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Math-131
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Assignment 6 written (#3, 4b, 4c on Matlab Publisher file)

4. $mx''(t) + dx'(t) + kx(t) = 0$ $x(t)$ is position of spring

$m =$ mass of spring $k =$ spring constant $d =$ damping coefficient

a) Rewrite as a system of first order DE's

- Let $U = x$ $V = x'$
- deriving we see that

$U' = x'$ $V' = x''$

$U' = x' = V$

- $U' = V$ $V' = x''$

- solve equation for $\cancel{x}''$

$mx''(t) = -dx'(t) - kx(t)$

$x''(t) = \frac{-dx'(t) - kx(t)}{m}$

- now plug in all variables into $U' = V$ and $V' = x''$

$U' = V$

$V' = \frac{-dx'(t) - kx(t)}{m}$ $\rightarrow$ $\boxed{U' = V \quad V' = \frac{-dV - kU}{m}}$

b) - Solve using euler_system code $m = 1\,kg$ $k = 4\,\frac{N}{m}$ $d = 0.5\,Ns/m$ $x(0) = -0.05\,m$ $0 \le t \le 10$ $N = 10{,}000$

- Solved the system and created a graph for the solution. Please see matlab publisher pdf submitted for code and output.

c) The time at which the spring no longer reaches a distance greater than 1cm from the center is 6.345 seconds.

5. $w_0 = \alpha \quad w_1 = \alpha_1$

$$w_{j+1} = -w_{j+1} + 3w_j - w_{j-1} + \Delta t F(t_{j+1}, t_j, t_{j-1}, w_{j+1}, w_j, w_{j-1})$$

a) The numerical scheme is a multi step method, more specifically, it is a 2 step method as it relies on w_j and w_{j-1}, the two previous solutions, to compute w_{j+1}. The method is implicit, because w_{j+1} appears on both sides of the equation.

b) To determine stability, find the characteristic function.
- Two step method, so highest degree is 2.

$$\lambda^2 = -\lambda^2 + 3\lambda - 1$$

$$2\lambda^2 - 3\lambda + 1 = 0$$

- Find roots of characteristic polynomial

$$(2\lambda - 1)(\lambda - 1) = 0$$

$2\lambda - 1 = 0 \qquad \lambda - 1 = 0$

$2\lambda = 1 \qquad \lambda = 1$

$$\lambda = \frac{1}{2}$$

$$\lambda = \frac{1}{2}, 1$$

- All roots are ≤ 1 so the root condition is satisfied and the method is stable. In addition, only one root is equal to 1, so the method is $\boxed{\text{strongly stable.}}$