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22D
Math-131

Assignment 2 Written Portion

4. show $g(x) = 2^{-x}$ has a unique fixed point on $[\frac{1}{3}, 1]$

(1) is $g(x)$ continuous on $[\frac{1}{3}, 1]$

- yes $g(x)$ is continuous

(2) does $g(x) \in [\frac{1}{3}, 1]$ when $x \in (\frac{1}{3}, 1)$?

- Find critical points

$g'(x) = -\ln(2)\,2^{-x} = (-0.693)2^{-x}$

$0 = -0.693\,2^{-x}$

$2^{-x} = 0$

- No critical points

evaluate endpoints

$g(\frac{1}{3}) = 2^{-\frac{1}{3}} = 0.7937$

$g(1) = 2^{-1} = 0.5$

$g(x) \in [0.5, 0.7937]$ for $x \in [\frac{1}{3}, 1]$
thus there exists a fixed point on $[\frac{1}{3}, 1]$

is $|g'(x)| \leq 1$ for all $x \in [\frac{1}{3}, 1]$

$|-(0.693)2^{-x}| \leq 1$

$0.6931(2)^{-x} \leq 1$

The largest value of $g'(x)$ will occur at $\frac{1}{3}$

$0.6931(2)^{-\frac{1}{3}} \leq 1$

$0.5501 \leq 1$ condition holds ✓

Since $|g'(x)| \leq 1$ for all $x \in [\frac{1}{3}, 1]$, then there exists one unique fixed point on $g(x)$ on the interval $[\frac{1}{3}, 1]$

5. Let $f(x) = x^2 - 6$

a) Use Newton's method to find x_1 if $x_0 = 2$

$x_0 = 2$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ $\qquad x_1 = 2 - \dfrac{-2}{4}$

$f(2) = (2)^2 - 6 = -2$ $\qquad x_1 = 2\frac{4}{2} + \frac{1}{2}$

$f'(x) = 2x$

$f'(2) = 2(2) = 4$ $\qquad \boxed{x_1 = \frac{5}{2}}$

b) Use the secant method to find x_2 if $x_0 = 3, x_1 = 2$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$f(x_0) = f(3) = 3^2 - 6 = 3$

$f(x_1) = f(2) = 2^2 - 6 = -2$

$$x_2 = 2 - \frac{(-2)(2-3)}{-2 - 3} = 2 - \frac{2}{-5} = \frac{10}{5} + \frac{2}{5} = \frac{12}{5}$$

$$\boxed{x_2 = \frac{12}{5}}$$