

Simple neural networks and logistic regression

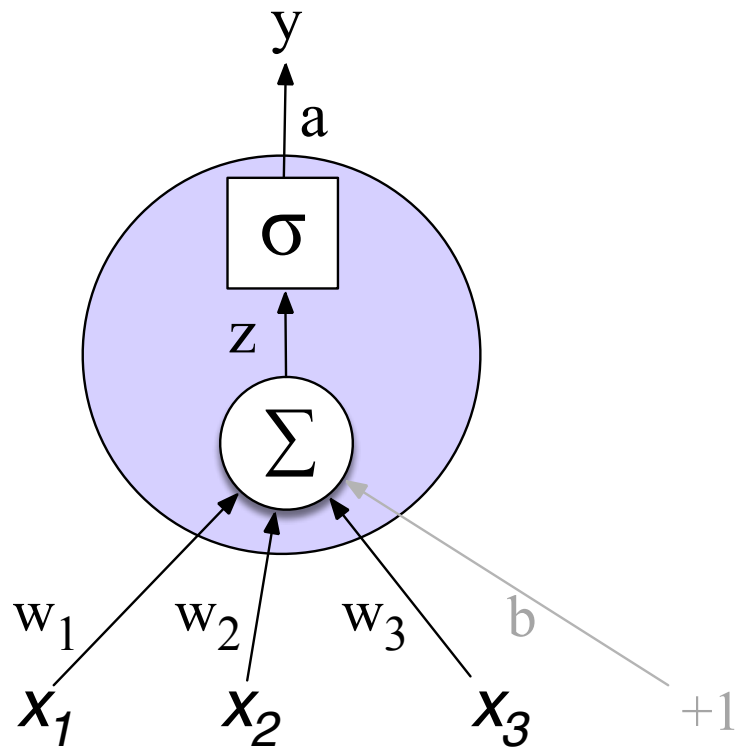
NLP Week 6

Thanks to Dan Jurafsky for most of the slides this week!

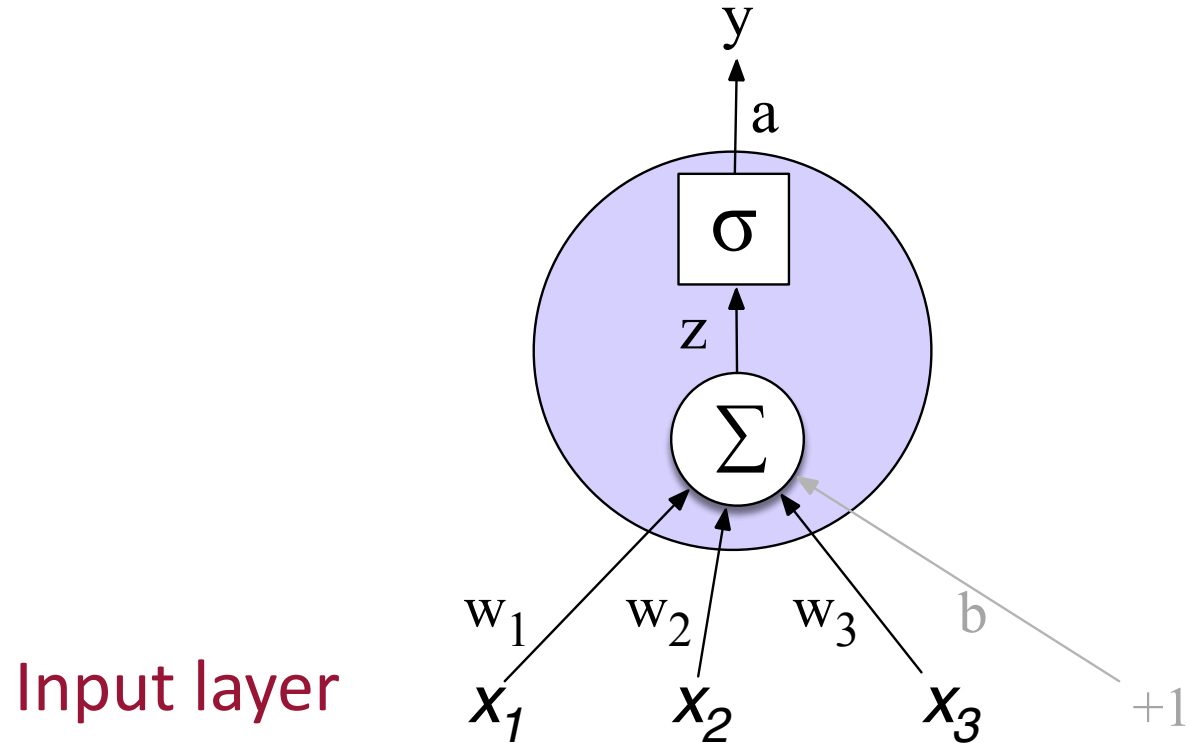
Plan for today

1. Neural Units
2. Simple neural network architectures and logistic regression
3. Training models and Back-propagation
4. *Group exercises*

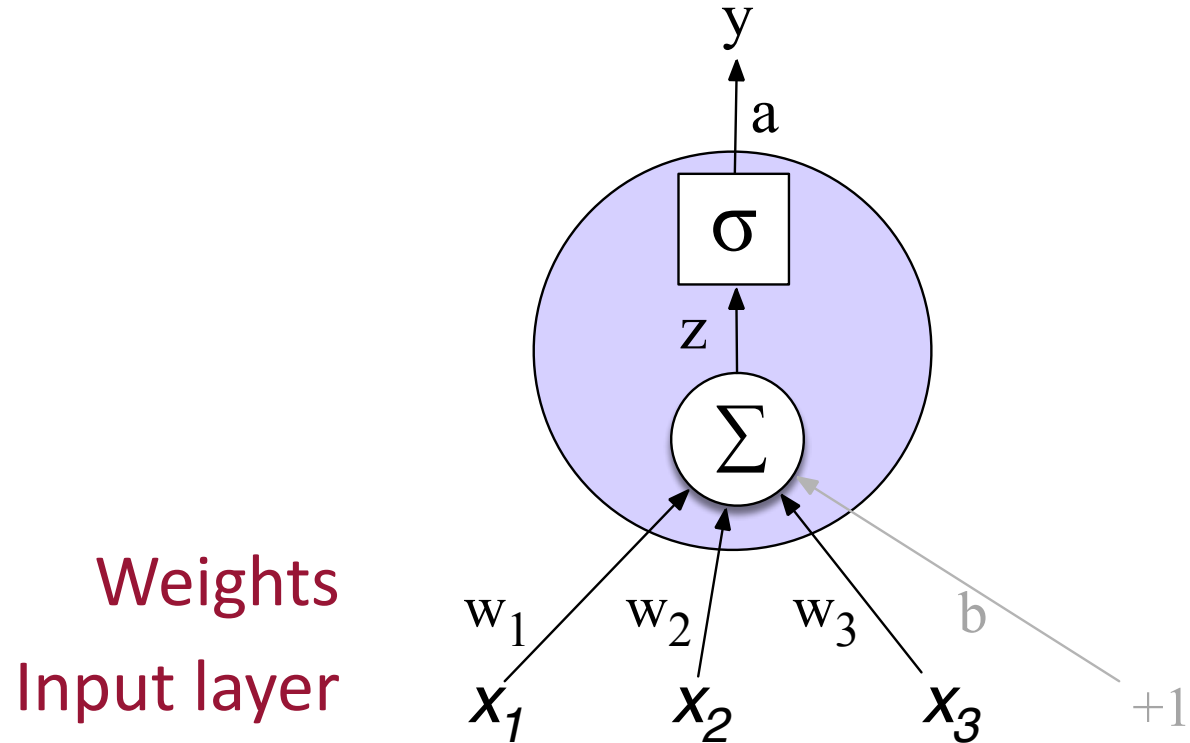
Neural Network Unit



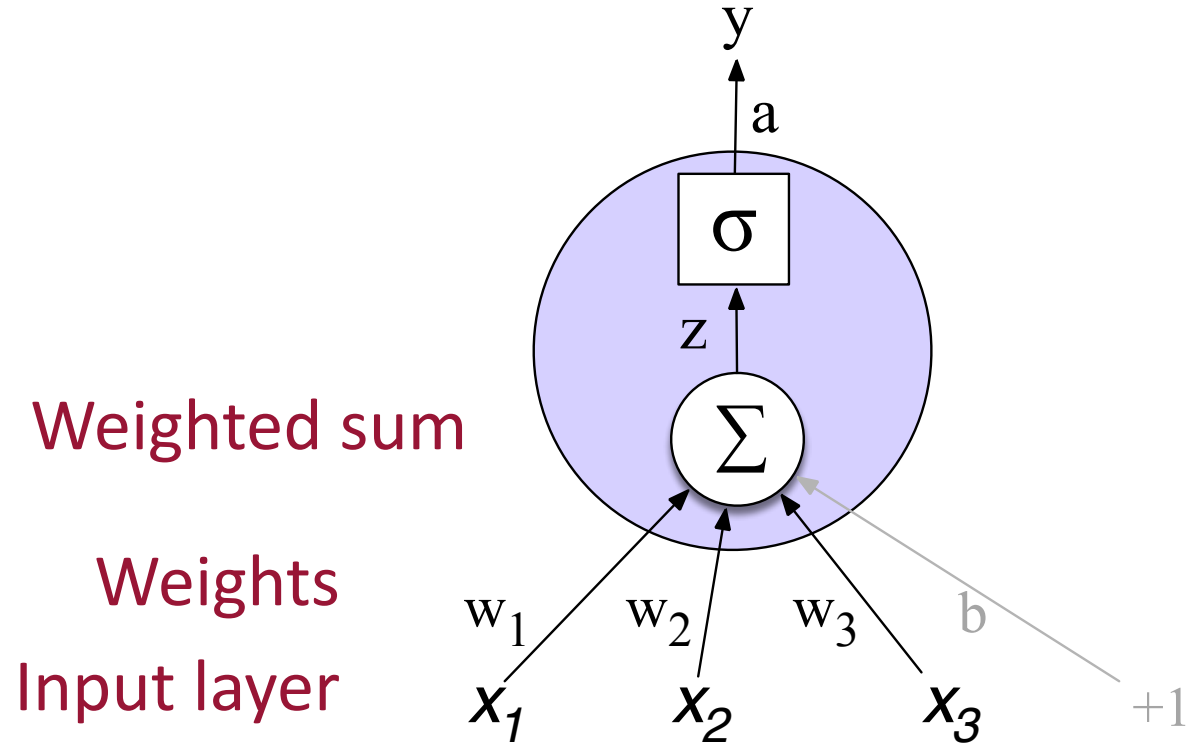
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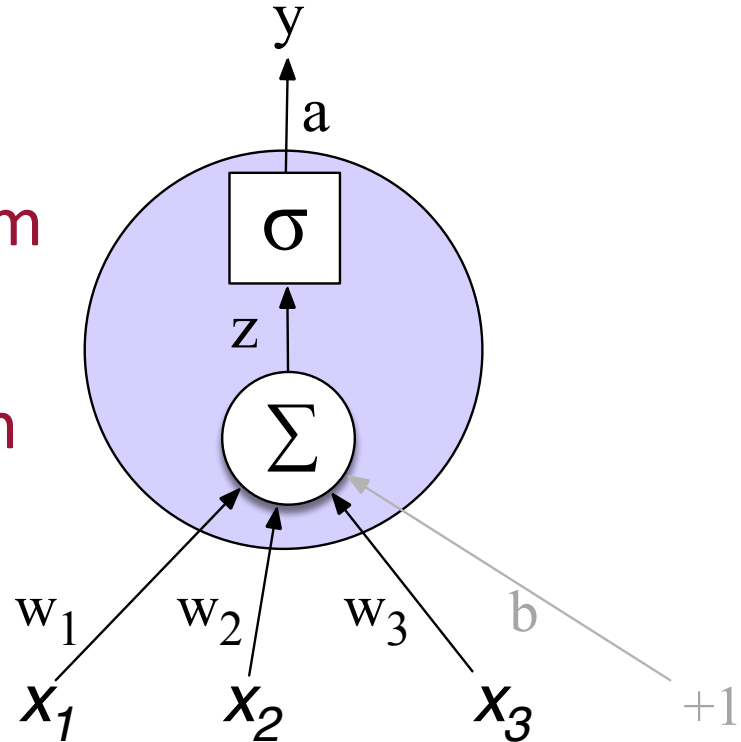
Neural Network Unit

Non-linear transform

Weighted sum

Weights

Input layer



Neural Network Unit

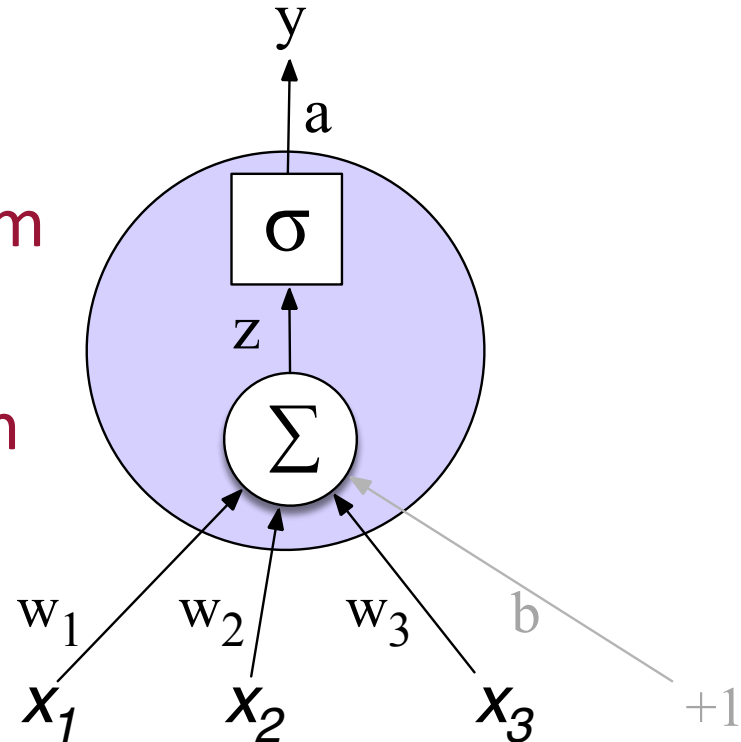
Output value

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Neural Network Unit

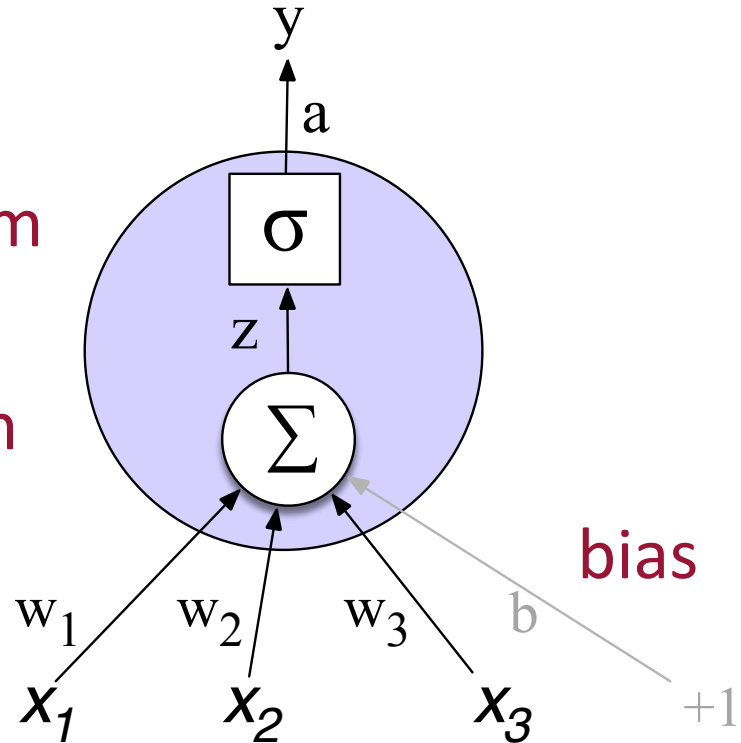
Output value

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Neural unit

Take weighted sum of inputs, plus a bias

$$z = b + \sum_i w_i x_i$$

$$z = w \cdot x + b$$

Instead of just using z , we'll apply a nonlinear activation function f :

