

MATH 131: Numerical Methods for scientists and engineers - Assignment 7

Homework Assignment 7 due by 11:59 PM, May 5, 2021. Test your answers in Matlab.

**For this assignment you are allowed 5 submissions per exercise.
Save each code as an .m file. We will use those codes later in class.**

Matlab Grader

1. (20 points) Create a function `gaussian_elimination` that performs the gaussian elimination of linear system of the form $A\vec{x} = \vec{b}$. The function should return the associated upper triangular matrix U and the modified right-hand side f . The function header should look something like

```
function [U,f] = gaussian_elimination(A,b)
```

Please make sure to consider the case when $a_{i,i} = 0$. In particular, you will need to swap rows with the next row that has a non-zero element in the i th column.

2. (20 points) Create a function `backward_substitution` that solve the system $U\vec{x} = \vec{f}$ when U is an upper triangular matrix. The function header should look something like

```
function x = backward_substitution(U,f)
```

Let us recall that the output has to be a **column** vector. *Hint: Look at your discussion coding assignment 6 to see how you did backwards substitution and generalize for any $n \times n$ matrix!*

3. (20 points) Create a function `forward_substitution` that solve the system $L\vec{y} = \vec{b}$, when L is a lower triangular matrix. The function header should look something like

```
function y = forward_substitution(L,b)
```

Let us recall that the output has to be a **column** vector.

4. (20 points) Create a function `LU_factorization` that performs the LU factorization of a matrix A using the Doolittle method discussed in class (where L has diagonals of 1). The function header should at least contain

```
function [L,U] = LU_factorization(A)
```

Catcourses

5. (20 points) Find the solution \bar{x} for $A\bar{x} = \bar{b}$ where $A = \begin{bmatrix} 1 & 1 & 2 & -5 \\ 2 & 5 & -1 & -9 \\ 2 & 1 & -1 & 3 \\ 1 & 3 & 2 & 7 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ -3 \\ -11 \\ -5 \end{bmatrix}$

- (a) via LU Factorization using the codes developed above for Matlab Grader.
- (b) via Gaussian Elimination using the code developed above for Matlab Grader.