Embeddings and vector semantics

NLP Week 5

Thanks to Dan Jurafsky for many of the slides and inspiration!

Plan for today

1. **Word meanings**
2. **Distributed representations for words and their properties**
3. **Three types:**
    1. **Sparse term-document frequency vectors**
    2. **Word2Vec and dense vectors**
    3. **Contextual word embeddings**
4. ***Group exercises***

Word meanings

What do words mean?

N-gram models we've seen so far

- Words are just strings (or indices w_i in a vocabulary list)
- That's not very satisfactory!

Introductory logic classes:

- The meaning of "dog" is DOG; cat is CAT

$$\forall x \; DOG(x) \longrightarrow MAMMAL(x)$$

Old linguistics joke by Barbara Partee in 1967:

- Q: What's the meaning of life?
- A: LIFE

Desiderata

What should a theory of word meaning do for us?

Let's look at some desiderata

From lexical semantics, the linguistic study of word meaning

Lemmas and senses

lemma

mouse (N)

sense

1. any of numerous small rodents...
2. a hand-operated device that controls a cursor...

Modified from the online thesaurus WordNet

A sense or "concept" is the meaning component of a word

Lemmas can be polysemous (have multiple senses)

Relations between senses: Synonymy

Synonyms have the same meaning in some or all contexts.

- filbert / hazelnut
- couch / sofa
- big / large
- automobile / car
- vomit / throw up
- water / H_20

Relations between senses: Synonymy

Note that there are probably no examples of perfect synonymy.

- Even if many aspects of meaning are identical
- Still may differ based on politeness, slang, register, genre, etc.

Relation: Synonymy?

water/H_20

"H_20" in a surfing guide?

big/large

my big sister != my large sister

je ne crois pas qu'il y ait de-
mot ſynonime dans aucune
Langue. Je le dis par con-

[I do not believe that there is a synonymous word in any language]

Abbé Gabriel Girard, 1718

LA JUSTESSE DE LA LANGUE FRANÇOISE,

OU

LES DIFFERENTES SIGNIFICATIONS DES MOTS QUI PASSENT POUR SYNONIMES.

Par M. l'Abbé GIRARD C. D. M. D. D. B.

A PARIS,
Chez LAURENT D'HOURY, Imprimeur-Libraire, au bas de la ruë de la Harpe, vis-à vis la ruë S. Severin, au Saint Eſprit.

M. DCC. XVIII.
Avec Approbation & Privilege du Roy.

The Linguistic Principle of Contrast

Difference in form → difference in meaning

Relation: Similarity

Words with similar meanings. Not synonyms, but sharing some element of meaning

`car, bicycle`

`cow, horse`

Ask humans how similar 2 words are

word1	word2	similarity
vanish	disappear	9.8
behave	obey	7.3
belief	impression	5.95
muscle	bone	3.65
modest	flexible	0.98
hole	agreement	0.3

SimLex-999 dataset (Hill et al., 2015)

Relation: Word relatedness

Also called "word association"

Words can be related in any way, perhaps via a semantic frame or field

- `coffee, tea`: **similar**
- `coffee, cup`: **related**, not similar

Semantic field

Words that

- cover a particular semantic domain
- bear structured relations with each other.

hospitals

surgeon, scalpel, nurse, anaesthetic, hospital

restaurants

waiter, menu, plate, food, menu, chef

houses

door, roof, kitchen, family, bed

Relation: Antonymy

Senses that are opposites with respect to only one feature of meaning

Otherwise, they are very similar!

`dark/light` `short/long` `fast/slow` `rise/fall`
`hot/cold` `up/down` `in/out`

More formally: antonyms can

- define a binary opposition or be at opposite ends of a scale
  - `long/short, fast/slow`
- Be *reversives*:
  - `rise/fall, up/down`

Connotation (sentiment)

- Words have **affective** meanings
  - Positive connotations (*happy*)
  - Negative connotations (*sad*)
- Connotations can be subtle:
  - Positive connotation: *copy, replica, reproduction*
  - Negative connotation: *fake, knockoff, forgery*
- Evaluation (sentiment!)
  - Positive evaluation (*great, love*)
  - Negative evaluation (*terrible, hate*)

Word meanings

To sum up

Concepts or word senses

- Have a complex many-to-many association with **words (eg. homonyms)**

Have relations with each other

- Synonymy
- Antonymy
- Similarity
- Relatedness
- Connotation

Computational models of word meaning

Can we build a theory of how to represent word meaning, that accounts for at least some of the desiderata?

We'll introduce **vector semantics**

The standard model of meaning in NLP!

Handles many of our goals!

Ludwig Wittgenstein, PI #43:

"The meaning of a word is its use in the language"

Let's define words by their usages

One way to define "usage":

words are defined by their environments (the words around them)

Zellig Harris (1954):

If A and B have almost identical environments we say that they are synonyms.

What does recent English borrowing *ongchoi* mean?

Suppose you see these sentences:

- Ong choi is delicious **sautéed with garlic**.
- Ong choi is superb **over rice**
- Ong choi **leaves** with salty sauces

And you've also seen these:

- ...spinach **sautéed with garlic over rice**
- Chard stems and **leaves** are **delicious**
- Collard greens and other **salty** leafy greens

Conclusion:

- Ongchoi is a leafy green like spinach, chard, or collard greens
  - We could conclude this based on words like "leaves" and "delicious" and "sauteed"

Distributed representations

Let's define the meaning of a word by its distribution in language use, meaning its neighboring words or grammatical environments.