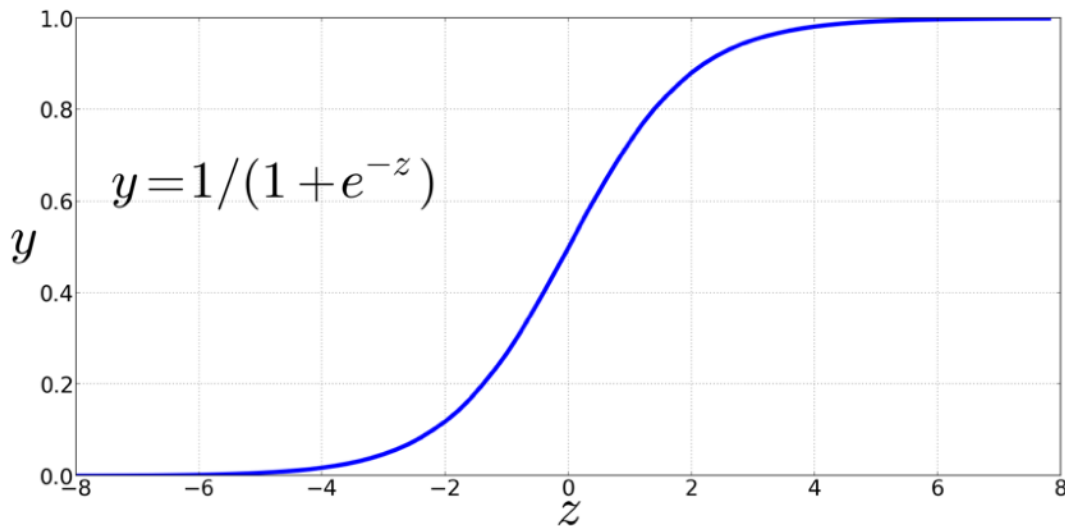
$$y = a = f(z)$$

Non-Linear Activation Functions

We're already seen the sigmoid for logistic regression:

Sigmoid

$$y = \sigma(z) = \frac{1}{1 + e^{-z}}$$



Final function the unit is computing

$$y = \sigma(w \cdot x + b) = \frac{1}{1 + \exp(-(w \cdot x + b))}$$

Final unit again

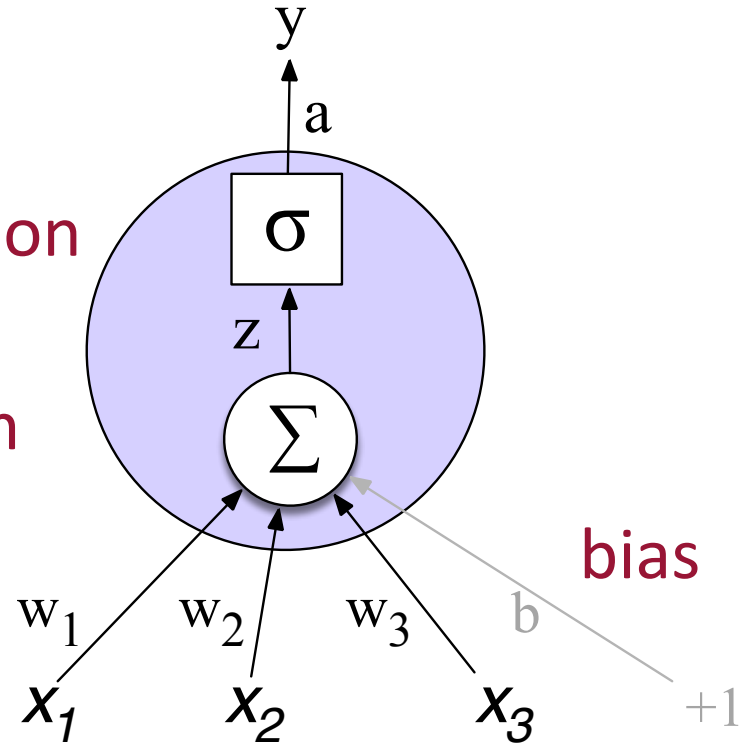
Output value

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An example

Suppose a unit has:

$$w = [0.2, 0.3, 0.9]$$

$$b = 0.5$$

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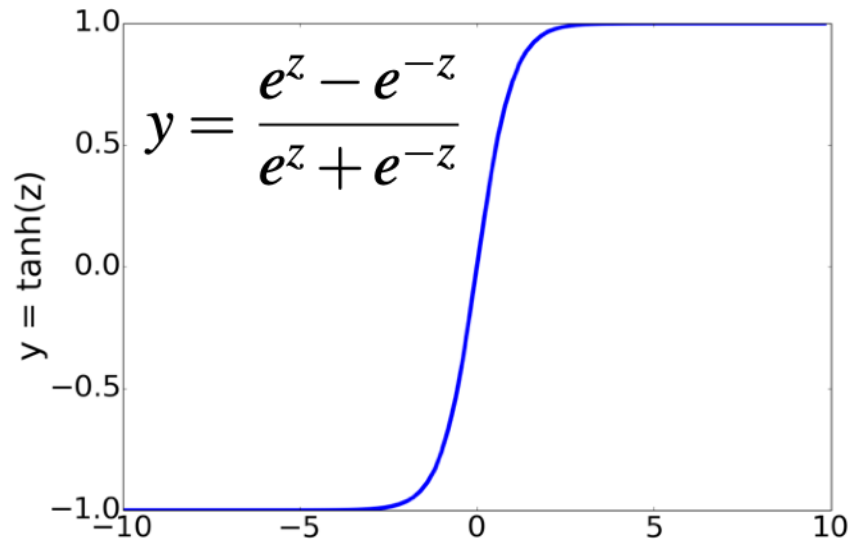
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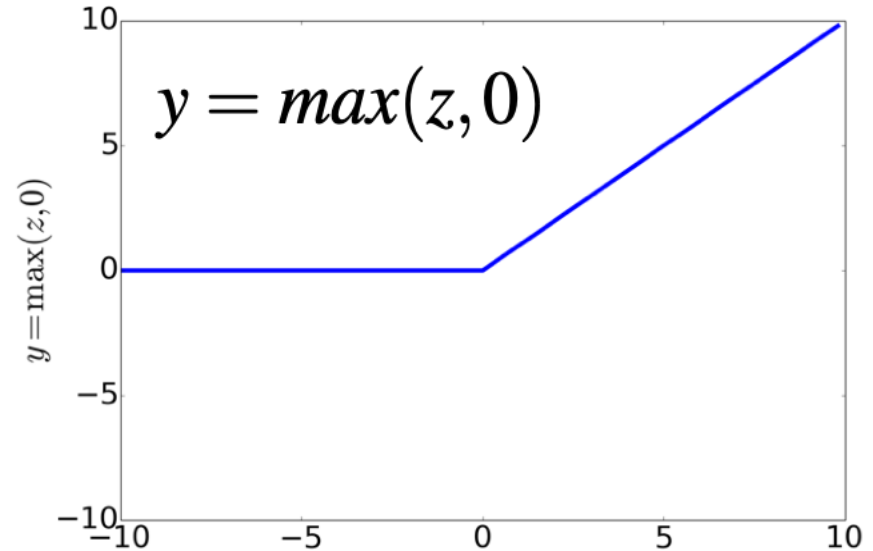
$$y = \sigma(w \cdot x + b) = \frac{1}{1 + e^{-(w \cdot x + b)}} = \frac{1}{1 + e^{-(.5 * .2 + .6 * .3 + .1 * .9 + .5)}} = \frac{1}{1 + e^{-0.87}} = .70$$

Non-Linear Activation Functions besides sigmoid

Most Common:



tanh

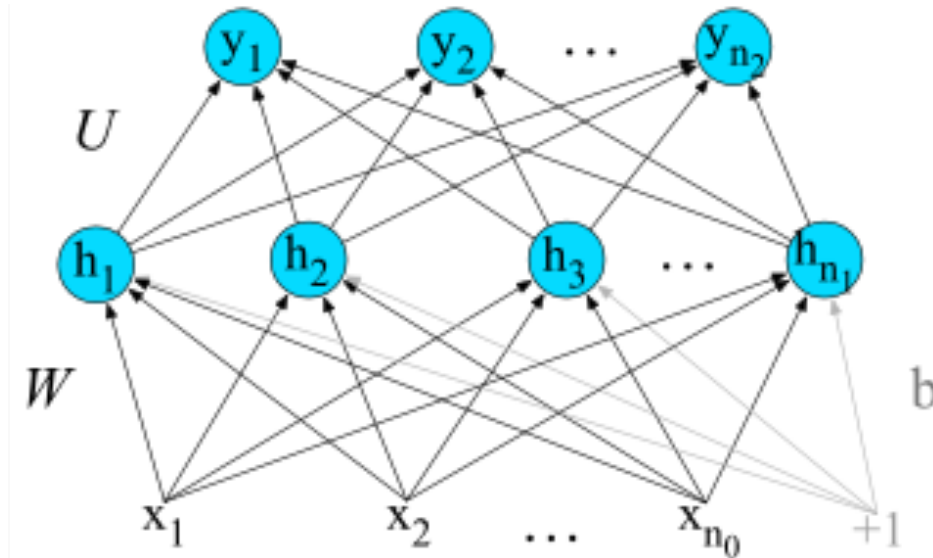


ReLU

Rectified Linear Unit

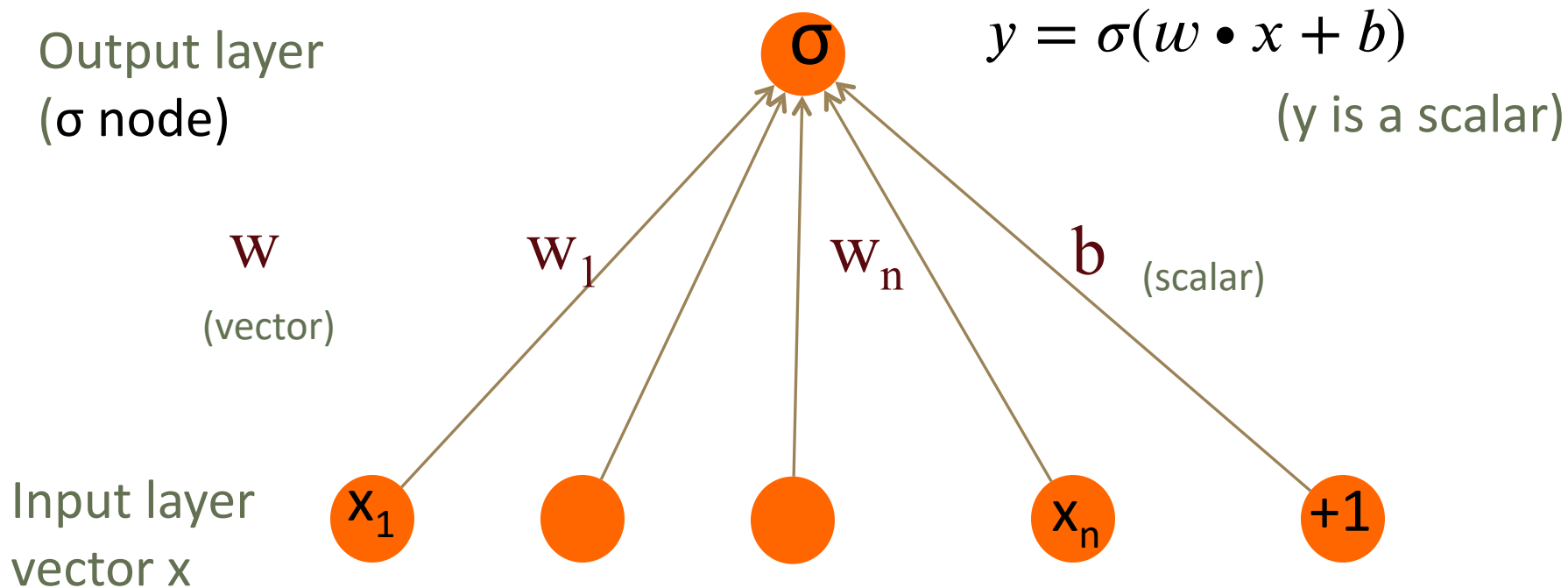
Feedforward Neural Networks

Can also be called **multi-layer perceptrons** (or **MLPs**) for historical reasons



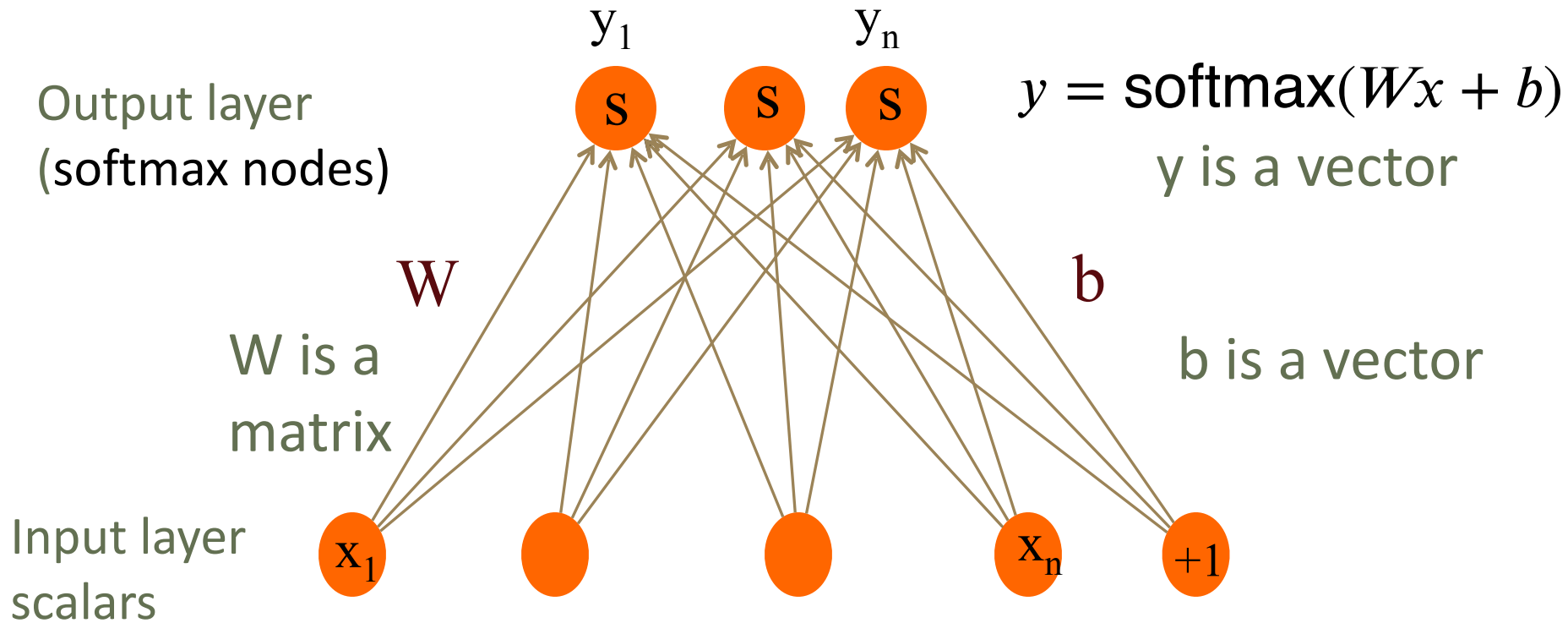
Binary Logistic Regression as a 1-layer Network

(we don't count the input layer in counting layers!)



