Hexagonal Pentagonal Number

A number which is simultaneously pentagonal and hexagonal. Let P_n denote the nth pentagonal number and H_m the mth hexagonal number, then a number which is both pentagonal and hexagonal satisfies the equation $P_n = H_m$, or

$$\frac{1}{2} n (3 n - 1) = m (2 m - 1). \quad (1)$$

Completing the square and rearranging gives

$$(6 n - 1)^2 - 3 (4 m - 1)^2 = -2. \quad (2)$$

Therefore, defining

$$x \equiv 6 n - 1 \quad (3)$$
$$y \equiv 4 m - 1 \quad (4)$$

gives the Pell-like equation

$$x^2 - 3 y^2 = -2 \quad (5)$$

The first few solutions are $(x, y) = (1, 1)$, (5, 3), (19, 11), (71, 41), (265, 153), (989, 571), ... These give the solutions $(n, m) = (1/3, 1/2)$, (1, 1), (10/3, 3), (12, 21/2), (133/3, 77/2), (165, 143), ..., of which the integer solutions are (1, 1), (165, 143), (31977, 27693), (6203341, 5372251), ... (Sloane's A046178 and A046179), corresponding to the pentagonal hexagonal numbers 1, 40755, 1533776805, 57722156241751, ... (Sloane's A046180).

SEE ALSO: Hexagonal Number, Pentagonal Number

REFERENCES:

Sloane, N. J. A. Sequences A046178, A046179, and A046180 in "The On-Line Encyclopedia of Integer Sequences."

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