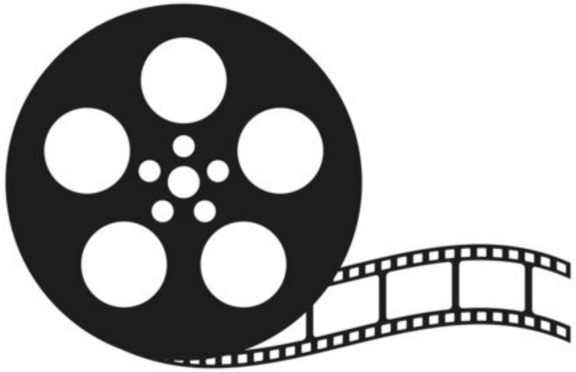
Vector semantics

Each word = a vector (not just "good" or "w_{45}")

Similar words are "**nearby in semantic space**"

We build this space automatically by seeing which words are **nearby in text**

Sentiment analysis



Positive or negative movie review?

- unbelievably disappointing
- Full of zany characters and richly applied satire, and some great plot twists
- this is the greatest screwball comedy ever filmed
- It was pathetic. The worst part about it was the boxing scenes.

Embeddings

Vector representations of word meaning are called an "embedding" because it's embedded into a space.

Every modern NLP algorithm uses embeddings as the representation of word meaning

Fine-grained model of meaning for similarity

Intuition: why vectors?

Consider sentiment analysis:

- With **words**, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires **exact same word** to be in training and test
- With **embeddings**:
  - Feature is a word vector
  - 'The previous word was vector [35,22,17…]
  - Now in the test set we might see a similar vector [34,21,14]
  - We can generalize to **similar but unseen** words!!!

Properties of embeddings

The kinds of neighbors depend on window size

Small windows (C= +/- 2) : nearest words are syntactically similar words in same taxonomy

- *Hogwarts* nearest neighbors are other fictional schools
  - *Sunnydale, Evernight, Blandings*

Large windows (C= +/- 5) : nearest words are related words in same semantic field

- *Hogwarts* nearest neighbors are Harry Potter world:
  - *Dumbledore, half-blood, Malfoy*

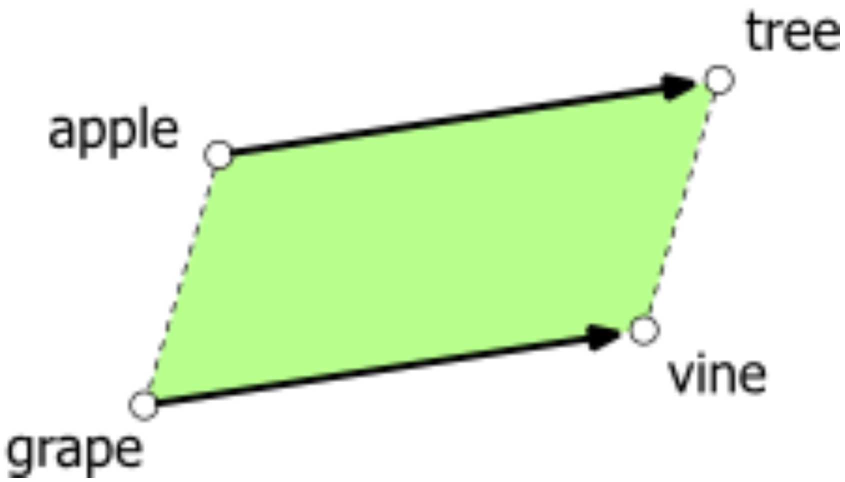
Properties of embeddings

Analogical relations

The classic parallelogram model of analogical reasoning (Rumelhart and Abrahamson 1973)

To solve: "*apple is to tree as grape is to ______*"

Add $\overrightarrow{\textit{tree}} - \overrightarrow{\textit{apple}}$ *to* $\overrightarrow{\textit{grape}}$ *to get* $\overrightarrow{\textit{vine}}$



Analogical relations via parallelogram

The parallelogram method can solve analogies with both sparse and dense embeddings (Turney and Littman 2005, Mikolov et al. 2013b)