Multinomial Logistic Regression as a 1-layer Network

Fully connected single layer network



Reminder: softmax: a generalization of sigmoid

$$\text{softmax}(z) = \left[\frac{\exp(z_1)}{\sum_{i=1}^k \exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^k \exp(z_i)}, \dots, \frac{\exp(z_k)}{\sum_{i=1}^k \exp(z_i)} \right]$$

$$\text{softmax}(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^k \exp(z_j)} \quad 1 \leq i \leq k$$

$$z = [0.6, 1.1, -1.5, 1.2, 3.2, -1.1]$$

$$\text{softmax}(z) = [0.055, 0.090, 0.006, 0.099, 0.74, 0.010]$$

Reminder: softmax: a generalization of sigmoid

For a vector z of dimensionality k , the softmax is:

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Example:

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Two-Layer Network with scalar output

Output layer
(σ node)

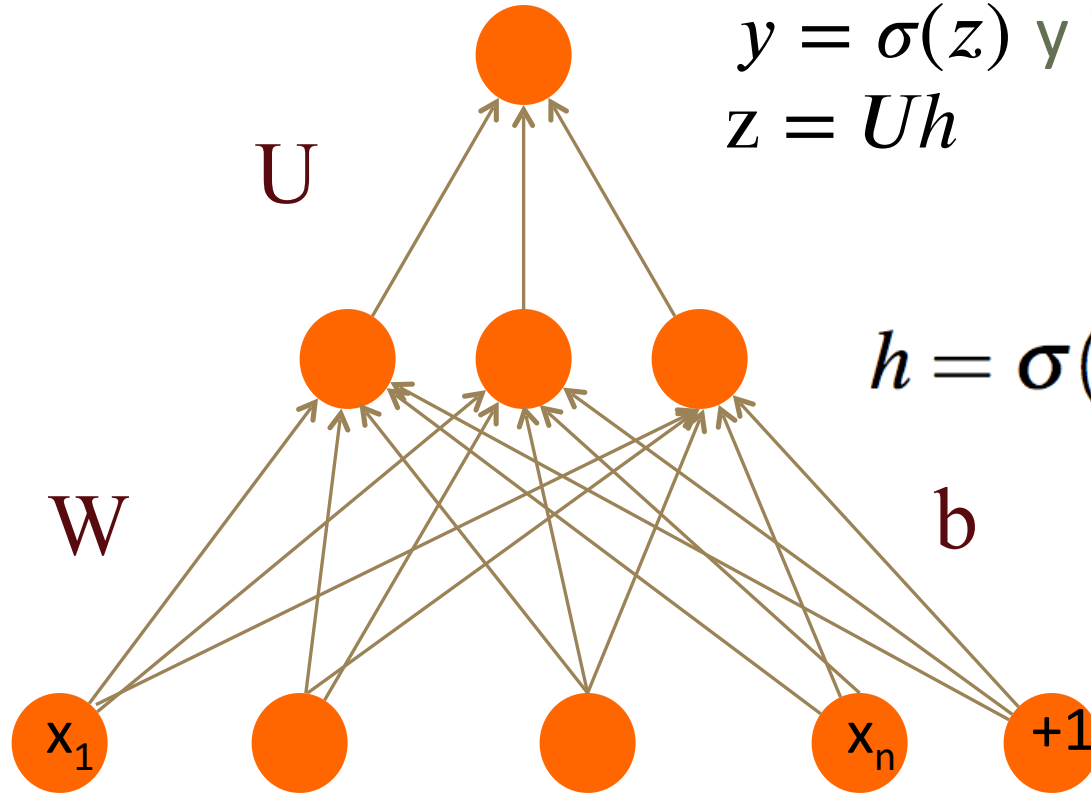
$$y = \sigma(z) \text{ } y \text{ is a scalar}$$
$$z = Uh$$

hidden units
(σ node)

$$h = \sigma(Wx + b)$$

Could be ReLU
Or tanh

Input layer
(vector)



Two-Layer Network with scalar output

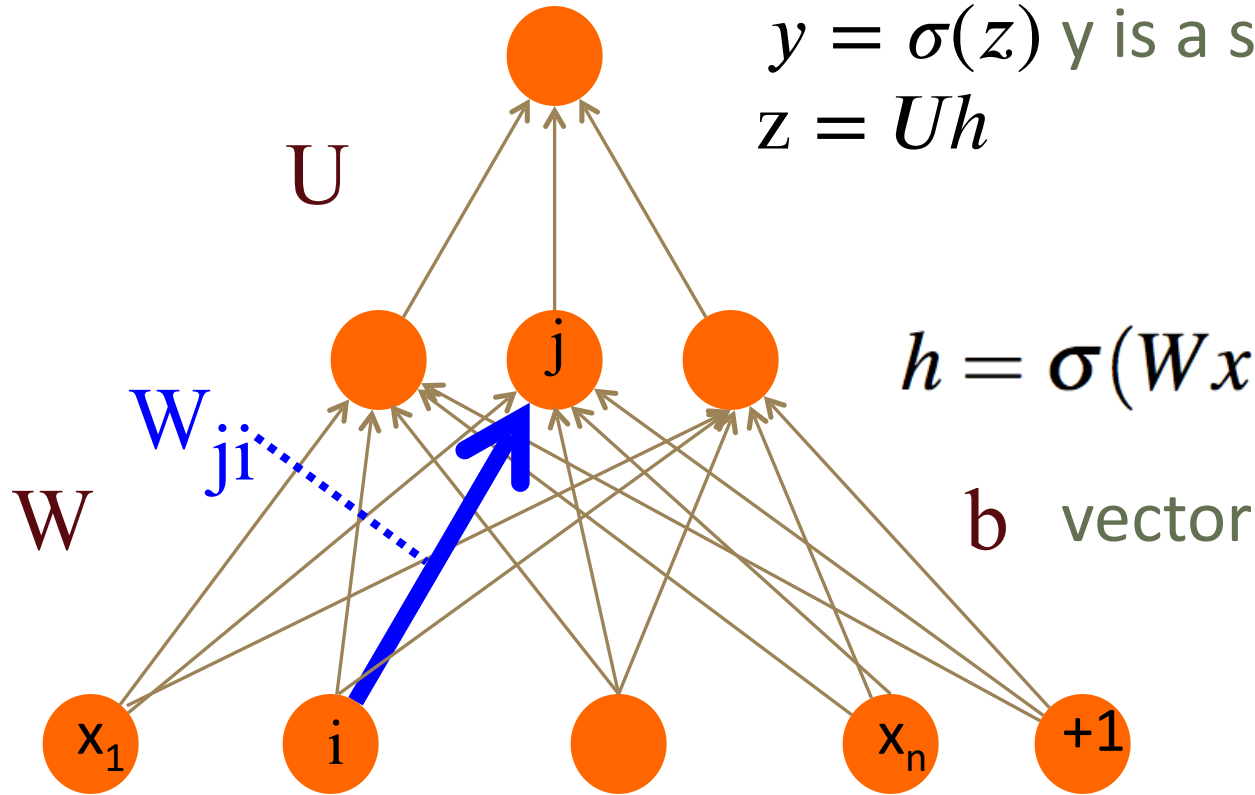
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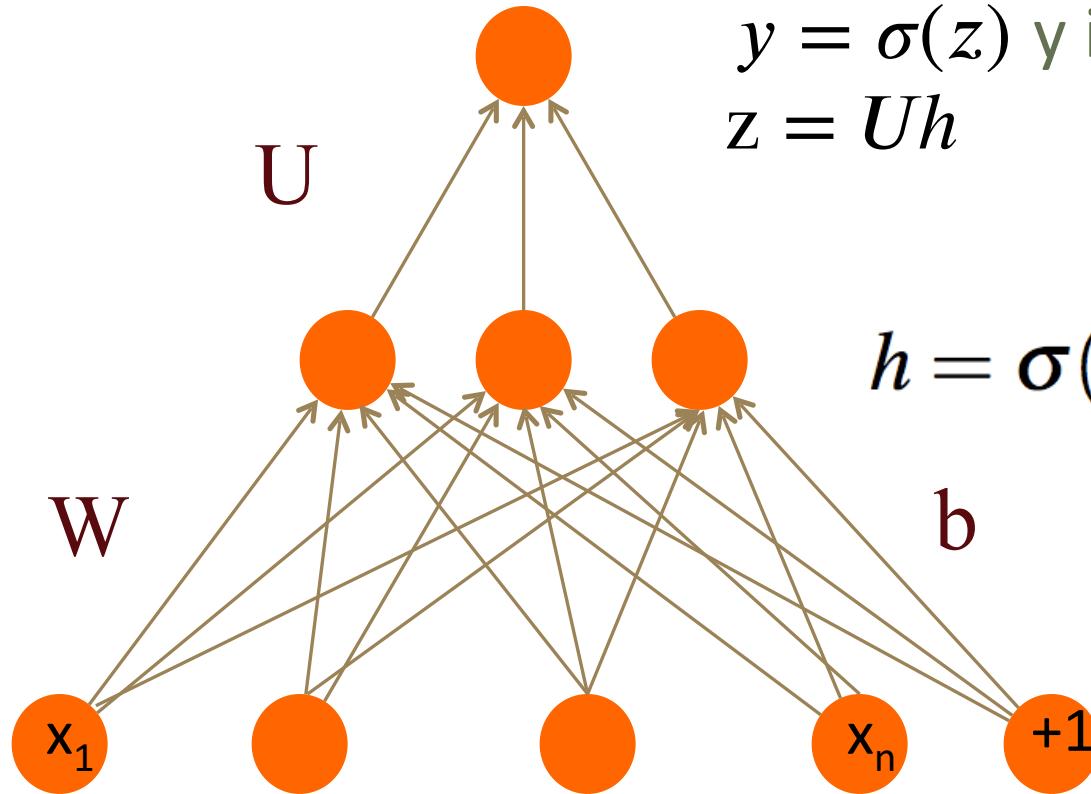
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Two-Layer Network with softmax output

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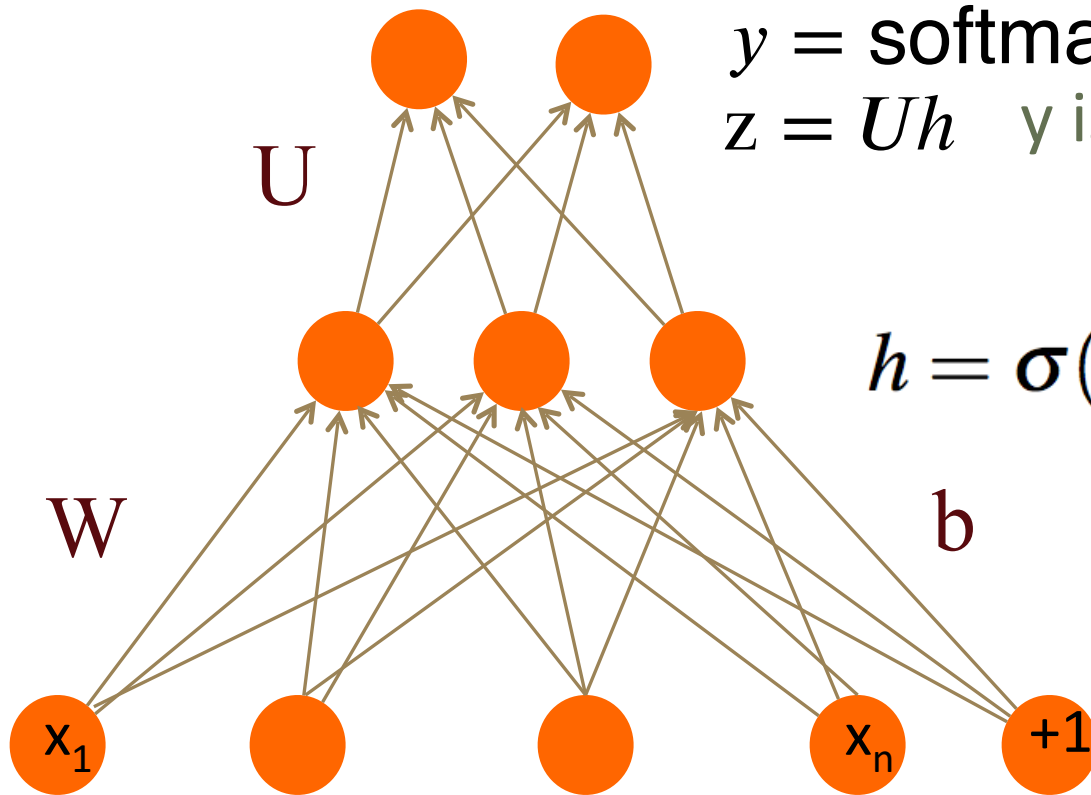
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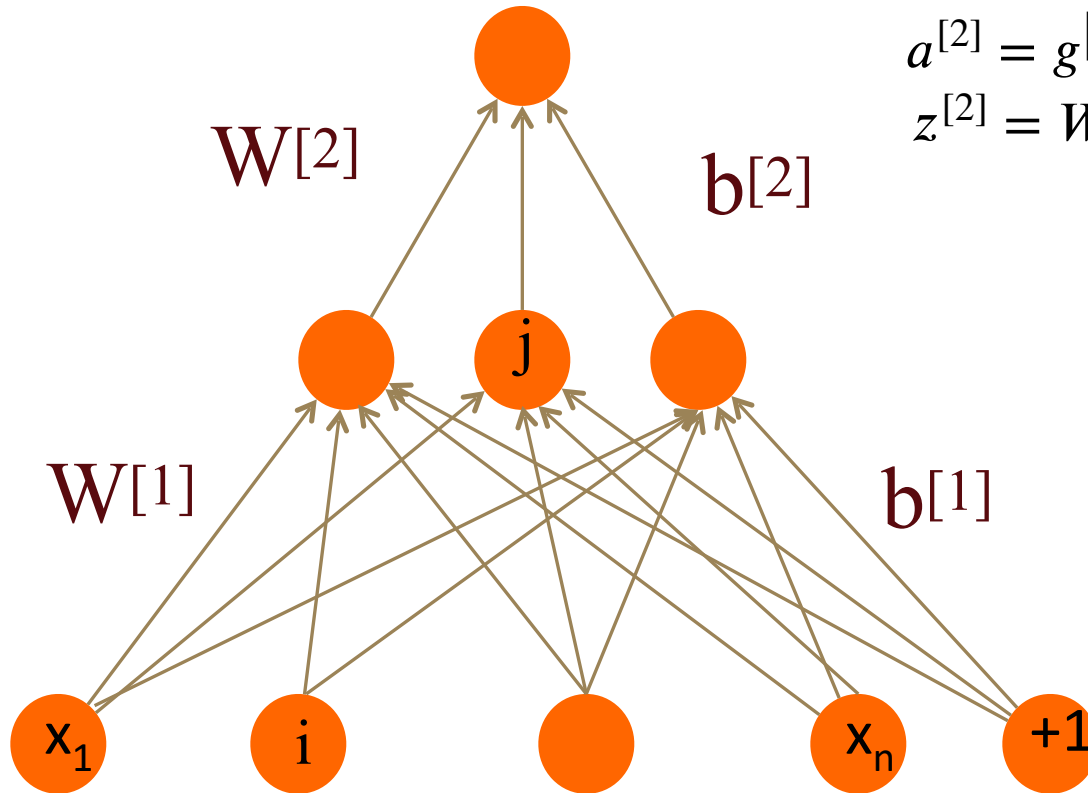
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Multi-layer Notation



$$y = a^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]}) \quad \text{sigmoid or softmax}$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[1]} = g^{[1]}(z^{[1]}) \quad \text{ReLU}$$

$$z^{[1]} = W^{[1]}a^{[0]} + b^{[1]}$$

$$a^{[0]}$$

Multi Layer Notation

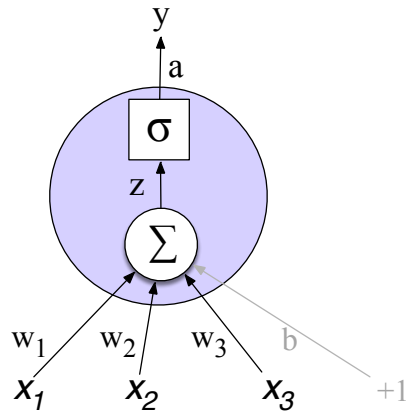
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$$\hat{y} = a^{[2]}$$



