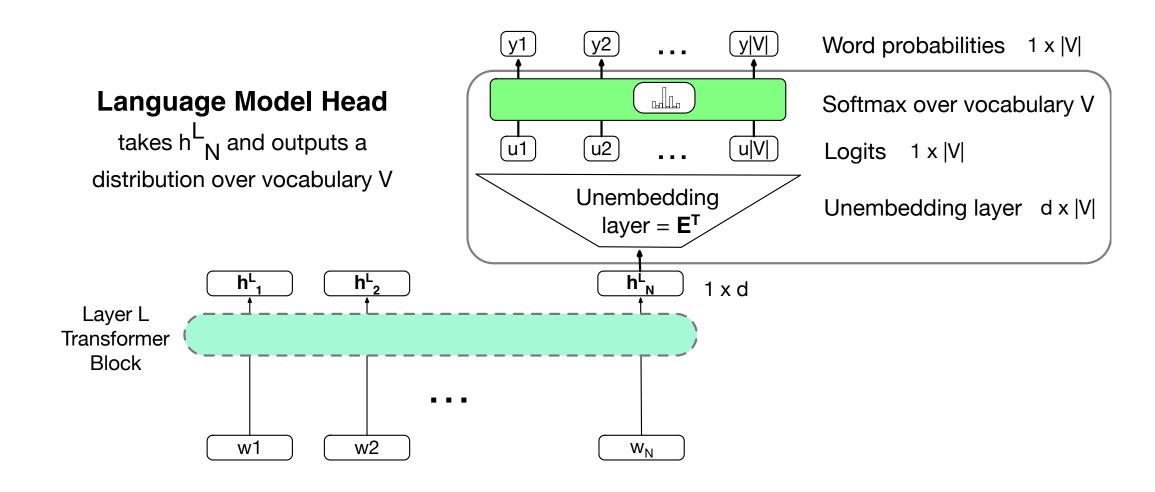
Each X_i is just the sum of word and position embeddings



Reminder: transformer architecture

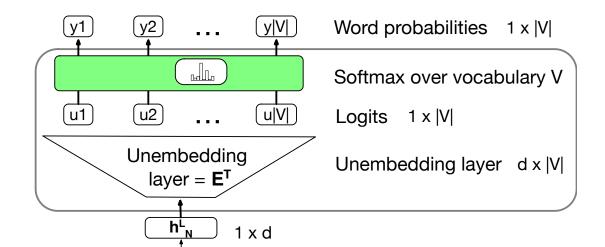


Language modeling head



Language modeling head

Unembedding layer: linear layer projects from \mathbf{h}_N^L (shape $1 \times d$) to logit vector



 W_N

Why "unembedding"? **Tied** to **E**^T

Weight tying, we use the same weights for two different matrices

Unembedding layer maps from an embedding to a $1 \times |\mathcal{V}|$ vector of logits

Language modeling head

1 x d

 W_N

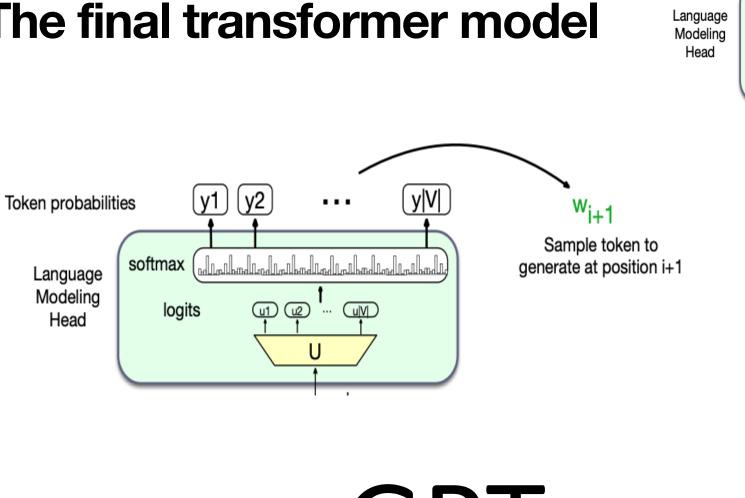
Logits, the score vector \mathbf{u}

One score for each of the $|\mathcal{V}|$ possible words in the vocabulary \mathcal{V} . Shape $1 \times |\mathcal{V}|$.