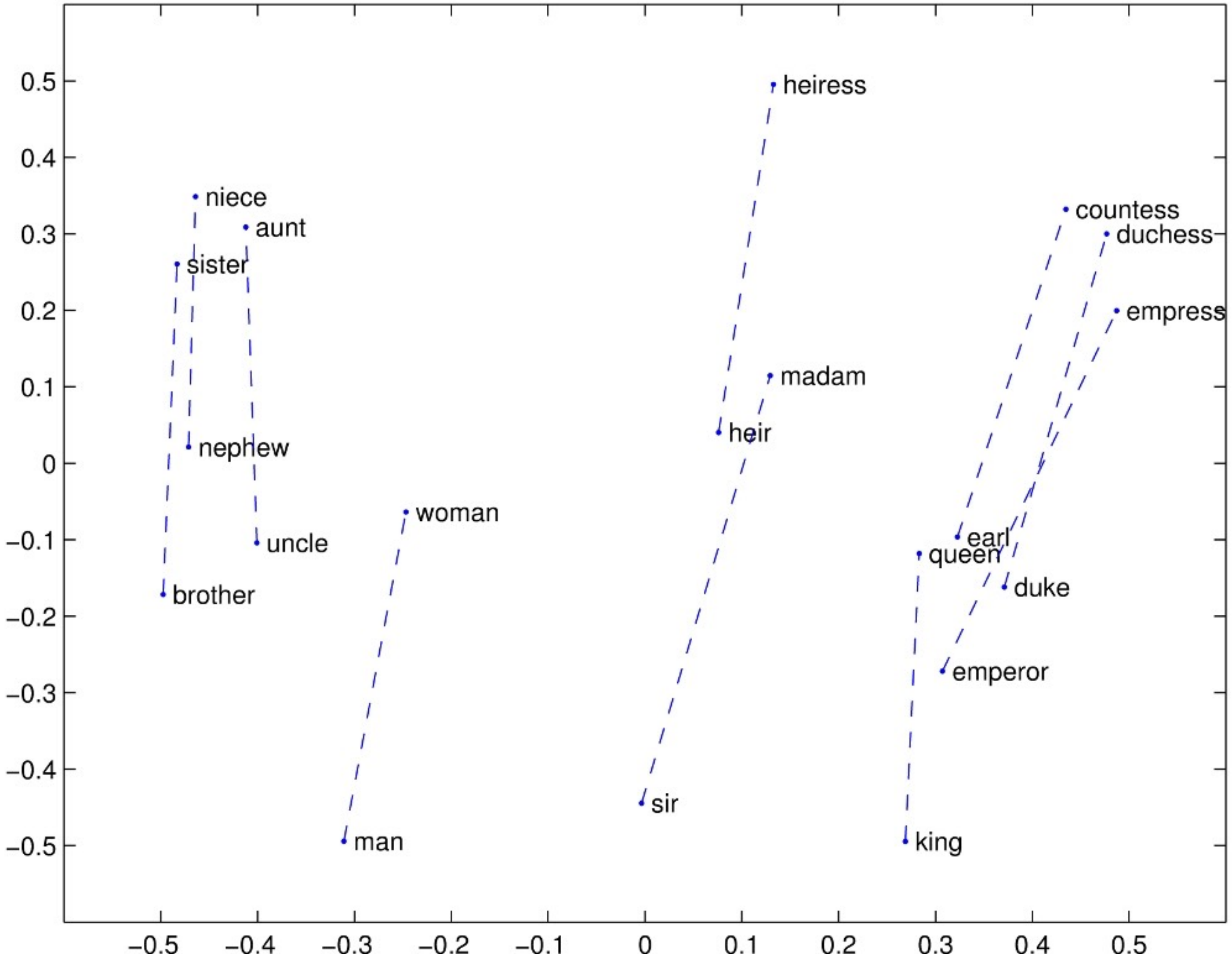
$\overrightarrow{\text{king}} - \overrightarrow{\text{man}} + \overrightarrow{\text{woman}}$ is close to $\overrightarrow{\text{queen}}$

$\overrightarrow{\text{Paris}} - \overrightarrow{\text{France}} + \overrightarrow{\text{Italy}}$ is close to $\overrightarrow{\text{Rome}}$

For a problem a:a*::b:b*, the parallelogram method is:

$$\hat{b}^* = \underset{x}{\operatorname{argmax}} \ \text{distance}(x, a^* - a + b)$$

Structure in GloVE Embedding space



Caveats with the parallelogram method

It only seems to work for frequent words, small distances and certain relations (relating countries to capitals, or parts of speech), but not others. (Linzen 2016, Gladkova et al. 2016, Ethayarajh et al. 2019a)

Understanding analogy is an open area of research (Peterson et al. 2020)

Embeddings as a window onto historical semantics

Train embeddings on different decades of historical text to see meanings shift

~30 million books, 1850-1990, Google Books data

William L. Hamilton, Jure Leskovec, and Dan Jurafsky. 2016. Diachronic Word Embeddings Reveal Statistical Laws of Semantic Change. Proceedings of ACL.

Embeddings reflect cultural bias!

Bolukbasi, Tolga, Kai-Wei Chang, James Y. Zou, Venkatesh Saligrama, and Adam T. Kalai. "Man is to computer programmer as woman is to homemaker? debiasing word embeddings." In *NeurIPS*, pp. 4349-4357. 2016.

Ask "Paris : France :: Tokyo : x"

- x = Japan

Ask "father : doctor :: mother : x"

- x = nurse

Ask "man : computer programmer :: woman : x"

- x = homemaker

Algorithms that use embeddings as part of e.g., hiring searches for programmers, might lead to bias in hiring

Embedding types

- **There are three main types of embeddings used in NLP and they are created using three different methods:**