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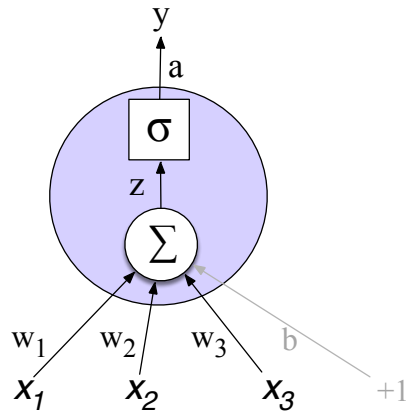
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for i in $1..n$

$$z^{[i]} = W^{[i]} a^{[i-1]} + b^{[i]}$$

$$a^{[i]} = g^{[i]}(z^{[i]})$$

$$\hat{y} = a^{[n]}$$

Replacing the bias unit

Let's switch to a notation without the bias unit

Just a notational change

1. Add a dummy node $a_0=1$ to each layer
2. Its weight w_0 will be the bias
3. So input layer $a^{[0]}_0=1$,
 - And $a^{[1]}_0=1$, $a^{[2]}_0=1, \dots$

Replacing the bias unit

Instead of:

$$x = x_1, x_2, \dots, x_{n_0}$$

$$h = \sigma(Wx + b)$$

$$h_j = \sigma \left(\sum_{i=1}^{n_0} W_{ji} x_i + b_j \right)$$

We'll do this:

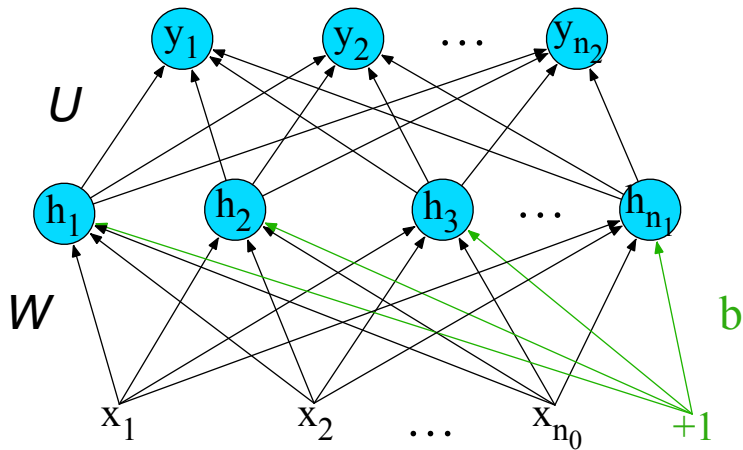
$$x = x_0, x_1, x_2, \dots, x_{n_0}$$

$$h = \sigma(Wx)$$

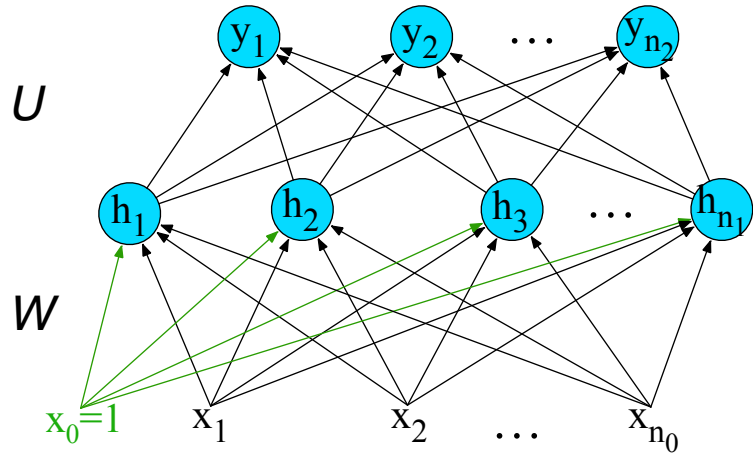
$$\sigma \left(\sum_{i=0}^{n_0} W_{ji} x_i \right)$$

Replacing the bias unit

Instead of:



We'll do this:



Use cases for feedforward networks

Use cases for feedforward networks

Let's consider 2 (simplified) sample tasks:

1. Text classification
2. Language modeling

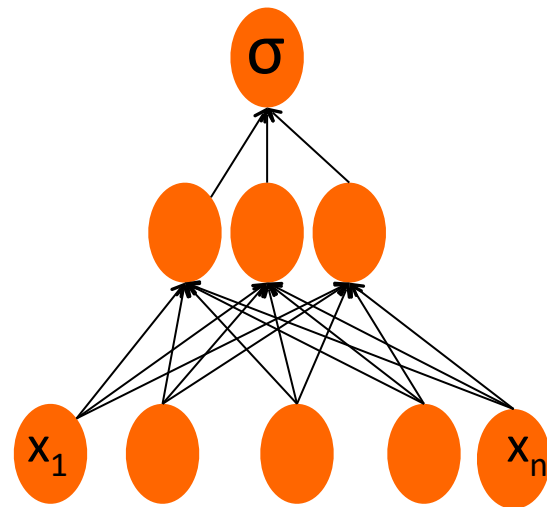
State of the art systems use more powerful neural architectures, but simple models are useful to consider!

Classification: Sentiment Analysis

We could do exactly what we did with logistic regression

Input layer are binary features as before

Output layer is 0 or 1



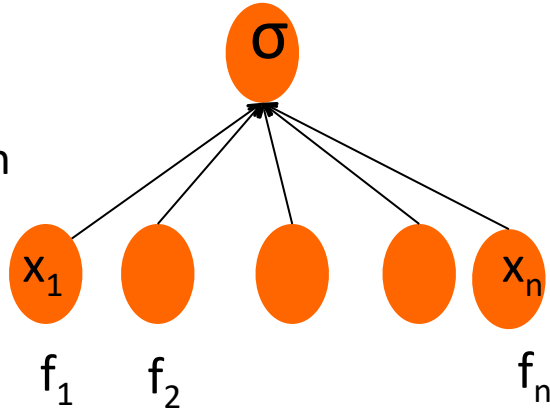
Sentiment Features

Var	Definition
x_1	count(positive lexicon) \in doc)
x_2	count(negative lexicon) \in doc)
x_3	$\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$
x_4	count(1st and 2nd pronouns \in doc)
x_5	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$
x_6	log(word count of doc)

Feedforward nets for simple classification

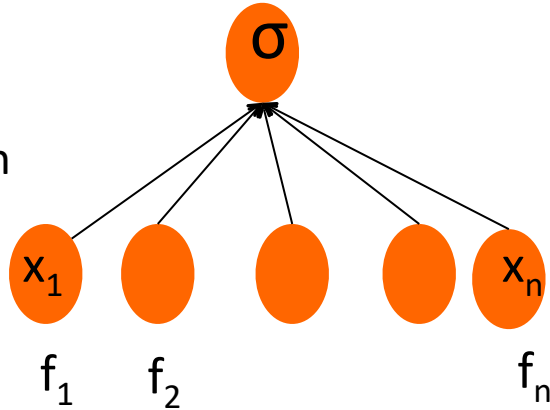
Feedforward nets for simple classification

Logistic
Regression

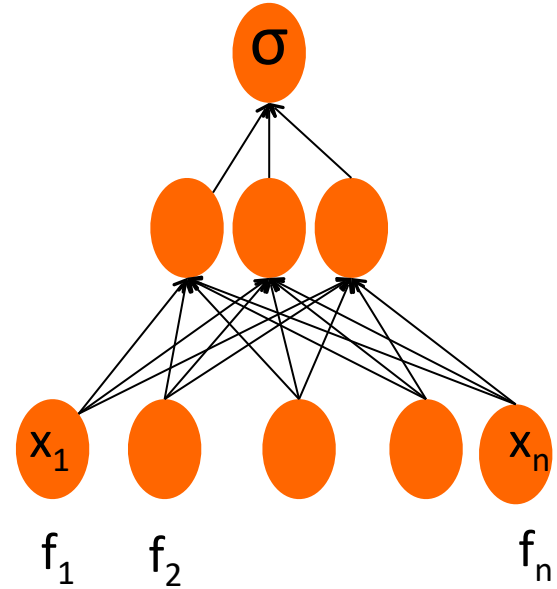


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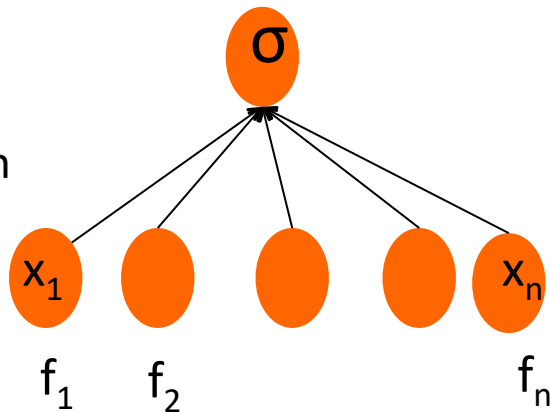


2-layer
feedforward
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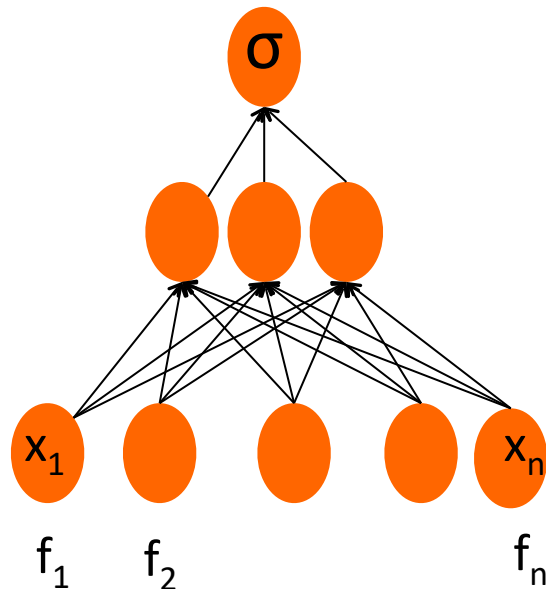


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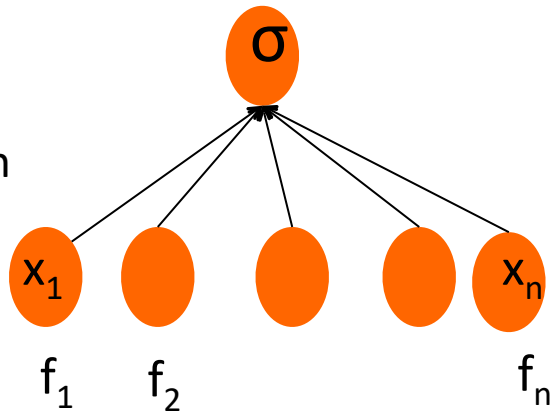
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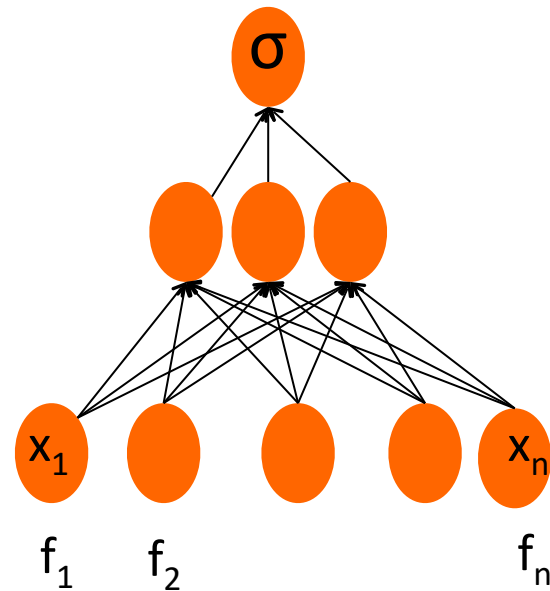
Just adding a hidden layer to logistic regression

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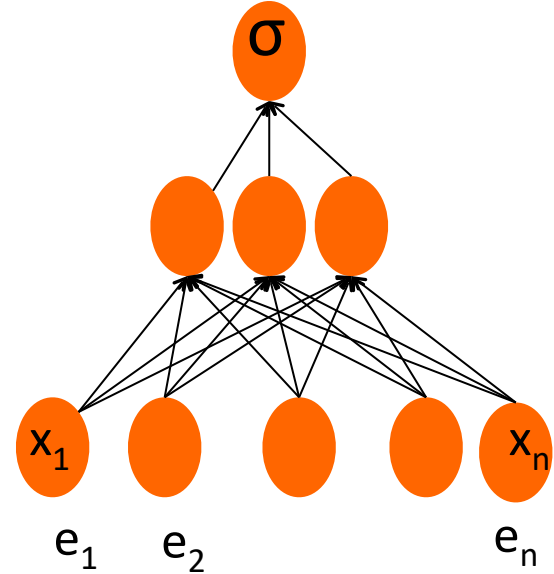


Just adding a hidden layer to logistic regression

- allows the network to use non-linear interactions between features which may (or may not) improve performance.

Even better: representation learning

The real power of deep learning comes from the ability to **learn** features from the data

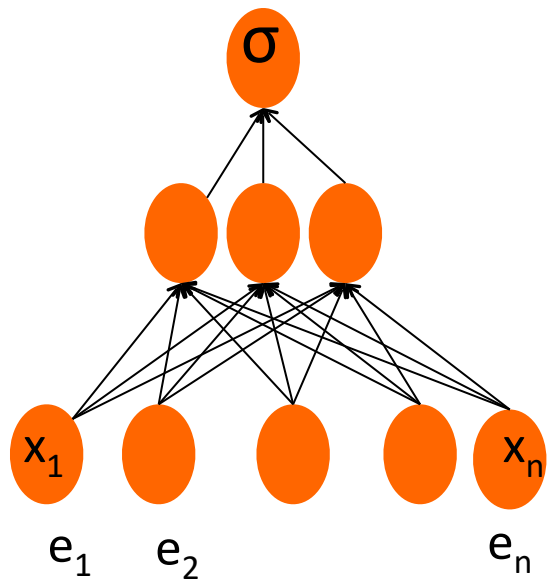


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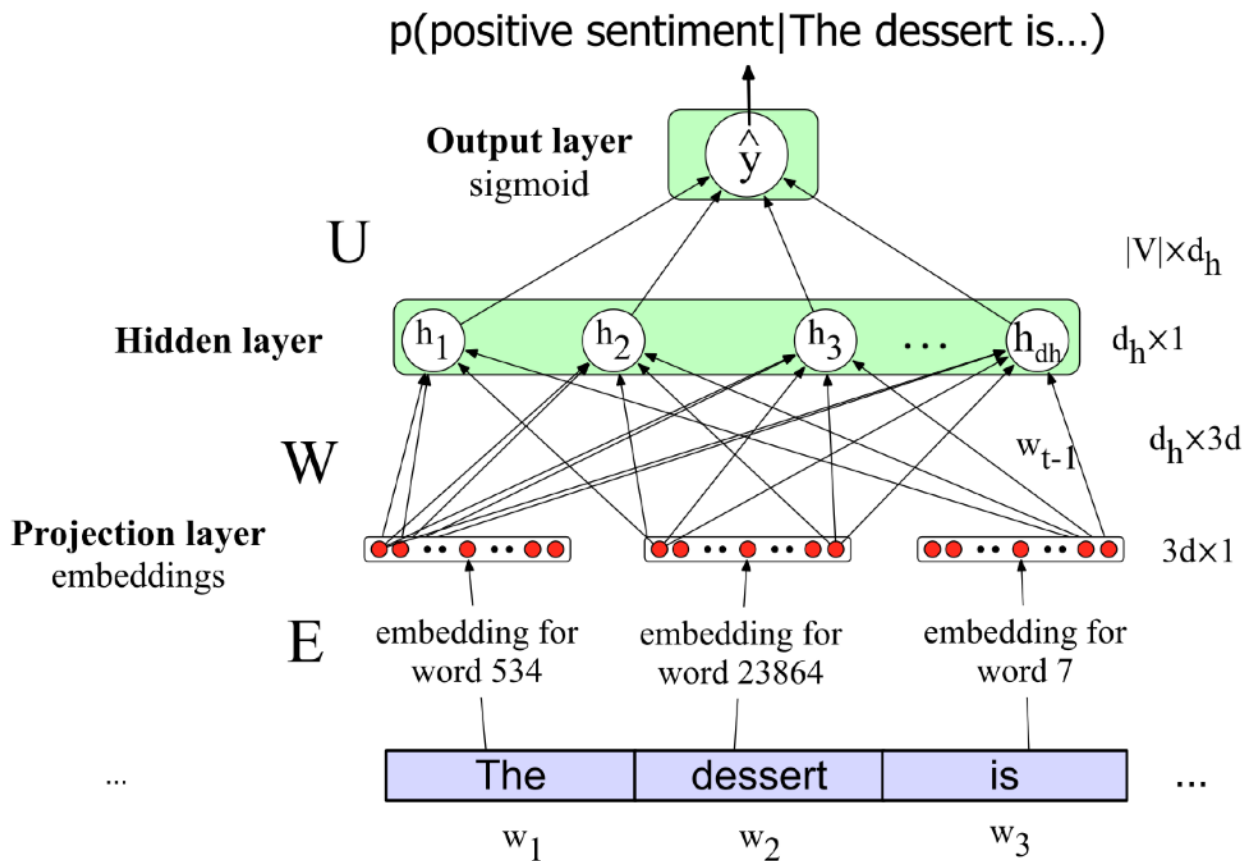
The real power of deep learning comes from the ability to **learn** features from the data

Instead of using hand-built human-engineered features for classification

Use learned representations like embeddings!

