MATH 131: Numerical Methods for scientists and engineers - Assignment 2

Homework Assignment 2 due by 11:59 PM, February 15, 2021. Test your answers in Matlab
For this assignment you are allowed **5 submissions per exercise.**
Save each code as an .m file. We will use those codes later in class.

Note: In Matlab Grader, you have the option to run a *pretest* which will not count towards your 5 submissions per exercise. To the left of the 'Submit' button, there is a button called 'Run Pretest'. For each of the problems listed below, I have set one pretest to make sure certain keywords are included in your code. Try running the pretest before your initial submission.

Exercises in Matlab Grader:

1. (25 pts) Write a `MATLAB` function, called `bisection_method` that inputs a function f, two numbers a, b, an error tolerance, `tol`, and a maximum number of iterations, N, and finds a root c of f in the interval $[a, b]$ using the bisection method. Your function should compute a bound on the error, and stop when the error is less than the tolerance, or if the number of iterations exceeds N - whichever happens first. The function header should look like

   ```
   function [c,n,err] = bisection_method(f,a,b,tol,N)
   ```

   Hint: The function should contain a 'while' loop, looking something like:

   ```
   while err > tol && n < N
   ```

   Make sure to check for the conditions necessary to apply the bisection method! Look back at the class notes for these conditions. If the conditions are not met, make sure to return c as an empty vector, the error as infinite, and the number of iterations as zero.

   Test Case: Recall that we did an example by hand, approximating the zero on the interval [1,2] for the function $f(x) = x^3 + 4x^2 - 10$. Do your answers match with the by-hand answers? To test this case, type in the command line:
   `bisection_method(@(x)(x.^3+4*x.^2-10),1,2,10^-10,3)`

2. (25 pts) Write a `MATLAB` function, called `fixed_point_iteration` that inputs a function, g, an initial guess x_0, an error tolerance, `tol`, and a maximum number of iterations, N, and outputs the fixed point of g, obtained using the fixed point iteration, starting with x_0. Your function should have an error defined by $E = |x_n - x_{n-1}|$, and stop when the error is less than the tolerance, or if the number of iterations exceeds N - whichever happens first. Your function header should look something like:

   ```
   function [c,n,err] = fixed_point_iteration(g,x0,tol,N)
   ```

   Test Case: Recall that we did an example by hand, approximating the solution for $x = e^{-x}$ with an initial guess of 3 and a total of 2 iterations. Do your answers match with the by-hand answers? To test this, type the following into the command line:
   `fixed_point_iteration(@(x)(exp(-x)),3,10^-10,2)`

3. (25 pts) Write a `MATLAB` function, called `Newtons_method` that inputs a function, f, it's derivative f', an initial guess x_0, an error tolerance, `tol`, and a maximum number of iterations, N, and outputs the root of f obtained using Newton's method (denoted by c), starting with x_0. Your function should have an error defined by `err`$= |x_n - x_{n-1}|$, and stop when the error is less than the tolerance, or if the number of iterations exceeds N - whichever happens first. Your function header should look something like:

```
function [c,n,err] = Newtons_method(f,fp,x0,tol,N)
```

where `n` is the last iteration when you stop.

Test Case: Recall that we did an example by hand, approximating the solution for $f(x) = x^2 - 3$ with an initial guess of 2 and a total of 3 iterations. Do your answers match with the by-hand answers? To test this, type the following into the command line:

```
Newtons_method(@(x)(x.^2-3),@(x)(2*x),2,10^-10,3)
```

Exercises due on CatCourses:

4. (10 pts) Consider the function $g(x) = 2^{-x}$. Use the theorem shown in class to show that the function has a unique fixed point in the interval $\left[\frac{1}{3}, 1\right]$. Show all work.
5. (15 pts) Let $f(x) = x^2 - 6$.
   (a) Use Newton's method to find x_1 if $x_0 = 2$.
   (b) Use the secant method to find x_2 if x_0=3 and $x_1 = 2$