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Number Theory > Special Numbers > Figurate Numbers > Polygonal Numbers >

Hexagonal Pentagonal Number

A number which is simultaneously [pentagonal](#) and [hexagonal](#). Let P_n denote the n th [pentagonal number](#) and H_m the m th [hexagonal number](#), then a number which is both pentagonal and hexagonal satisfies the equation $P_n = H_m$, or

$$\frac{1}{2}n(3n-1) = m(2m-1). \quad (1)$$

Completing the square and rearranging gives

$$(6n-1)^2 - 3(4m-1)^2 = -2. \quad (2)$$

Therefore, defining

$$x \equiv 6n-1 \quad (3)$$

$$y \equiv 4m-1 \quad (4)$$

gives the Pell-like equation

$$x^2 - 3y^2 = -2 \quad (5)$$

The first few solutions are $(x, y) = (1, 1), (5, 3), (19, 11), (71, 41), (265, 153), (989, 571), \dots$. These give the solutions $(n, m) = (1/3, 1/2), (1, 1), (10/3, 3), (12, 21/2), (133/3, 77/2), (165, 143), \dots$, of which the integer solutions are $(1, 1), (165, 143), (31977, 27693), (6203341, 5372251), \dots$ (Sloane's [A046178](#) and [A046179](#)), corresponding to the pentagonal hexagonal numbers 1, 40755, 1533776805, 57722156241751, ... (Sloane's [A046180](#)).

SEE ALSO: [Hexagonal Number](#), [Pentagonal Number](#)

REFERENCES:

Sloane, N. J. A. Sequences [A046178](#), [A046179](#), and [A046180](#) in "The On-Line Encyclopedia of Integer Sequences."

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