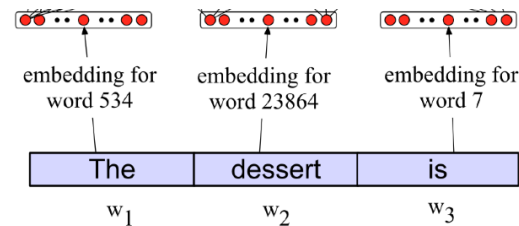


Neural Net Classification with embeddings as input features!



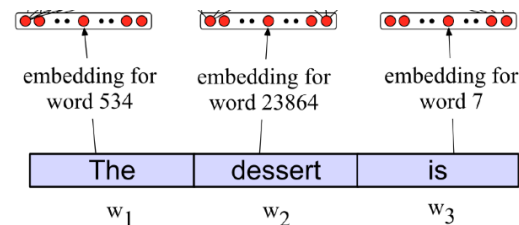
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This assumes a fixed size length (3)!



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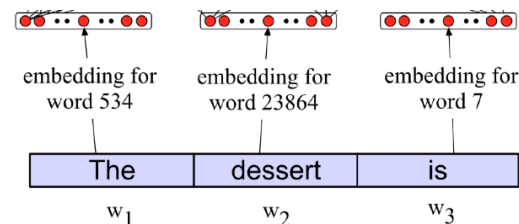


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Some simple solutions (more sophisticated solutions later)



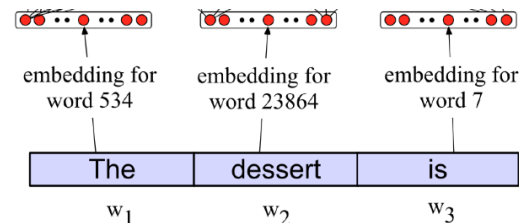
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 - If shorter then pad with zero embeddings



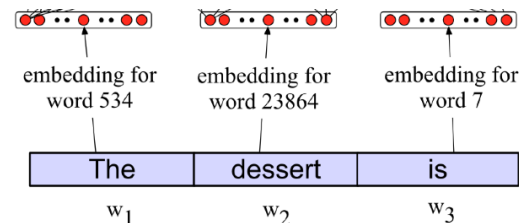
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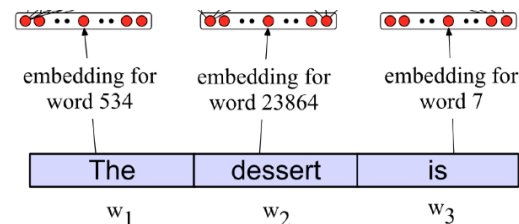


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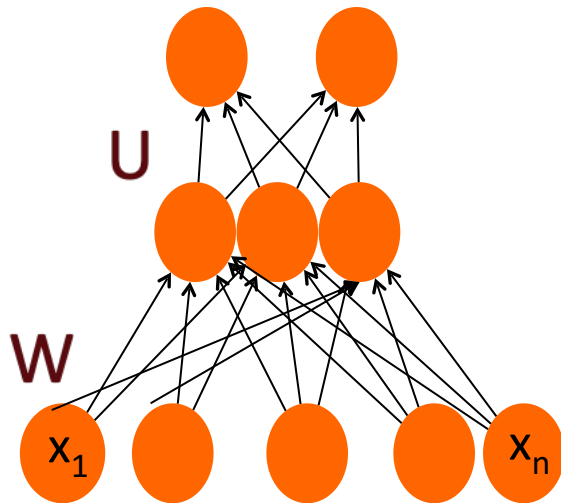
1. Make the input the length of the longest review
 - If shorter then pad with zero embeddings
 - Truncate if you get longer reviews at test time
2. Create a single "sentence embedding" (the same dimensionality as a word) to represent all the words
 - Take the mean of all the word embeddings
 - Take the element-wise max of all the word embeddings
 - For each dimension, pick the max value from all words

Reminder: Multiclass Outputs

What if you have more than two output classes?

- Add more output units (one for each class)
- And use a “softmax layer”

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^k e^{z_j}} \quad 1 \leq i \leq D$$



Neural Language Models (LMs)

Language Modeling: Calculating the probability of the next word in a sequence given some history.

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- But neural network LMs far outperform n-gram language models

State-of-the-art neural LMs are based on more powerful neural network technology like Transformers

But **simple feedforward LMs** can do almost as well!

Simple feedforward Neural Language Models

Task: predict next word w_t

given prior words $w_{t-1}, w_{t-2}, w_{t-3}, \dots$

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Solution: Sliding windows (of fixed length)

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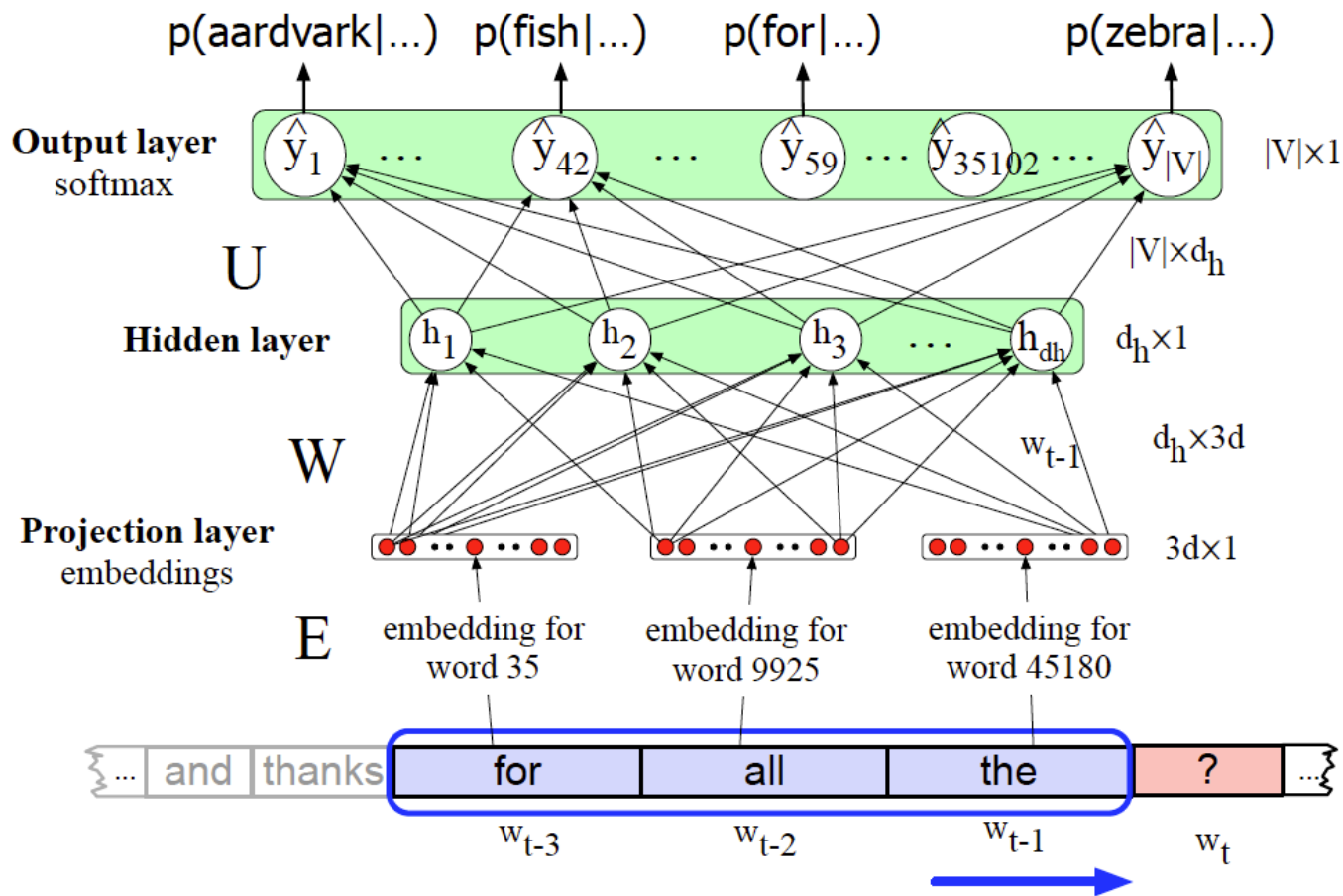
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$$P(w_t | w_1^{t-1}) \approx P(w_t | w_{t-N+1}^{t-1})$$

Neural Language Model



Why Neural LMs work better than N-gram LMs

Training data:

We've seen: I have to make sure that the cat gets fed.

Never seen: dog gets fed

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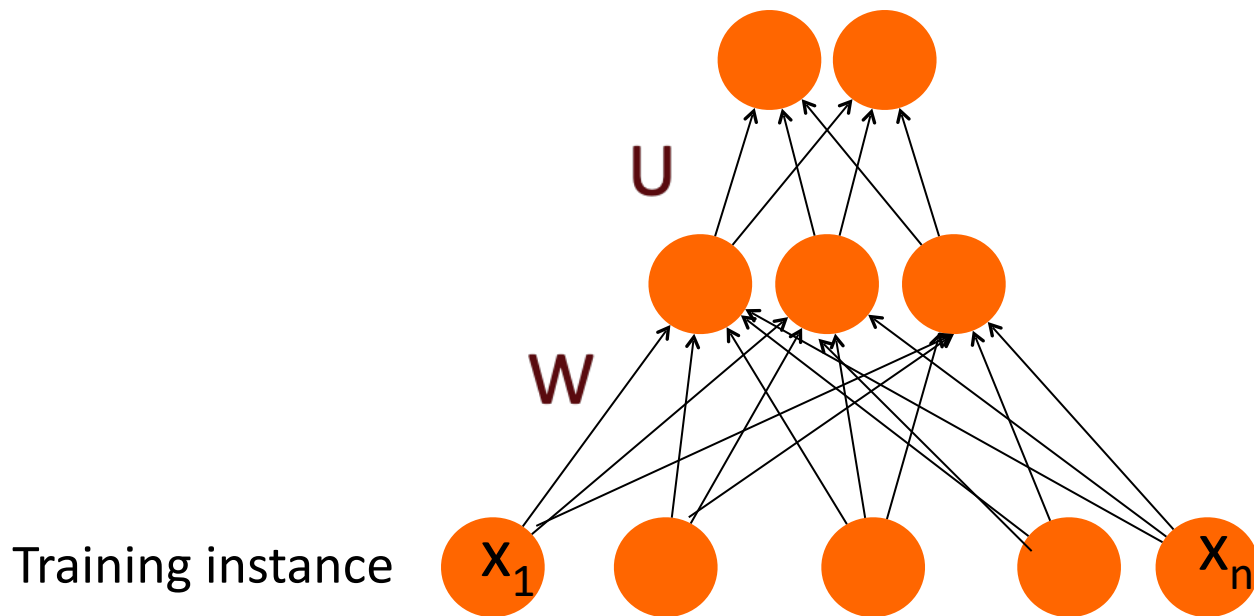
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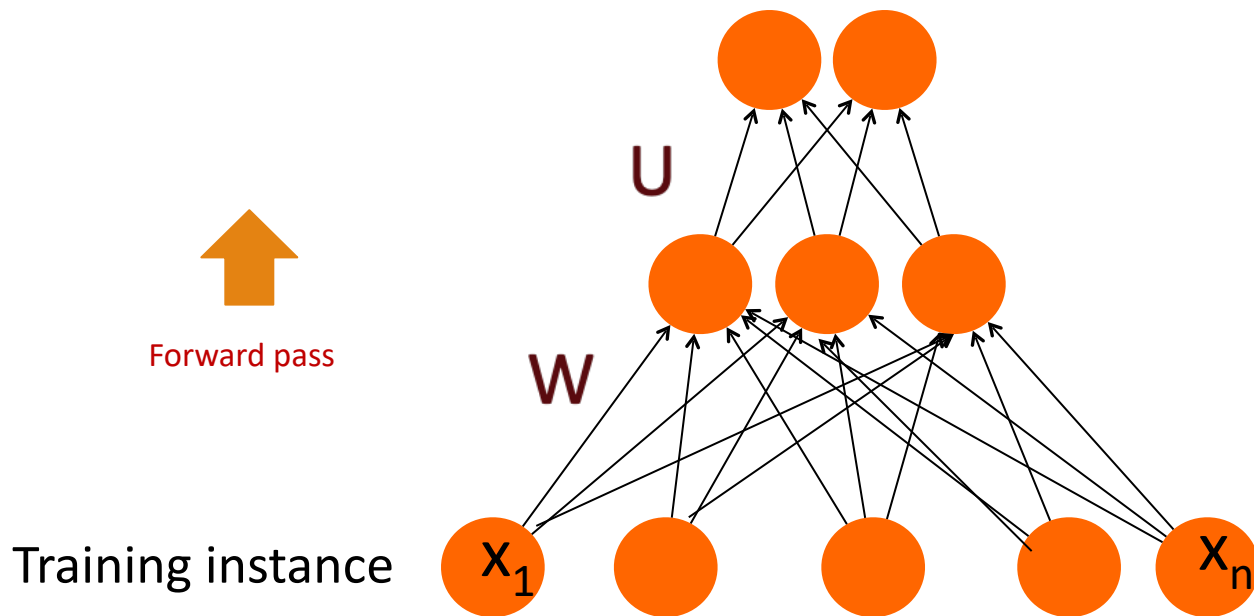
Neural LM can use similarity of "cat" and "dog" embeddings to generalize and predict "fed" after dog

Great, but how do we train, or fit, a neural network model to data?

