Emmanuel Versus
Math 131
022D

Assignment 3 #5

5. $(\frac{1}{4}, 25.2)\ (\frac{1}{2}, 49.2)\ (1, 96.4)\ (1.25, 119.4)$

Using lagrange interpolating polynomials:

a) Create an interpolating polynomial, predict time at $\frac{3}{4}$ mile mile, compare to actual time of 1:13

$$L_0 = \frac{(x-0.5)}{(0.25-0.5)}\frac{(x-1)}{(0.25-1)}\frac{(x-1.25)}{(0.25-1.25)}$$

$$L_1 = \frac{(x-0.25)}{(0.5-0.25)}\frac{(x-1)}{(0.5-1)}\frac{(x-1.25)}{(0.5-1.25)}$$

$$L_2 = \frac{(x-0.25)}{(1-0.25)}\frac{(x-0.5)}{(1-0.5)}\frac{(x-1.25)}{(1-1.25)}$$

$$L_3 = \frac{(x-0.25)}{(1.25-0.25)}\frac{(x-0.5)}{(1.25-0.5)}\frac{(x-1)}{(1.25-1)}$$

-.229

$$P(x) = (25.2)\left(\frac{(x-0.5)(x-1)(x-1.25)}{-0.1875}\right) + (49.2)\left(\frac{(x-0.25)(x-1)(x-1.25)}{0.09375}\right)$$
$$(96.4)\left(\frac{(x-0.25)(x-0.5)(x-1.25)}{-0.09375}\right) + (119.4)\left(\frac{(x-0.25)(x-0.5)(x-1)}{0.1875}\right)$$

$$P(\tfrac{3}{4}) = (-134.4)(0.03125) + (524.8)(0.0625) + (-1028.267)(-.0625) + (636.8)(-.03125)$$

$P(\frac{3}{4}) = 72.9667$ seconds

$P(\frac{3}{4}) =$ 1 minute 12.97 seconds

Actual time 1 minute 13 seconds

$$\text{Relative error} = \frac{|72.9667 - 73|}{73} \times 100$$

$= .046\ \%$ error

This was a very accurate interpolation.

b) Use derivative of interpolating polynomial to estimate speed at end of the race. :(

First do some simplifying

$P(x) = (-134.4)(x-0.5)(x-1)(x-1.25) + (524.8)(x-0.25)(x-1)(x-1.25)$
$(-1028.267)(x-0.25)(x-0.5)(x-1.25) + (636.8)(x-0.25)(x-0.5)(x-1)$

- Multiply all the x terms

$$P(x) = (-134.4)(x^3 - 2.75x^2 + 2.375x - 0.625) + (524.8)(x^3 - 2.5x^2 + 1.8125x - 0.3125)$$
$$+ (-1028.267)(x^3 - 2x^2 + 1.0625x - 0.15625) + (636.8)(x^3 - 1.75x^2 + 0.875x - .125)$$

Take the derivative

$$P'(x) = (-134.4)(3x^2 - 5.5x + 2.375) + (524.8)(3x^2 - 5x + 1.8125) +$$
$$(-1028.267)(3x^2 - 4x + 1.0625) + (636.8)(3x^2 - 3.5x + 0.875)$$

Plug in $x = 1.25$ (end of the race)

$$P'(1.25) = (-134.4)(0.1875) + (524.8)(0.25) + (-1028.267)(0.75)$$
$$+ (636.8)(1.1875)$$

$P'(1.25) = 90.99$ seconds / mile to get miles / second take inverse,

$1/90.99$

0.0109 miles / second

or $\boxed{39.56 \text{ miles / hour}}$