MATH 131: Numerical Methods for scientists and engineers - Assignment 5

Homework Assignment 5 due by 11:59 PM, April 7, 2021. Test your answers in Matlab.
For this assignment you are allowed 5 submissions per exercise.
Save each code as an .m file. We will use those codes later in class.

Matlab Grader

1. (20 points) Write a Matlab function called `euler_timestep` that solves an IVP using Euler's method. The header should look like

   ```
   function [y,t] = euler_timestep(f,t0,tf,alpha,N)
   ```

   where N is the number of intervals used, so that $\Delta t = \frac{t_f - t_0}{N}$. Note that the output should be y, a vector that contains the evaluation of the solution at all time steps and t, a vector of the time variable.

2. (20 points) Write a Matlab function called `rk2` that solves an IVP using the Runge-Kutta method of order 2. The header should look like

   ```
   function [y,t] = rk2(f,t0,tf,alpha,N)
   ```

   where N is the number of intervals used, so that $\Delta t = \frac{t_f - t_0}{N}$. Note that the output should be y, a vector that contains the evaluation of the solution at all time steps and t, a vector of the time variable.

3. (20 points) Write a Matlab function called `rk4` that solves an IVP using the Runge-Kutta method of order 4. The header should look like

   ```
   function [y,t] = rk4(f,t0,tf,alpha,N)
   ```

   where N is the number of intervals used, so that $\Delta t = \frac{t_f - t_0}{N}$. Note that the output should be y, a vector that contains the evaluation of the solution at all time steps and t, a vector of the time variable.

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4 (20 points) Is the following IVP well-posed? If so, what is the Lipschitz constant? Show all work.

$$y'(t) = -\tfrac{4}{t^3}y - t\ln t, \quad 1 \le t \le 4, \quad y(1) = -1.$$

5 (20 points) Find the equation for the Taylor Method of order 3, and it's associated local truncation error. What order is the method? Show all work and perform all steps. Do **not** just use the formula for order n derived in class – instead derive using Taylor Polynomials.