    1. Sparse embeddings and term-document frequency

    2. Dense embeddings and Word2Vec algorithms

    3. Contextual word embeddings and language modeling

Sparse embeddings and frequencies

Term-document matrix

Each document is represented by a vector of words

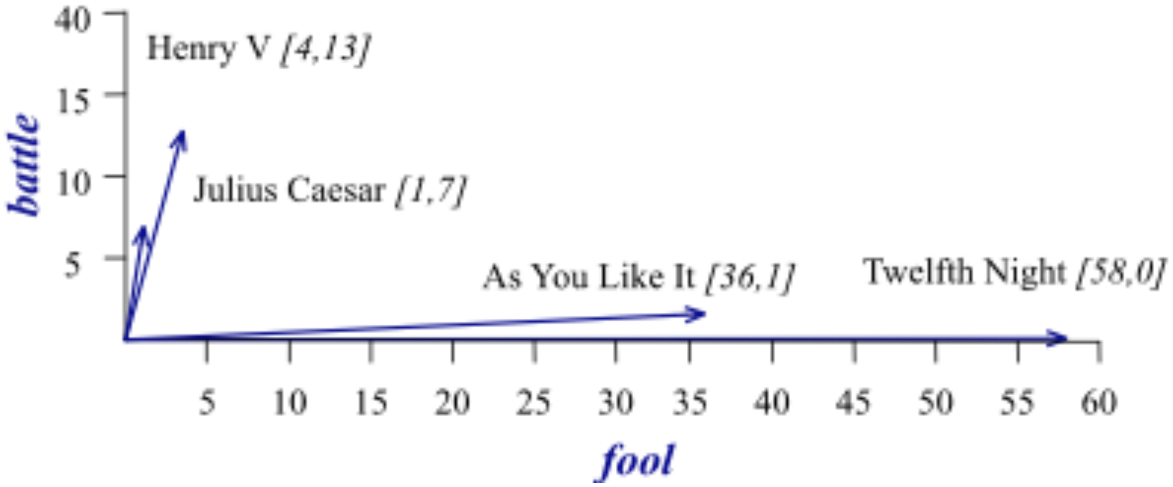
	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Vectors are the basis of information retrieval

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Vectors are similar for the two comedies

Comedies have more *fools* and *wit* and fewer *battles*.



Idea for word meaning: Words can be vectors too!

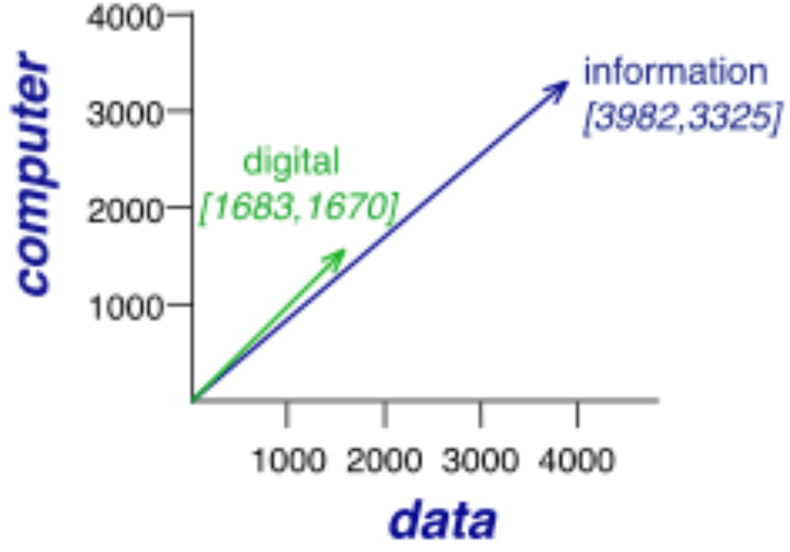
	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

battle is "the kind of word that occurs in Julius Caesar and Henry V"

fool is "the kind of word that occurs in comedies, especially Twelfth Night"

More common: word-word matrix (or "term-context matrix")

Two **words** are similar in meaning if their context vectors are similar



is traditionally followed by	**cherry**	pie, a traditional dessert
often mixed, such as	**strawberry**	rhubarb pie. Apple pie
computer peripherals and personal	**digital**	assistants. These devices usually
a computer. This includes	**information**	available on the internet

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...

But raw frequency is a bad representation

- The co-occurrence matrices we have seen represent each cell by word frequencies.
- Frequency is clearly useful; if *sugar* appears a lot near *apricot*, that's useful information.
- But overly frequent words like *the*, *it,* or *they* are not very informative about the context
- It's a paradox! How can we balance these two conflicting constraints?

Two common solutions for word weighting

tf-idf: (term frequency x inverse document frequency)

tf-idf value for word t in document d:

$$w_{t,d} = \mathrm{tf}_{t,d} \times \mathrm{idf}_t$$

Words like "the" or "it" have very low idf

PMI: (Pointwise mutual information)

- $$\mathrm{PMI}(w_1, w_2) = log \frac{p(w_1, w_2)}{p(w_1)p(w_2)}$$

See if words like "good" appear more often with "great" than we would expect by chance

Term frequency (tf) in the tf-idf algorithm

We could imagine using raw count:

$$\text{tf}_{t,d} = \text{count}(t,d)$$

But instead of using raw count, we usually squash a bit:

$$\text{tf}_{t,d} = \begin{cases} 1 + \log_{10} \text{count}(t,d) & \text{if } \text{count}(t,d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Document frequency (df)

df_t *is* the number of documents *t* occurs in.

(note this is not collection frequency: total count across all documents)

"*Romeo*" is very distinctive for one Shakespeare play:

	Collection Frequency	Document Frequency
Romeo	113	1
action	113	31

Inverse document frequency (idf)

$$\text{idf}_t = \log_{10}\left(\frac{N}{\text{df}_t}\right)$$

N is the total number of documents in the collection

Word	df	idf
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

What is a document?

Could be a play or a Wikipedia article

But for the purposes of tf-idf, documents can be **anything**; we often call each paragraph a document!

Final tf-idf weighted value for a word

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Raw counts:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

tf-idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.246	0	0.454	0.520
good	0	0	0	0
fool	0.030	0.033	0.0012	0.0019
wit	0.085	0.081	0.048	0.054

TF-IDF

- Information Retrieval workhorse!
- A common baseline model
- **Sparse** vectors
- Words are represented by (a simple function of) the **counts** of nearby words

Embedding types

- **There are three main types of embeddings used in NLP and they are created using three different methods:**

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  2. Dense embeddings and Word2Vec algorithms

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Dense embeddings

Sparse versus dense vectors

tf-idf (or PMI) vectors are

- **long** (length |V|= 20,000 to 50,000)
- **sparse** (most elements are zero)

Alternative: learn vectors which are

- **short** (length 50-1000)
- **dense** (most elements are non-zero)

Sparse versus dense vectors

Why dense vectors?