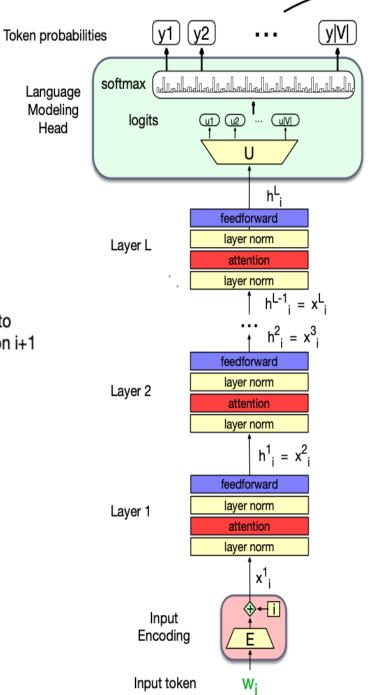
Softmax turns the logits into probabilities over vocabulary. Shape $1 \times |\mathcal{V}|$.

$$\mathbf{u} = \mathbf{h}_N^L \mathbf{E}^{\top}$$
$$\mathbf{y} = \operatorname{softmax}(\mathbf{u})$$

The final transformer model







 w_{i+1}

Sample token to

generate at position i+1

LM loss

The LM head takes output of final transformer layer L, multiplies it by unembedding layer and turns into probabilities:

$$\mathbf{u}_i = E\mathbf{h}_i^L$$
 $\mathbf{y}_i = \operatorname{softmax}(\mathbf{u}_i)$

The loss is the probability of the next word, given output h_i^L :

$$\mathcal{L}_{LM}(x_i) = -\log P(x_{i+1} \mid \mathbf{h_i}^L) = -\log \hat{\mathbf{y}}[\mathbf{x_{i+1}}]$$

We get the gradients by taking the average of this loss over the batch

$$\mathcal{L}_{LM} = -\frac{1}{|\mathcal{B}|} \sum_{s \in \mathcal{B}} \frac{1}{|s|} \sum_{i \in s} \log P(x_{i+1} \mid \mathbf{h}_i^L)$$

Language Models are Unsupervised Multitask Learners

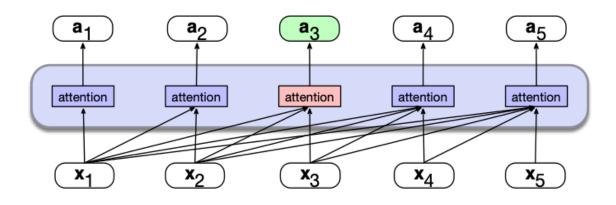
GPT-2 (<u>Radford et al., 2019</u>)

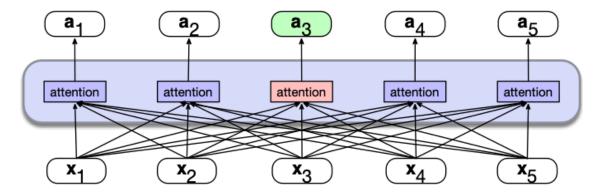
- Trained on ~40GB of text crawled from the internet
- Input context window N=1024 tokens, and model dimensionality $\{d=768, d=1024, d=1280, d=1600\}$
- $\{L=12, L=24, L=36, L=48\}$ layers of transformer blocks
- The resulting models have around {117M, 335M, 762M, 1542M} parameters

Masked Language Modeling

- We've seen autoregressive (causal, left-to-right) LMs.
- But what about tasks for which we want to peak at future tokens?
 - Especially true for tasks where we map each input token to an output token
- Bidirectional encoders use unmasked self-attention to
 - map sequences of input embeddings $\mathbf{x}_1, \dots, \mathbf{x}_n$
 - to sequences of output embeddings of the same length $\mathbf{h}_1, \dots, \mathbf{h}_n$
 - where the output vectors have been contextualized using information from the entire input sequence.

Bidirectional Self-Attention





a) A causal self-attention layer

b) A bidirectional self-attention layer

We just remove the mask

Casual self-attention

Ν

q1·k1	-∞	-8	-∞
q2·k1	q2•k2	-8	-∞
q3•k1	q3·k2	q3·k3	-8
q4·k1	q4·k2	q4•k3	q4•k4

N

$$\mathbf{A} = \operatorname{softmax} \left(\mathbb{M} \left(\frac{\mathbf{Q} \mathbf{K}^{\top}}{\sqrt{d^k}} \right) \right)$$

Bidirectional self-attention

	q1•k1	q1·k2	q1•k3	q1•k4
N I	q2•k1	q2•k2	q2•k3	q2•k4
N	q3•k1	q3•k2	q3•k3	q3•k4
	q4•k1	q4•k2	q4•k3	q4•k4

Ν

$$\mathbf{A} = \operatorname{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^{\top}}{\sqrt{d^k}}\right)$$