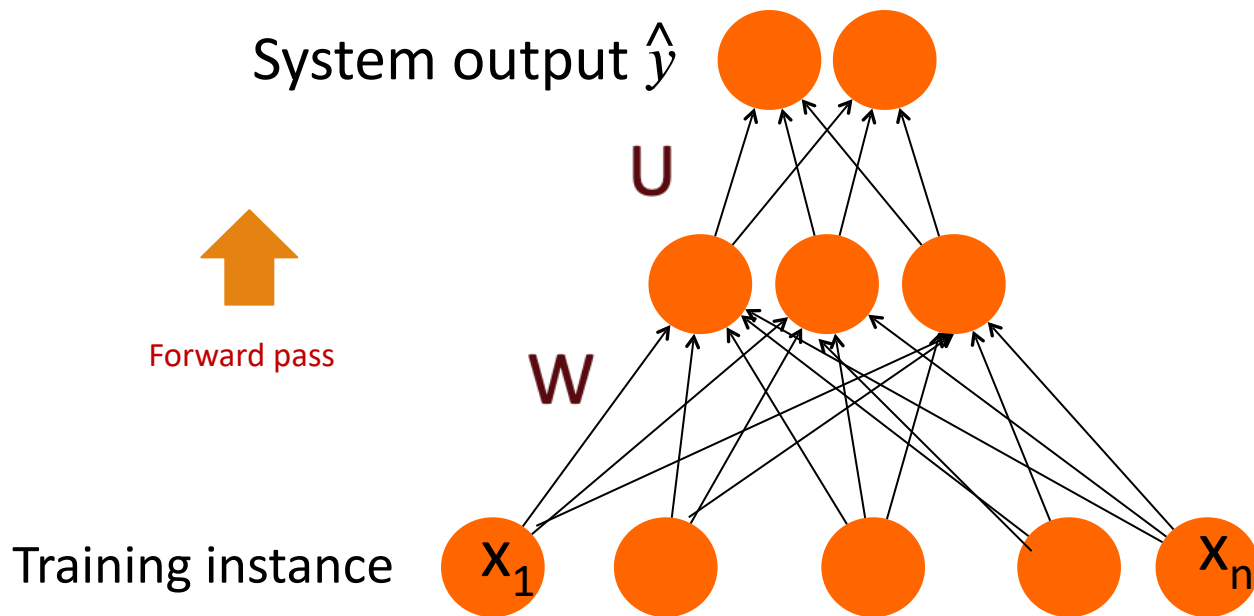
Intuition: training a 2-layer Network



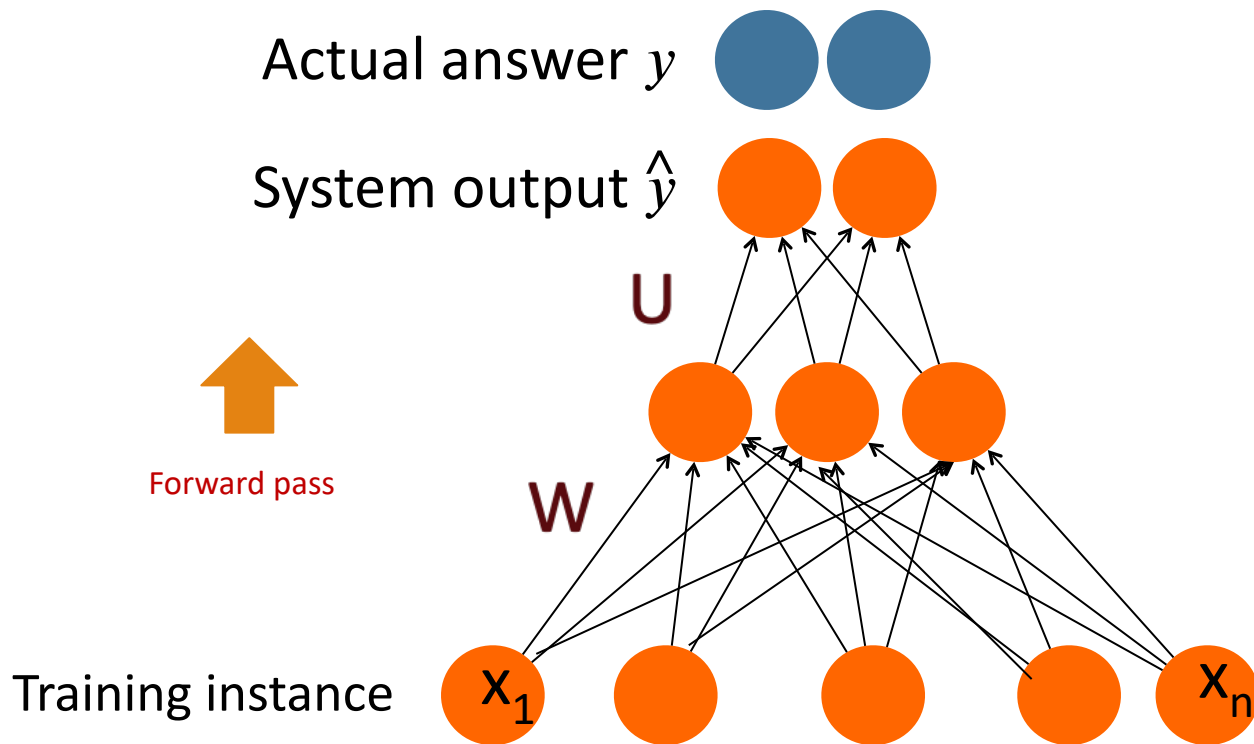
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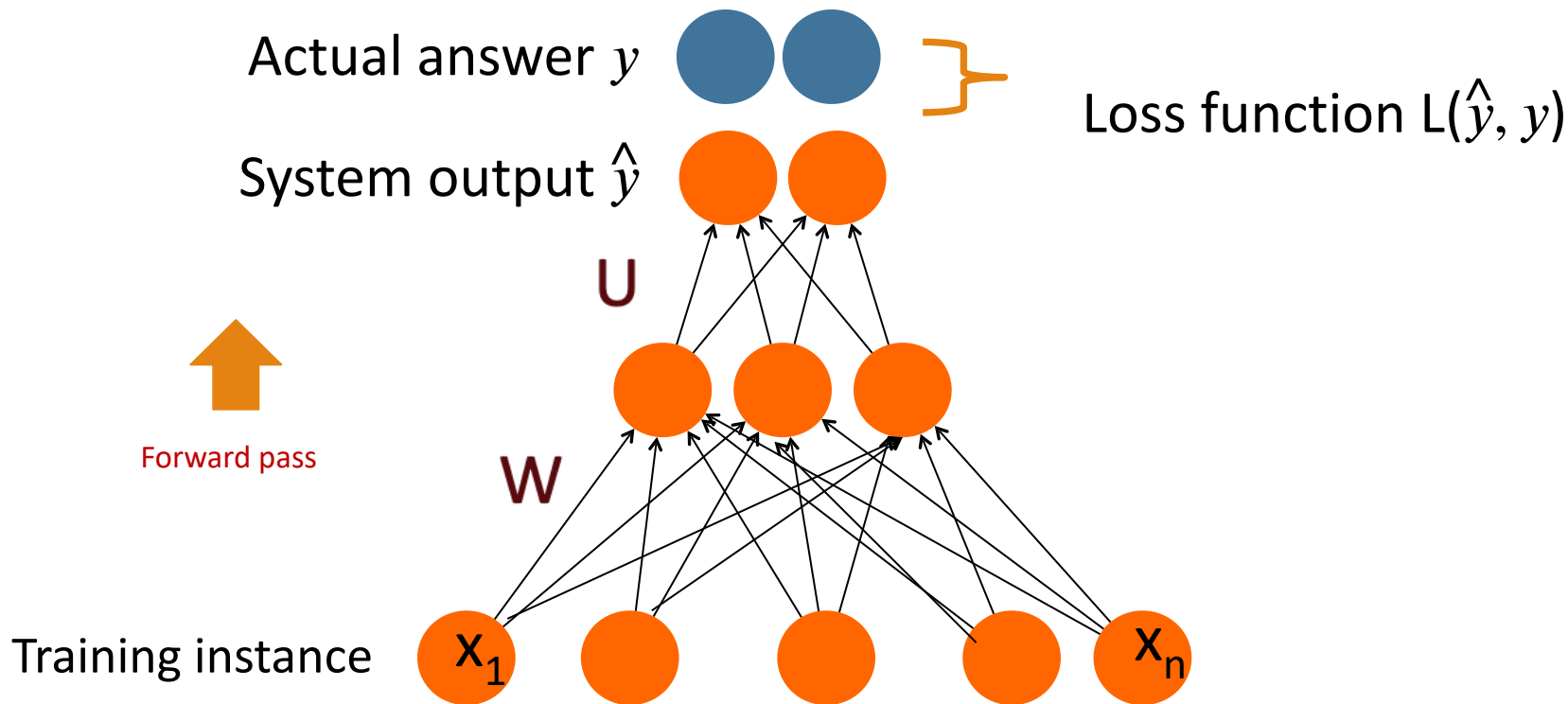
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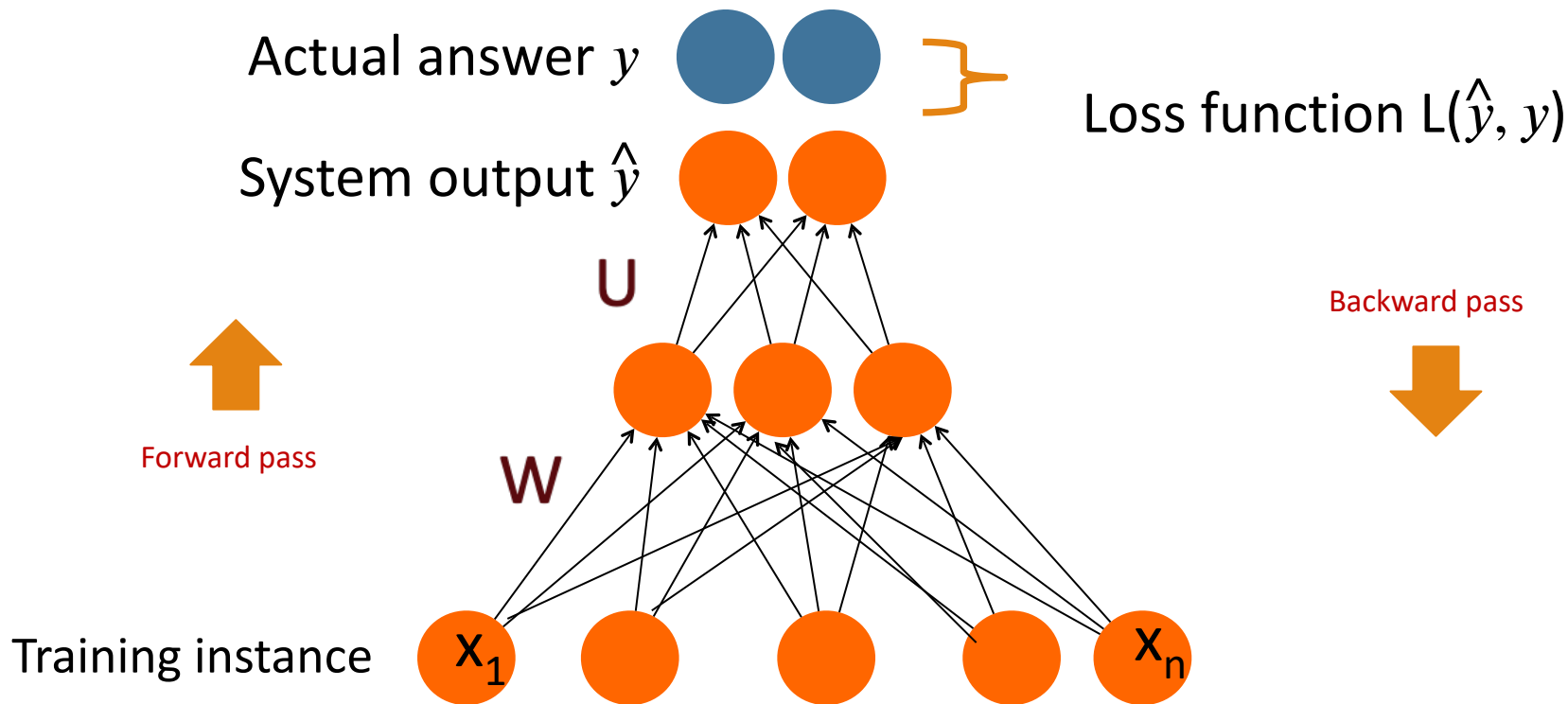
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Intuition: Training a 2-layer network

For every training tuple (x, y)

- Run *forward* computation to find our estimate \hat{y}
- Run *backward* computation to update weights:
 - For every output node
 - Compute loss L between true y and the estimated \hat{y}
 - For every weight w from hidden layer to the output layer
 - Update the weight
 - For every hidden node
 - Assess how much blame it deserves for the current answer
 - For every weight w from input layer to the hidden layer
 - Update the weight

Loss Function for binary logistic regression

A measure for how far off the current answer is to the right answer

Cross entropy loss for logistic regression:

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -\log p(y|x) = -[y \log \hat{y} + (1 - y) \log (1 - \hat{y})] \\ &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log (1 - \sigma(w \cdot x + b))] \end{aligned}$$

Gradient descent for weight updates

Use the derivative of the loss function with respect to weights $\frac{d}{dw}L(f(x; w), y)$

To tell us how to adjust weights for each training item

- Move them in the opposite direction of the gradient

$$w^{t+1} = w^t - \eta \frac{d}{dw}L(f(x; w), y)$$

Where did that derivative come from?

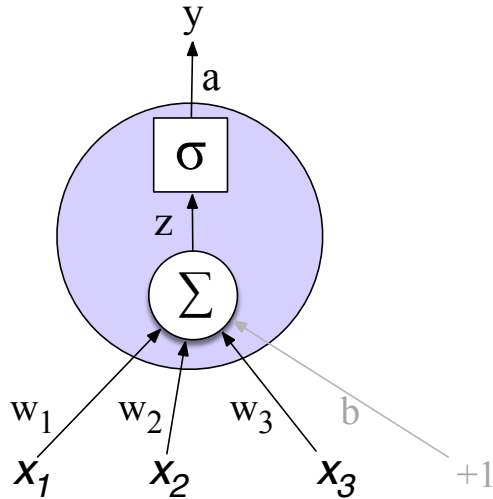
$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

Where did that derivative come from?

Using the chain rule! $f(x) = u(v(x))$ $\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$

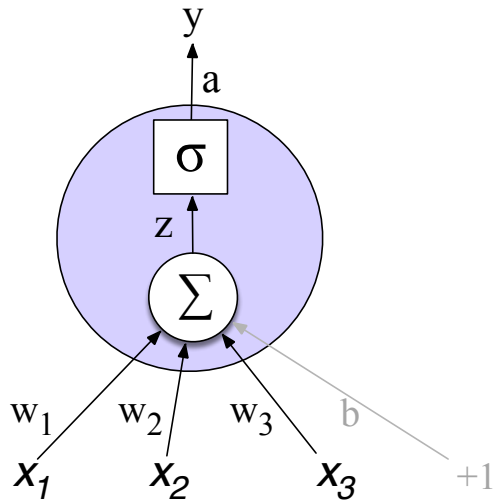
Where did that derivative come from?

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Where did that derivative come from?

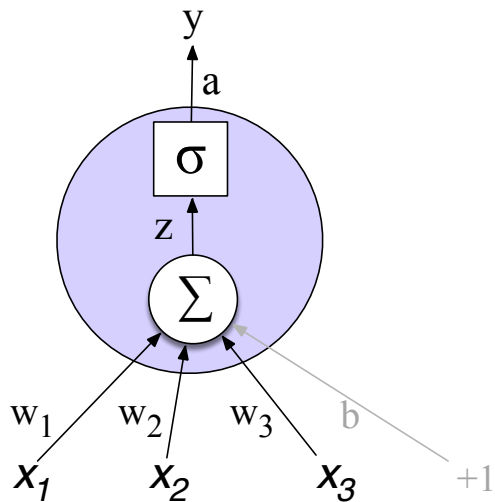
Using the chain rule! $f(x) = u(v(x))$ $\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$



$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial w_i}$$

Where did that derivative come from?