- Short vectors may be easier to use as **features** in machine learning (fewer weights to tune)
- Dense vectors may **generalize** better than explicit counts
- Dense vectors may do better at capturing synonymy:
  - *car* and *automobile* are synonyms; but are distinct dimensions
    - a word with *car* as a neighbor and a word with *automobile* as a neighbor should be similar, but aren't
- **In practice, they work better**

Similarity in vector space

Dot product and cosine

The dot product between two vectors is a scalar:

$$\text{dot product}(\mathbf{v}, \mathbf{w}) = \mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

The dot product tends to be high when the two vectors have large values in the same dimensions

Dot product can thus be a useful similarity metric between vectors

Problem with raw dot-product

- Dot product favors long vectors
- Dot product is higher if a vector is longer (has higher values in many dimension)
- Vector length:

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

- Frequent words (of, the, you) have long vectors (since they occur many times with other words).
- So dot product overly favors frequent words

Alternative: cosine for computing word similarity

$$\text{cosine}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

Based on the definition of the dot product between two vectors a and b

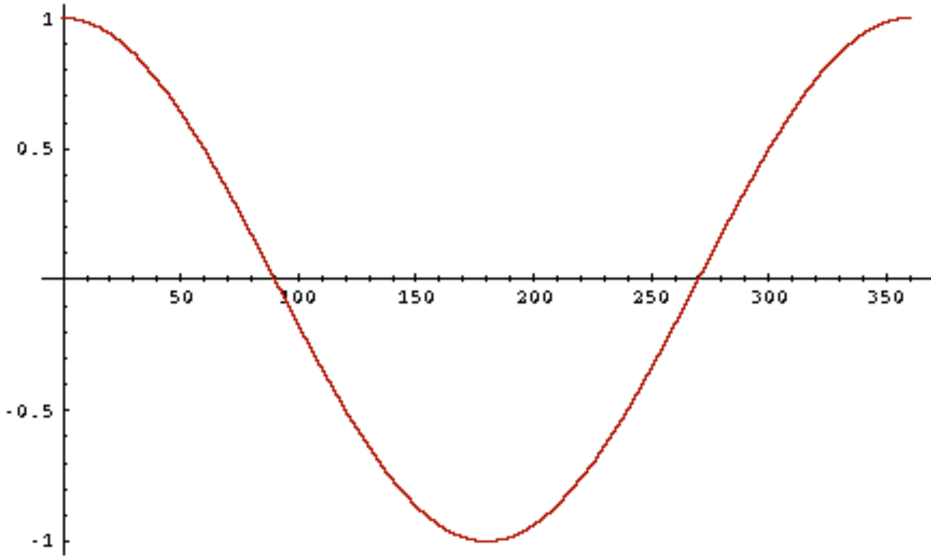
$$\begin{aligned} \mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos\theta \\ \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} &= \cos\theta \end{aligned}$$

Cosine as a similarity metric

-1: vectors point in opposite directions

+1: vectors point in same directions

0: vectors are orthogonal



But since raw frequency values are non-negative, the cosine for term-term matrix vectors like in tf-idf range from 0–1

Cosine examples

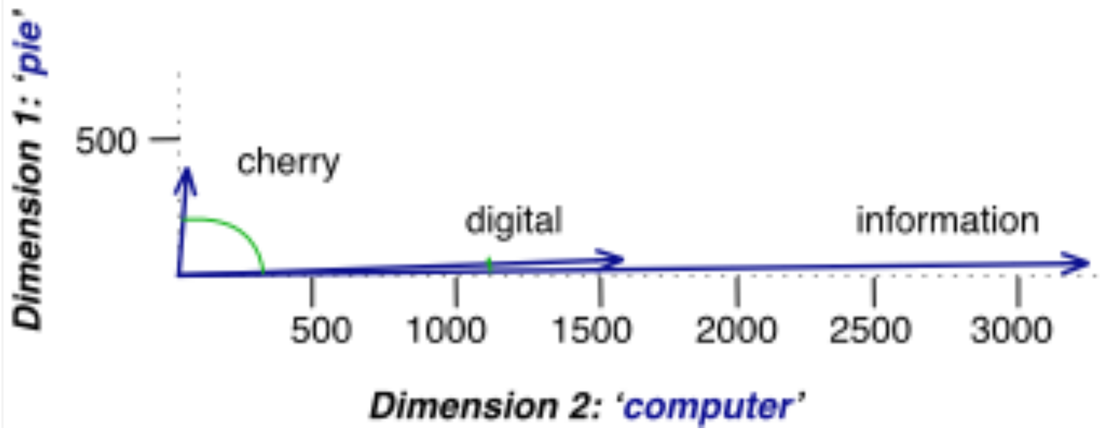
$$\text{cosine}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

$$\cos(\text{cherry},\text{information}) =$$
$$\frac{442*5+8*3982+2*3325}{\sqrt{442^2+8^2+2^2}\sqrt{5^2+3982^2+3325^2}} = .017$$

$$\cos(\text{digital},\text{information}) =$$
$$\frac{5*5+1683*3982+1670*3325}{\sqrt{5^2+1683^2+1670^2}\sqrt{5^2+3982^2+3325^2}} = .996$$

	pie	data	computer
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

Visualizing cosine distance as angles



Dense embeddings and Word2Vec

Instead of **counting** how often each word *w* occurs near "*apricot*"

- Train a classifier on a binary **prediction** task:
  - Is *w* likely to show up near "*apricot*"?