Masked training intuition

• For left-to-right (causal; decoder-only) LMs, the model tries to predict the last word from prior words:

The water of Walden Pond is so beautifully _____

- And we train it to improve its predictions.
- For bidirectional masked LMs, the model tries to predict one or more missing words from all the rest of the words:

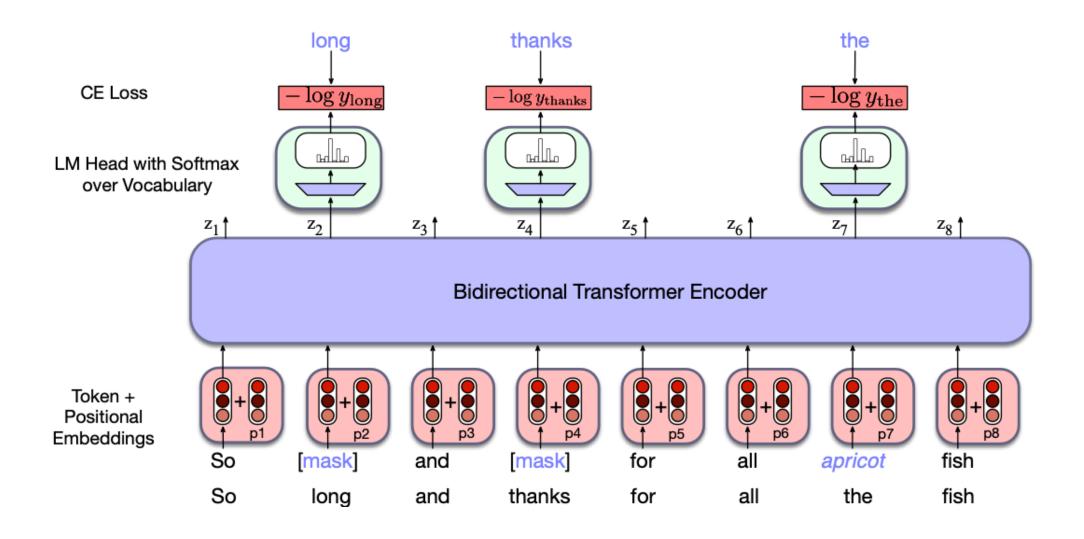
The of Walden Pond so beautifully

blue

- The model generates a probability distribution over the vocabulary for each missing token
- We use the cross-entropy loss from each of the model's predictions to drive the learning process.

Bidirectional Transformer





MLM training in BERT

15% of the tokens are randomly chosen to be part of the masking Example: "Lunch was **delicious**", if delicious was randomly chosen:

Three possibilities:

- 80%: Token is replaced with special token [MASK]
 Lunch was delicious -> Lunch was [MASK]
- 2. 10%: Token is replaced with a random token (sampled from unigram prob)

Lunch was delicious -> Lunch was gasp

3. 10%: Token is unchanged

Lunch was delicious -> Lunch was delicious

MLM loss

The LM head takes output of final transformer layer L, multiplies it by unembedding layer and turns into probabilities:

$$\mathbf{u}_i = E\mathbf{h}_i^L$$
 $\mathbf{y}_i = \operatorname{softmax}(\mathbf{u}_i)$

E.g., for the x_i corresponding to "long", the loss is the probability of the correct word *long*, given output h_i^L):

$$\mathscr{L}_{\mathsf{MLM}}(x_i) = -\log P(x_i \mid \mathbf{h}_i^L)$$

We get the gradients by taking the average of this loss over the batch

$$\mathcal{L}_{\mathsf{MLM}} = -\frac{1}{|\mathcal{B}|} \sum_{s \in \mathcal{B}} \frac{1}{|\mathcal{M}_s|} \sum_{i \in \mathcal{M}_s} \log P(x_i \mid \mathbf{h}_i^L)$$

Bidirectional Encoder Representations from Transformers

BERT (Devlin et al., 2019)

- 30,000 English-only tokens (WordPiece tokenizer)
- Input context window N=512 tokens, and model dimensionality d=768
- L=12 layers of transformer blocks, each with A=12 (bidirectional) multihead-attention layers.
- The resulting model has about 100M parameters.

XLM-RoBERTa (Conneau et al., 2020)

- 250,000 multilingual tokens (SentencePiece Unigram LM tokenizer)
- Input context window N=512 tokens, model dimensionality d=1024
- L=24 layers of transformer blocks, with A=16 multihead attention layers each
- The resulting model has about 550M parameters.

[15 minute break]

Implementing the Transformer!

Team up!

Open exercises/week 8 in your course folder and start writing/running code!