Using the chain rule! $f(x) = u(v(x))$ $\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$

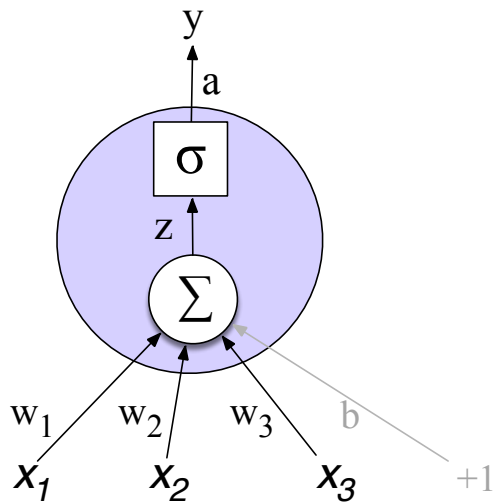


Derivative of the Loss

$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial w_i}$$

Where did that derivative come from?

Using the chain rule! $f(x) = u(v(x))$ $\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$



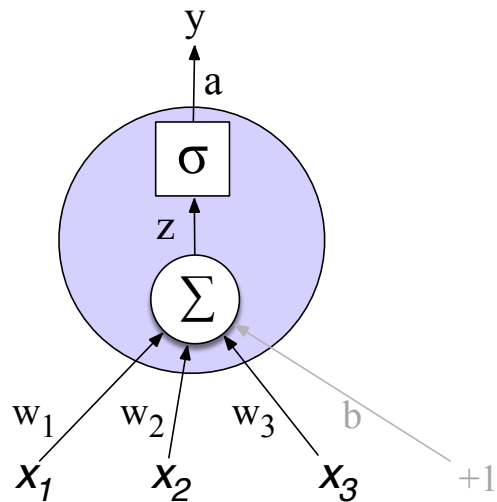
Derivative of the Activation

Derivative of the Loss

$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial w_i}$$

Where did that derivative come from?

Using the chain rule! $f(x) = u(v(x))$ $\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$



Derivative of the weighted sum

Derivative of the Activation

Derivative of the Loss

$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial w_i}$$

How can I find that gradient for every weight in the network?

These derivatives on the prior slide only give the updates for one weight layer: the last one!

What about deeper networks?

- Lots of layers, different activation functions?

Solution:

- Computation graphs and backward differentiation!

Why Computation Graphs

For training, we need the derivative of the loss with respect to each weight in every layer of the network

- But the loss is computed only at the very end of the network!

Solution: **error backpropagation** (Rumelhart, Hinton, Williams, 1986)

- **Backprop** is a special case of backward differentiation which relies on computation graphs.

Computation Graphs

A computation graph represents the process of computing a mathematical expression

Example: $L(a, b, c) = c(a + 2b)$

Computations:

$$d = 2 * b$$

$$e = a + d$$

$$L = c * e$$

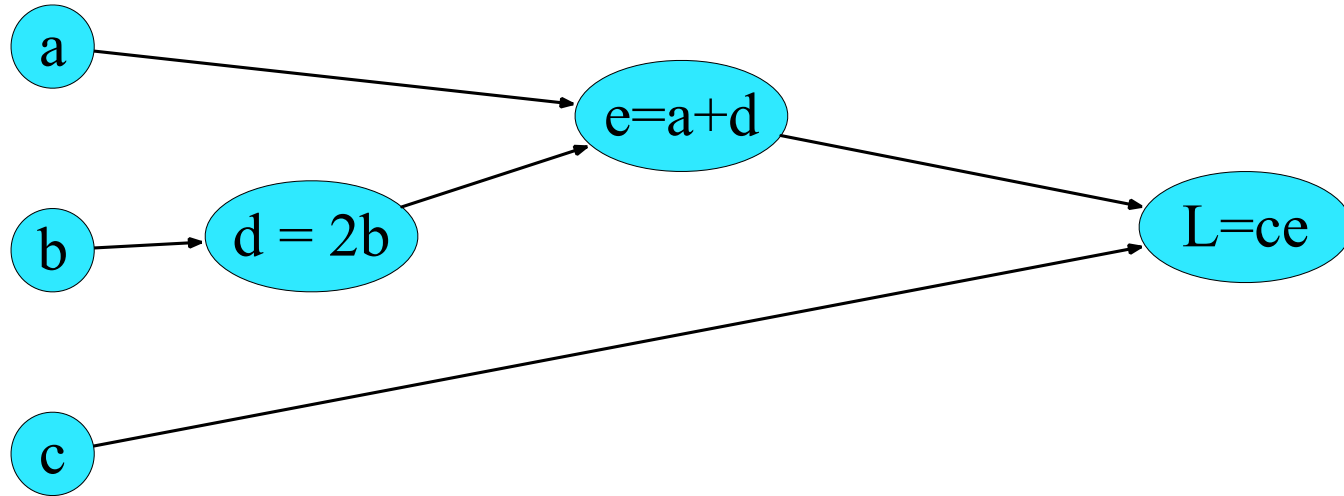
Example: $L(a, b, c) = c(a + 2b)$

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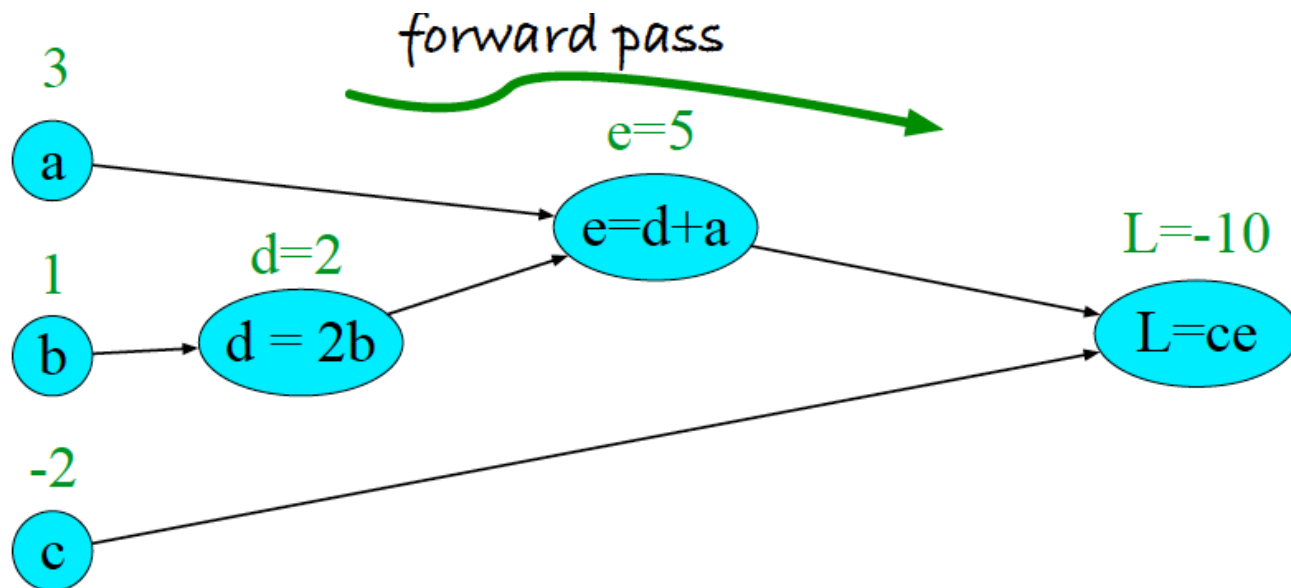
Example: $L(a, b, c) = c(a + 2b)$

$$d = 2 * b$$

Computations:

$$e = a + d$$

$$L = c * e$$



Backwards differentiation in computation graphs

The importance of the computation graph comes from the backward pass

This is used to compute the derivatives that we'll need for the weight update.

Example $L(a, b, c) = c(a + 2b)$

$$d = 2 * b$$

$$e = a + d$$

$$L = c * e$$

We want: $\frac{\partial L}{\partial a}$, $\frac{\partial L}{\partial b}$, and $\frac{\partial L}{\partial c}$

The derivative $\frac{\partial L}{\partial a}$, tells us how much a small change in a affects L .

The chain rule

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

The chain rule

Computing the derivative of a composite function:

$$f(x) = u(v(x)) \qquad \frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

$$f(x) = u(v(w(x))) \qquad \frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Example $L(a, b, c) = c(a + 2b)$

$$d = 2 * b$$

$$e = a + d$$

$$L = c * e$$

$$\frac{\partial L}{\partial c} = e$$

$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial e} \frac{\partial e}{\partial a}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial e} \frac{\partial e}{\partial d} \frac{\partial d}{\partial b}$$

Example $L(a,b,c) = c(a + 2b)$

$$d = 2 * b$$

$$e = a + d$$

$$L = c * e$$

$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial e} \frac{\partial e}{\partial a}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial e} \frac{\partial e}{\partial d} \frac{\partial d}{\partial b}$$

$$L = ce : \quad \frac{\partial L}{\partial e} = c, \frac{\partial L}{\partial c} = e$$

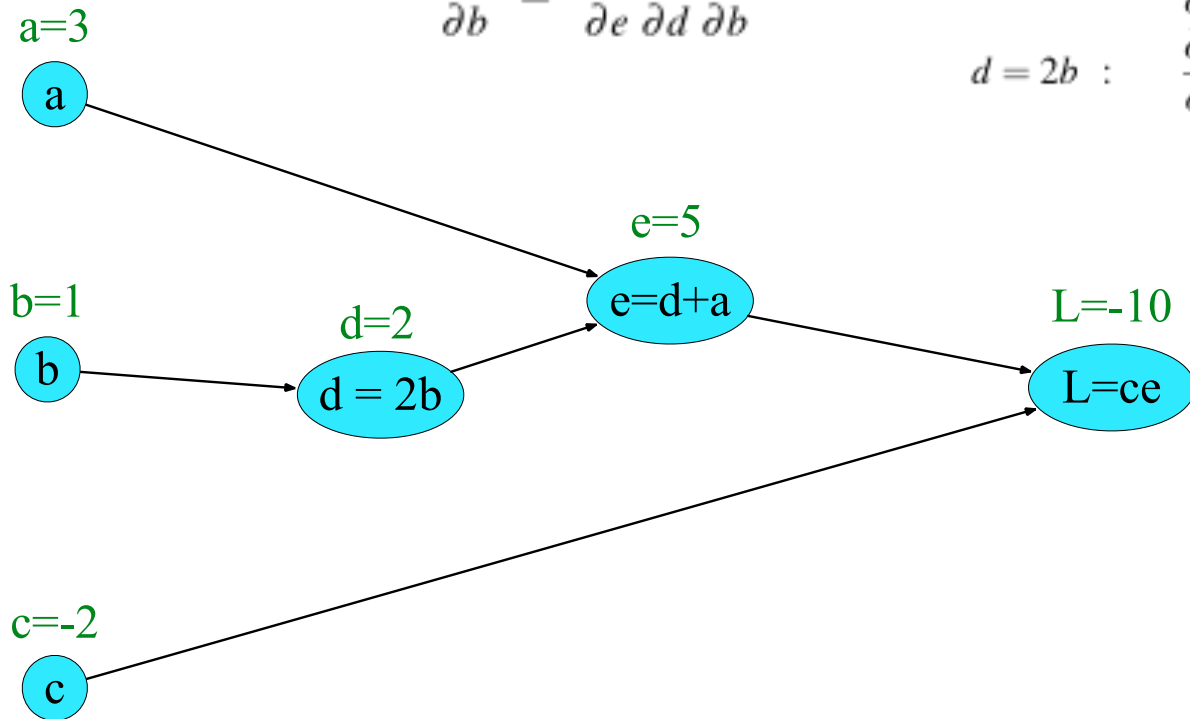
$$e = a + d : \quad \frac{\partial e}{\partial a} = 1, \frac{\partial e}{\partial d} = 1$$

$$d = 2b : \quad \frac{\partial d}{\partial b} = 2$$

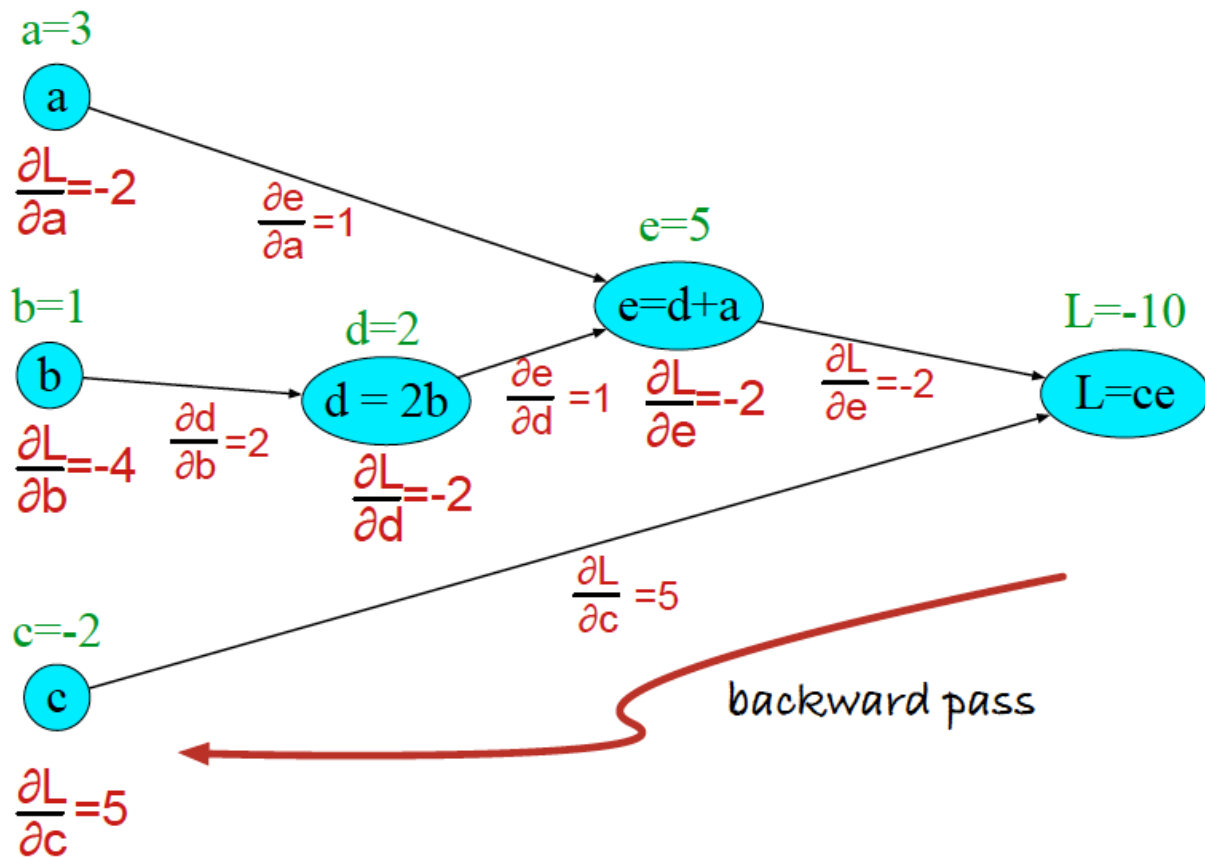
Example

$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial e} \frac{\partial e}{\partial a}$$
$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial e} \frac{\partial e}{\partial d} \frac{\partial d}{\partial b}$$