We don't actually care about this task

- But we'll take the learned classifier weights as the word embeddings

Big idea: **self-supervision**:

- No need for human labels
- Bengio et al. (2003); Collobert et al. (2011)

A self-supervised approach

Approach: predict if candidate word *c* is a "neighbor"

1. Treat the target word *t* and a neighboring context word *c* as **positive examples**.
2. Randomly sample other words in the lexicon to get negative examples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the learned weights as the embeddings

Skip-gram classifier

A probabilistic classifier, given

- a target word *w*
- its context window of *L* words $c_{1:L}$

Estimates probability that w occurs in this window based on similarity of w (embeddings) to $c_{1:L}$ (embeddings).

To compute this, we just need embeddings for all the words.

Skip-Gram Classifier

(assuming a +/- 2 word window)

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

Goal: train a classifier that is given a candidate (**w**ord, **c**ontext) pair

(apricot, jam)

(apricot, aardvark)

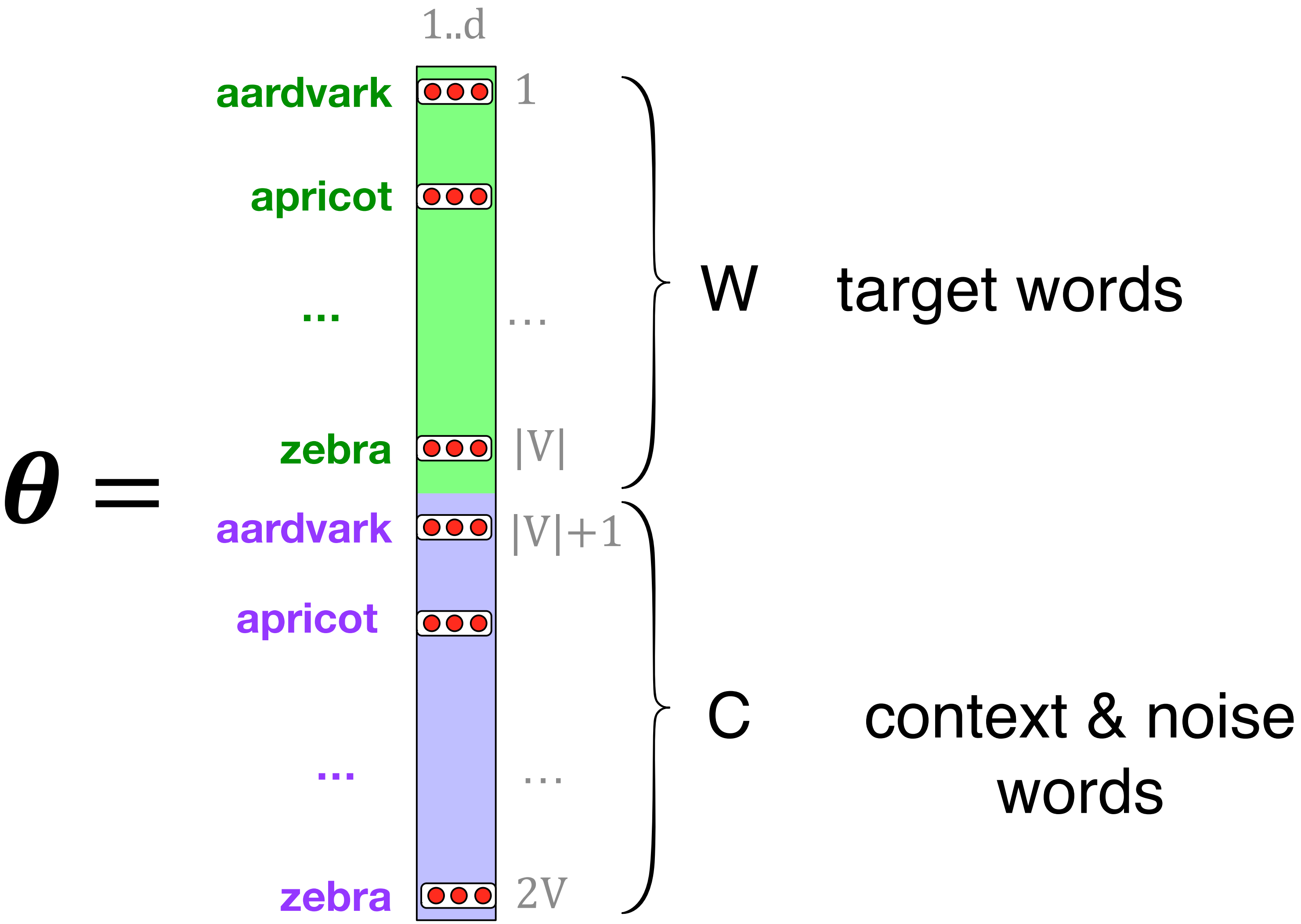
…

And assigns each pair a probability:

$P(+|w, c)$

$P(-|w, c) = 1 - P(+|w, c)$

Embeddings we'll need: a set for w, a set for c



Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +	
t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

For each positive example we'll grab k negative examples, sampling by frequency

Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch...

