MATH 131: Numerical Methods for scientists and engineers - Assignment 4

Homework Assignment 4 due by 11:59 PM, March 15, 2021. Test your answers on Matlab.
For this assignment you are allowed 5 submissions per exercise.
Save each code as an .m file. We will use those codes later in class.

Matlab Grader

1. (20 points) Create a function called `composite_trapezoid` that inputs a function, a pair of endpoints, a, b, and a number n of subintervals, and outputs the approximation to the integral of f from a to b using the composite trapezoid rule. Your function header should look like this

   ```
   function I = composite_trapezoid(f,a,b,n)
   ```

2. (20 points) Create a function called `composite_simpsons` that inputs a function, a pair of endpoints, a, b, and a number n, and outputs the approximation to the integral of f from a to b using the Composite Simpson's rule on $n+1$ points (n subintervals). Your function header should look like this

   ```
   function I = composite_simpsons(f,a,b,n)
   ```

   Note: If the incorrect number of subintervals is supplied, return I as the empty vector.

3. (20 points) Consider the function $f(x) = (x^2 - 3x + 2)\arctan x$. Estimate the derivative of the function at the point $x = 0$ using the forward diference formula, the 3-point midpoint formula and the 5-point midpoint formula for all of the following values of h: $h = 10^{-n}$, $n = 1, 3, 6$. Make a `loglog` plot of your ***absolute error*** verses h (use the same figure for all methods).

   For $n = 1, 3, 6$ call `FD1`, `FD3`, `FD6` the result obtain with the forward difference, `CD31`, `CD33`, `CD36` the result obtain with the 3 point centered difference, and `CD51`, `CD53`, `CD56` the result obtain with the 5 point centered difference.

   Find the absolute error present for each estimate (create a vector FD for the error generated by forward differences, a vector CD3 for the error generated by 3-point midpoint, and a vector CD5 for the error generated by 5-point midpoint). Plot your absolute error for all three methods on the same graph using a log-log plot.

4. (20 points) Create a function called `derivative_3point` that inputs a function, f, and a vector, x, and outputs/returns the approximation of the derivative of f at all x values using the 3-point derivative formula. Your function header should look like this

   ```
   function fp = derivative_3point(f,x)
   ```

   You will use the 3-point midpoint formulas for all of the interior x values, but the 3-point endpoint formula for the first and last entries of the vector, x.

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5 (20 points)Consider the function $f(x) = \sin(x)$. **SHOW ALL CODE AND WORK – answers without supporting work will be given zero points.**

   (a) Estimate the derivative of the function at the point $x = 1$ using 3-point midpoint formula for the following values of h: $h = 10^{-n}$ for n going from 16 by integers to 1 (e.g., $h = 10^{-1}, 10^{-2}, ..., 10^{-16}$). Plot the step size h versus the relative error in a loglog plot. On the same figure, plot the estimated truncation error for those same h values in red. (*Hint: Try plotting the 3rd derivative to figure out for which ξ value the the third derivative will be maximum at!*)

(b) What do you notice as h grows smaller? Is the error decreasing with h? Why or why not? Does the actual error follow the truncation error estimate? If not, what do you think causes this discrepancy?