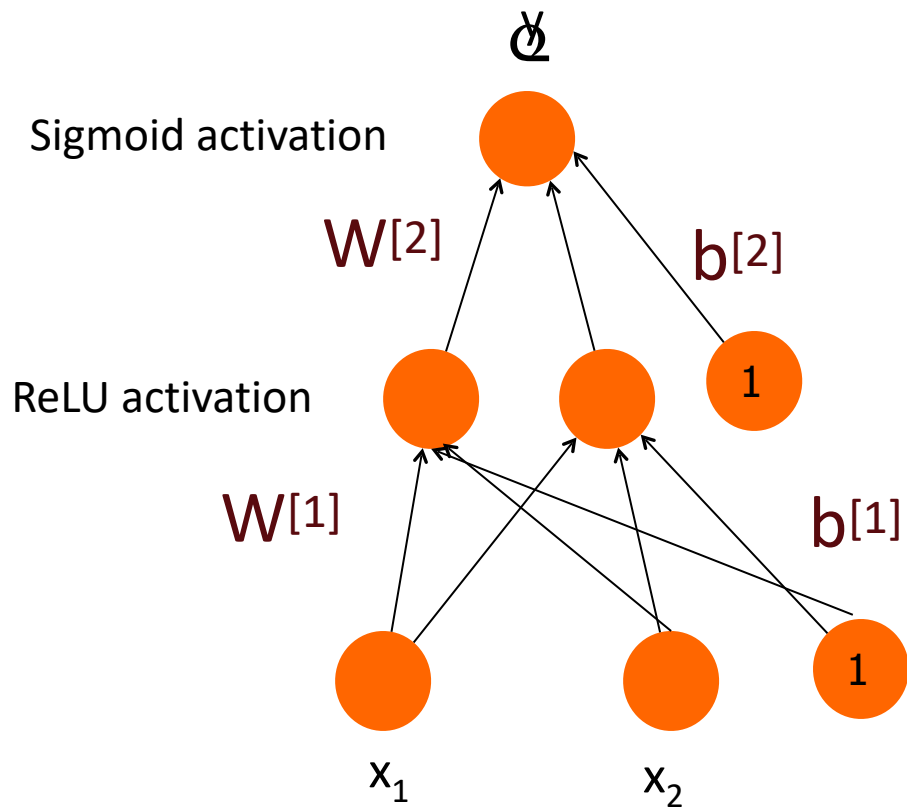
$$L = ce : \quad \frac{\partial L}{\partial e} = c, \frac{\partial L}{\partial c} = e$$
$$e = a + d : \quad \frac{\partial e}{\partial a} = 1, \frac{\partial e}{\partial d} = 1$$
$$d = 2b : \quad \frac{\partial d}{\partial b} = 2$$



Example



Backward differentiation on a two layer network



$$z^{[1]} = W^{[1]}\mathbf{x} + b^{[1]}$$

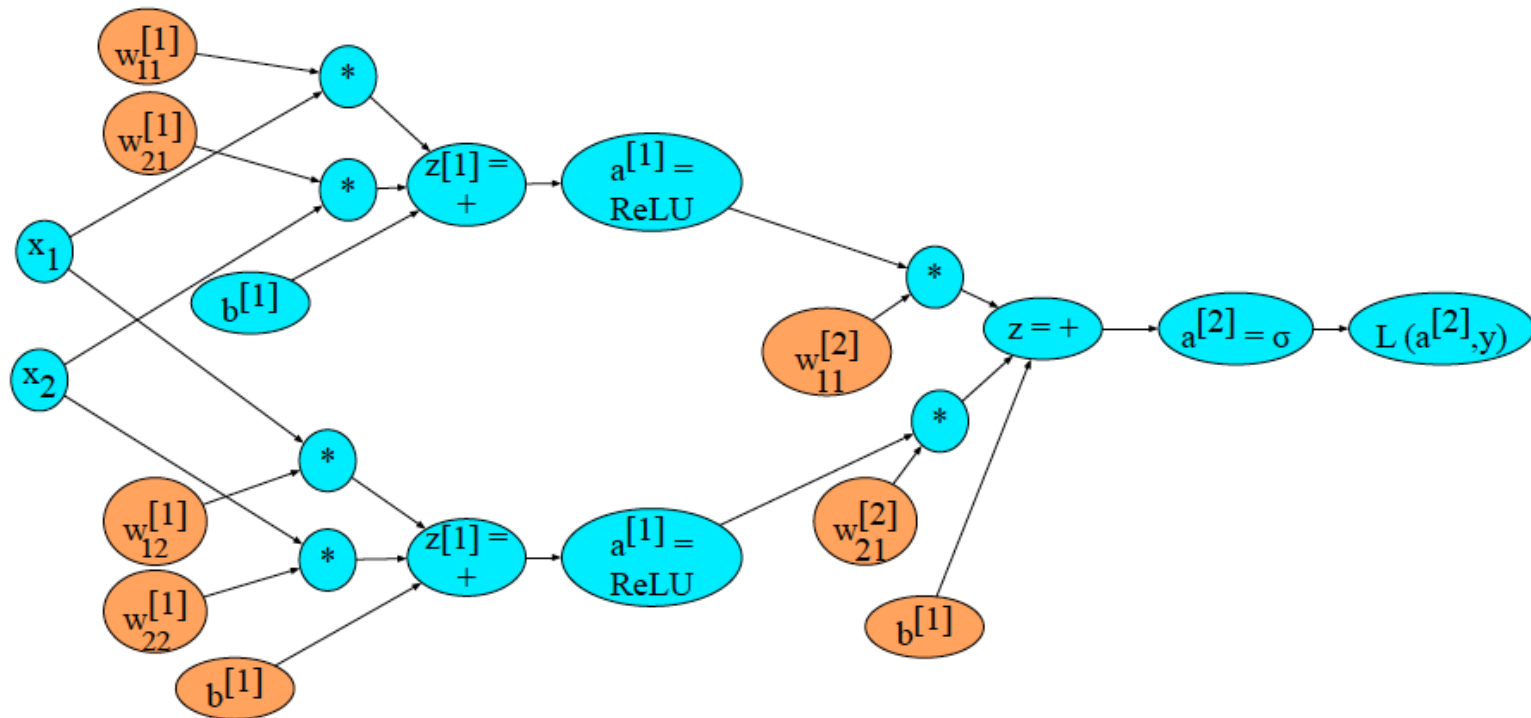
$$a^{[1]} = \text{ReLU}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

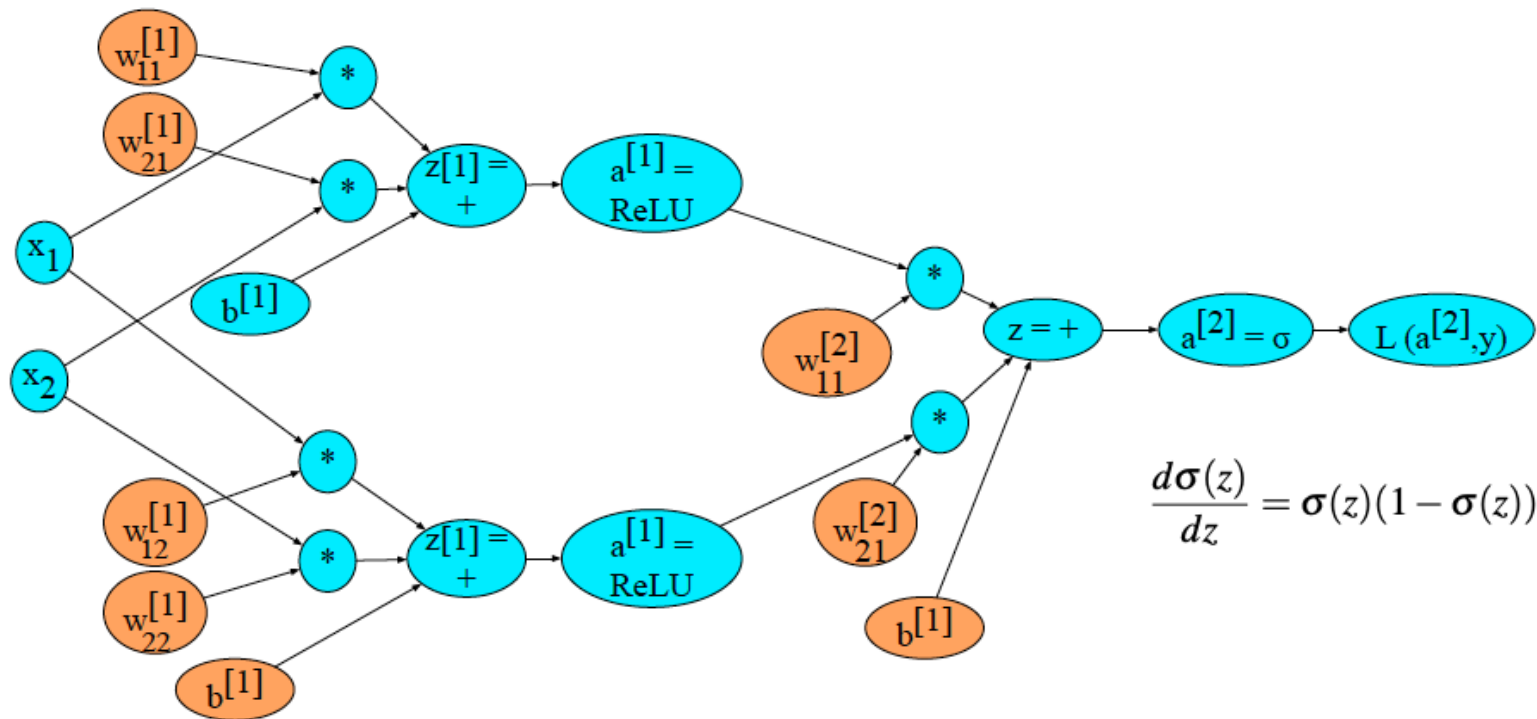
$$a^{[2]} = \sigma(z^{[2]})$$

$$\hat{y} = a^{[2]}$$

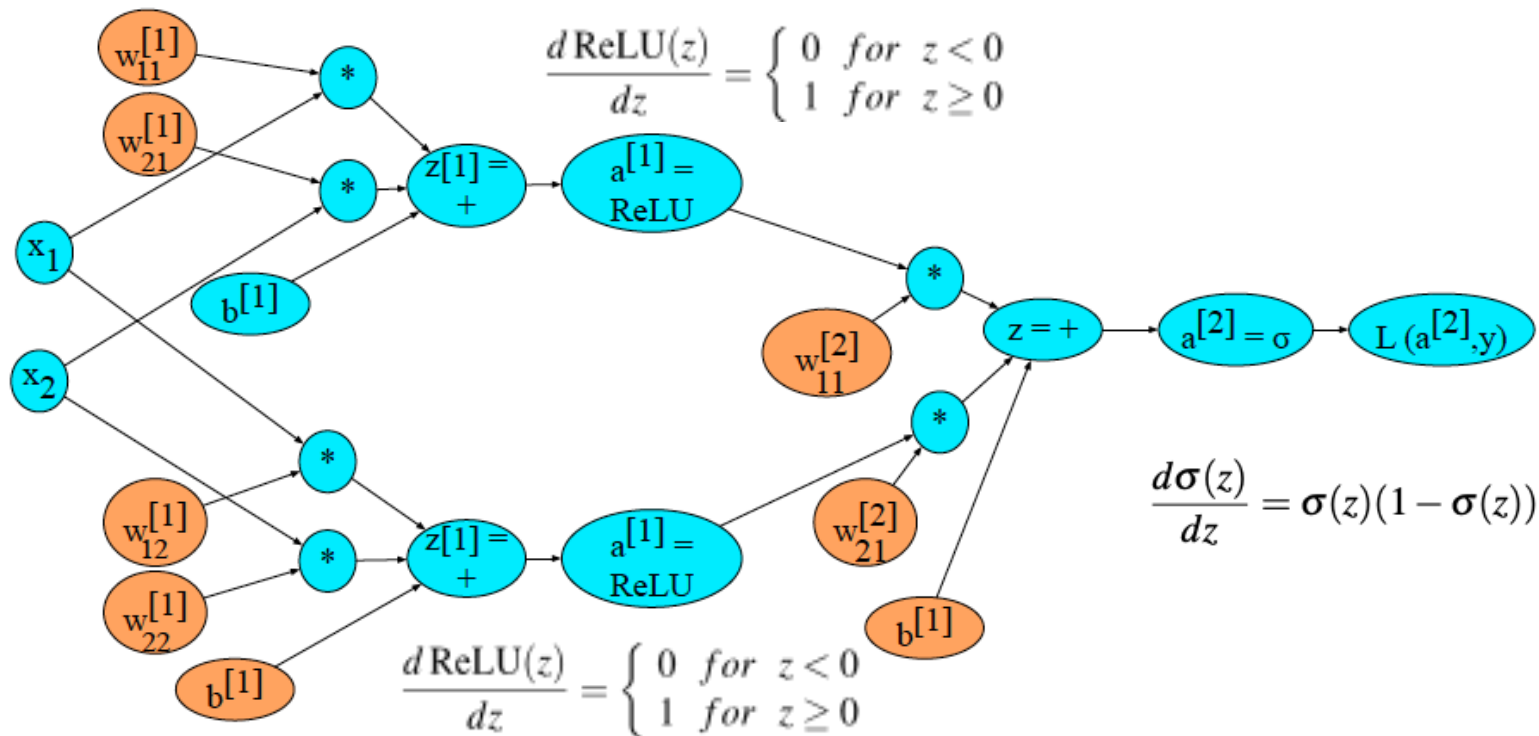
Backward differentiation on a two layer network



Backward differentiation on a two layer network



Backward differentiation on a two layer network



Summary

For training, we need the derivative of the loss with respect to weights in early layers of the network

- But loss is computed only at the very end of the network!

Solution: **backward differentiation**

Given a computation graph and the derivatives of all the functions in it we can automatically compute the derivative of the loss with respect to these early weights.

Reminder: Gradient descent for weight updates

Use the derivative of the loss function with respect to weights $\frac{d}{dw}L(f(x; w), y)$

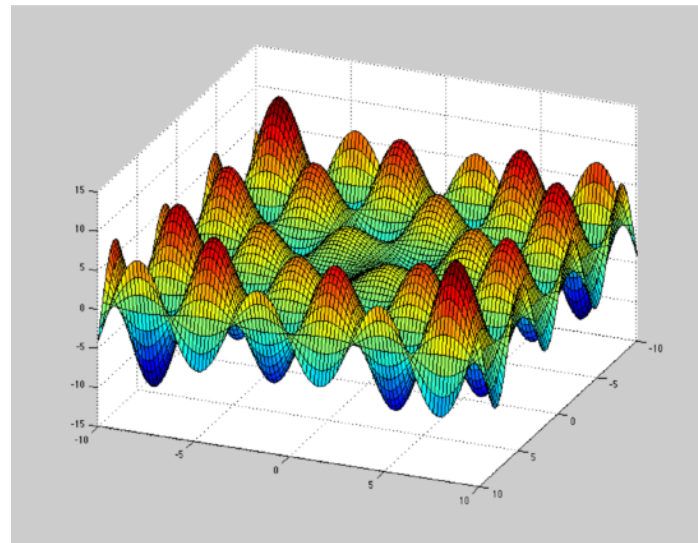
To tell us how to adjust weights for each training item

- Move them in the opposite direction of the gradient

$$w^{t+1} = w^t - \eta \frac{d}{dw}L(f(x; w), y)$$

Optimization

- NN optimization is a non-convex problem (i.e. there are many local optima)
- So how do we try to find the best one and avoid overfitting?



Hyperparameters

Weights (**W**, **b**) are the model parameters which will update during training, hyperparameters are those which we set from the start and do not update but affect the computational graph.

Hyperparameters

Here are some hyperparameters which will affect model optimization:

- Learning rate
- Number of epochs
- Minibatch size
- Number of layers
- Hidden layer sizes
- ...

[15 minute break]

Working with NN models!

Team up!

Open exercises/week 6 in your course folder and start writing/running code!