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

negative examples -

t	c	t	c
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

Similarity is computed from dot product

Remember: two vectors are similar if they have a high dot product

- Cosine is just a normalized dot product

So:

- Similarity(w,c) $\propto$ w · c

We'll need to normalize to get a probability

- (cosine isn't a probability either)

Turning dot products into probabilities

$\mathrm{sim(w,c)} \approx \mathrm{w} \cdot c$

To turn this into a probability

We'll use the sigmoid from logistic regression:

$$P(+|w,c) = \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$$

$$\begin{aligned} P(-|w,c) &= 1 - P(+|w,c) \\ &= \sigma(-c \cdot w) = \frac{1}{1+\exp(c \cdot w)} \end{aligned}$$

Dealing with larger contexts in Skip-Gram Classifier

$$P(+|w,c) \;=\; \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$$

This is for one context word, but we have lots of context words.
We'll assume independence and just multiply them:

$$P(+|w,c_{1:L}) \;=\; \prod_{i=1}^{L} \sigma(c_i \cdot w)$$

$$\log P(+|w,c_{1:L}) \;=\; \sum_{i=1}^{L} \log \sigma(c_i \cdot w)$$

Learning objective (loss function)

Given the set of positive and negative training instances, and an initial set of embedding vectors

The goal of learning is to adjust those word vectors such that we:

- **Maximize** the similarity of the target word, context word pairs (w, c_{pos}) drawn from the positive data
- **Minimize** the similarity of the (w, c_{neg}) pairs drawn from the negative data.

Loss function for one w *with* c_{pos}, c_{neg1} ...c_{negk}

Maximize the similarity of the target with the actual context words, and minimize the similarity of the target with the *k* negative sampled non-neighbor words.

$$
\begin{aligned}
L_{\mathrm{CE}} &= -\log\left[P(+|w,c_{pos})\prod_{i=1}^{k}P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos})+\sum_{i=1}^{k}\log P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos})+\sum_{i=1}^{k}\log\left(1-P(+|w,c_{neg_i})\right)\right] \\
&= -\left[\log\sigma(c_{pos}\cdot w)+\sum_{i=1}^{k}\log\sigma(-c_{neg_i}\cdot w)\right]
\end{aligned}
$$

Learning/training the classifier

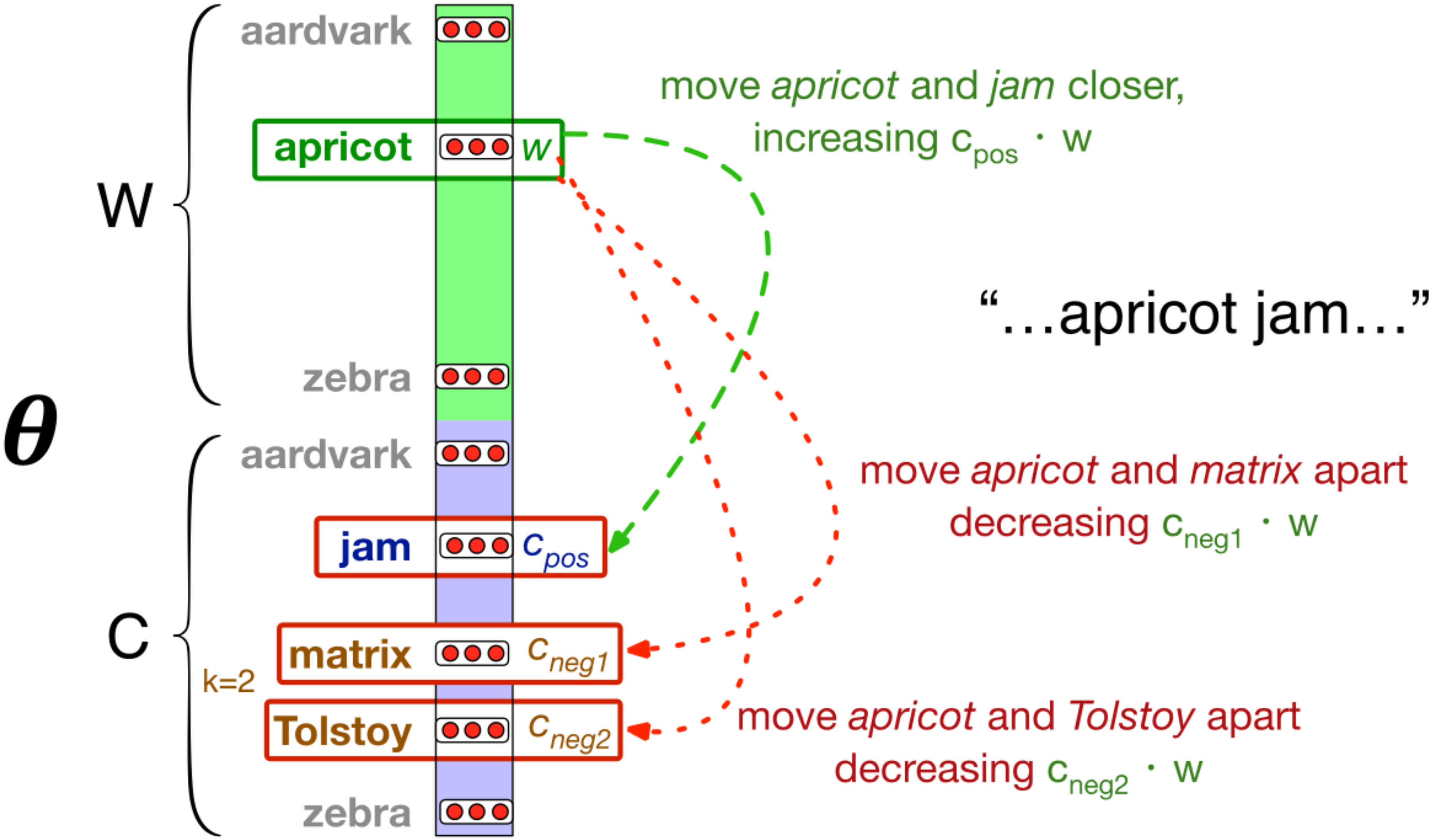
How to learn? **Stochastic gradient descent**!

We'll adjust the word weights to make the positive pairs more likely and the negative pairs less likely, over the entire training set.

Gradient descent

- At each step
  - Direction: We move in the reverse direction from the gradient of the loss function
  - Magnitude: we move the value of this gradient weighted by a **learning rate** η
  - Higher learning rate means move *w* faster

Intuition of one step of gradient descent



Two sets of embeddings

The classifier learns two sets of embeddings

Target embeddings matrix W

Context embedding matrix C

It's common to just add them together, representing word *i* as the vector $w_i + c_i$

Summary: How to learn word2vec (skip-gram) embeddings

Start with |V| random d-dimensional vectors as initial embeddings

Train a classifier based on embedding similarity

- Take a corpus and take pairs of words that co-occur as positive examples
- Take pairs of words that don't co-occur as negative examples
- Train the classifier to distinguish these by slowly adjusting all the embeddings to improve the classifier performance
- Throw away the classifier code and keep the embeddings.

Embedding types

- **There are three main types of embeddings used in NLP and they are created using three different methods:**

    1. Sparse embeddings and term-document frequency

    2. **Dense embeddings and Word2Vec algorithms**

    3. Contextual word embeddings and language modeling

Contextual embeddings

Words are ambiguous

A **word sense** is a discrete representation of one aspect of meaning

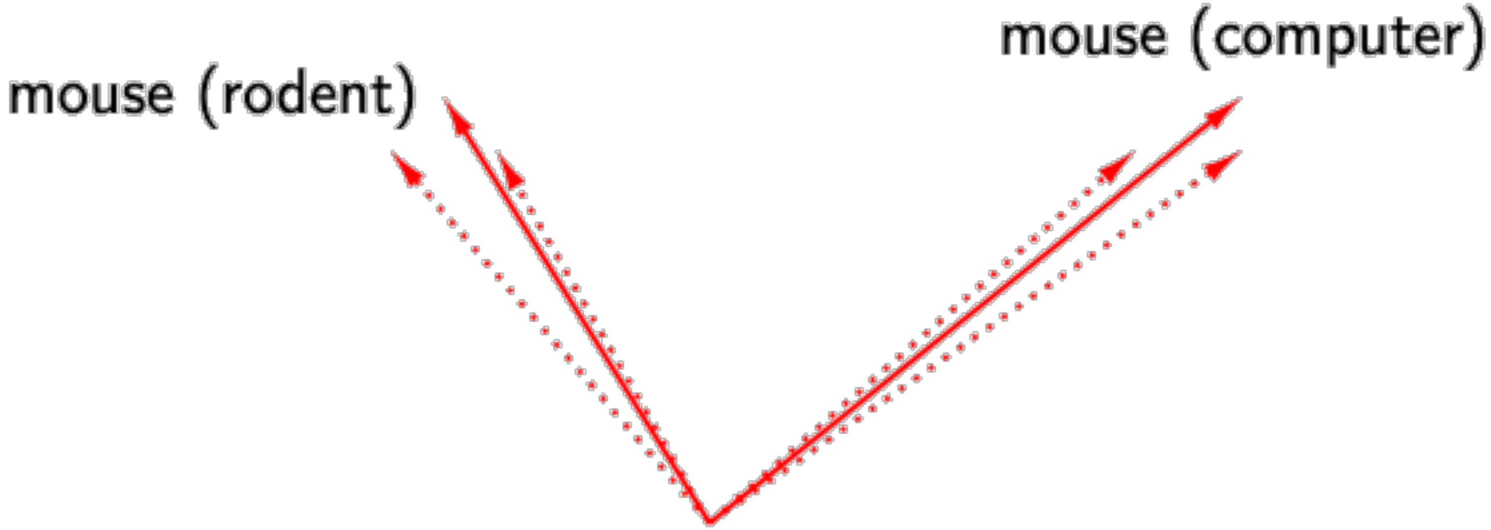
mouse^1 : .... a *mouse* controlling a computer system in 1968.
mouse^2 : .... a quiet animal like a *mouse*
bank^1 : ...a *bank* can hold the investments in a custodial account ...
bank^2 : ...as agriculture burgeons on the east *bank*, the river ...

Contextual embeddings offer a continuous high-dimensional model of meaning that is more fine grained than discrete senses.

Static vs Contextual Embeddings

Static embeddings represent **word types** (dictionary entries)

Contextual embeddings represent **word instances** (one for each time the word occurs in any context/sentence)



Contextual embeddings

- Intuition: a representation of meaning of a word should be different in different contexts!
- **Contextual Embedding**: each word has a different vector that expresses different meanings depending on the surrounding words
- How to compute contextual embeddings?
  - **Using a neural language model**

Embedding types

- **There are three main types of embeddings used in NLP and they are created using three different methods:**

    1. Sparse embeddings and term-document frequency

    2. Dense embeddings and Word2Vec algorithms

    3. **Contextual word embeddings and language modeling**

[15 minute break]

Working with word embeddings!

Team up!

Open exercises/week 4 in your course folder and start writing/running code!