GPS Signal for Sufficient Coherence with Multiple SDR Receivers System for Antenna Array Applications

1st Evariste Some
Dept. of Computer Science
University of Colorado Boulder
Boulder Colorado, USA

2nd Nadia Yoza Mitsuishi
Dept. of Physics
University of Colorado Boulder
Boulder Colorado, USA

2nd Mark Ruzindana
Independent Researcher
Independent Researcher
Oakland California, USA

2nd Ibrahim Ayad
System Engineer
Cisco Systems, Inc
San Jose, USA

2nd Kevin Gifford
Dept. of Computer Science
University of Colorado Boulder
Boulder Colorado, USA

2nd Jim Lansford
Dept. of Computer Science
Qualcomm Technologies Inc.
Greater Colorado Springs, USA

CORRESPONDING AUTHOR:
Evariste Some e-mail:
evariste.some@gmail.com

***Abstract*—This paper explores the requirements for phase stability of two or multiple software-defined radio (SDR) receivers using the global positioning system (GPS) signal. The illustration relies on a technique that features the benefits of combining the use of injection-locking techniques with GPS signals as a common clock source when applied to a low-cost software-defined radio to improve the accuracy of coherent multiple receivers. This is because coherent systems impose severe requirements on the frequency stability of the signal source at the receiver. This paper shows the benefits of using the highly stable GPS signal, experimentally illustrates the applicability of GPS signal as a common source to multiple SDR receivers, the implications to achieve sufficient coherence with multiple SDRs and its impact to increase the beam directivity precision. The beamforming demonstrated in the results proves the advantages of higher signal-to-noise ratio, interference prevention and rejection over co-channel interference. The phase stability to achieve sufficient coherence proves the feasibility of use for many applications, which include radio astronomy, radio frequency (RF) emitter detection and geo-localization, and beamforming.**

***Index Terms*—GPS receiver, coherent signals system, SDR, antenna array, injection locking, synchronization, correlation, beamforming, MIMO.**

I. Introduction

The Global Navigation Satellite System (GNSS) consists of a constellation of satellites orbiting the Earth in very specific trajectories. GNSS is often generically referred to as Global Positioning System (GPS), but that acronym actually refers specifically to the United States constellation. There are several GNSS constellations provided by governments around the world which include BeiDou-China, Galileo–European Union, GLONASS–Russia, GPS-United States [1].

The signals are sent over radio waves from GNSS satellites and have extremely accurate timestamps and other information encoded into them. This is enabled by the use of incredibly accurate and very expensive atomic clocks on the board of each satellite. Once the GNSS receiver has determined its position, it synchronizes its internal clock, much less accurate, with the satellite clocks. By maintaining that synchronization, the GNSS receiver clock is then considered to have a very accurate timing source. Nowadays, GNSS receiver-based clocks are used extensively in large infrastructures requiring precise time, reference frequency and time synchronization. Many industries now rely heavily on these highly accurate GNSS receiver-based clocks including banks, stock exchanges, telecommunications companies, digital terrestrial broadcast base stations, wireless communication network systems, and electricity suppliers.

The GPS serves as a radio navigation system, but has also become the dominant system for the distribution of time and frequency signals. Each satellite carries a rubidium and/or cesium atomic clock that provides the reference for both the carrier and code broadcasts. The satellite clocks are continuously adjusted to agree with the Coordinated Universal Time (UTC) as maintained by the United States Naval Observatory (USNO) [1], [2]. There are several types of time and frequency measurements that involve the GPS, including one-way, common-view, and carrier-phase measurements.

GPS receivers are often used in time-keeping applications, due to their availability, low price and high accuracy. Distributed industrial and technological systems request precise time synchronization between their sectors. Devices connected to each other by a communication link have to share the same time system in order to complete their tasks [3], [4]. The tightest synchronization requirements lead to the need of highly accurate clock settings, which can be accomplished by means of GPS [3], [5]. The components of the distributed system can easily use the readily available GPS time standard, wherever they are located.

Modified low-cost SDRs hardware are used to illustrate

979-8-3503-9214-2/24/$31.00 ©2024 IEEE

the applicability of GPS signals as a common signal source for multiple SDR receivers and the implications to achieve sufficient coherence for antenna array applications [6], [7].

In the remainder of this paper, Section 2 describes the overall architecture and functionality of the proposed system. Section 3 introduces the GPS receiver signal and GPS disciplined oscillator that provide the locked signal for sufficient coherency. Sections 4 and 5 demonstrate the theory of injection locking and the principle of coherent channels leading to coherent received signals. Finally, Sections 6 through 8 present the method to synchronize multi-channel signals, experiment results, and conclusion.

II. PROPOSED SYSTEM ARCHITECTURE

The proposed architecture consists of the hardware components assembled to build the coherent multi-receivers of sixteen channels using SDRs [6], [7].

Figure 1 presents the comprehensive SDR coherent systems architecture and the connectivity of different parts. Parts include a GPS antenna, GPS discipline oscillator (GPSDO), function generator, RF splitter, SDRs radios, and a USB hub. A synchronization signal is provided by the GPS satellites to the GPS receiver, which in turn generates 1 Pulse Per Second (PPS) to input the GPSDO. The GPSDO translates 1 PPS locked signal to 10 MHz locked signal. The output of the GPSDO 10 MHz signal is fed to the signal generator. The signal generator is then used to select a desired frequency compatible with the SDR local oscillator (LO). The signal generator output signal, still locked, goes through a RF power splitter. The individual outputs of the RF-splitter are then injected to the SDRs local oscillator via a physical connection. Figure 2 shows the 50Ω coaxial cable connected to the LO input pin, which is used to inject the external locked-signal. The 50Ω coaxial cable is used for source and load impedance matching and to minimize signal reflection or maximize power transfer. The standard impedance of the SDR radio used is 50Ω. For signal integrity, components in the circuitry are required to have the same impedance. If mismatched impedances are used between components, the signal will experience reflection, e.g. not all of the signal gets transmitted and some gets reflected back, causing a reduction in signal strength.

Figure 3 illustrates 16 RTL-SDRs PCB board modified and connected to the 16-port splitter, and then to the 16-port USB hub. USB adapters are used to spread out the 16 RTL-SDRs connected to the 16-port USB hub. The RTL-SDRs lay on a foam protective material that has been designed to contain and stabilize the RTL-SDRs.

III. GPS RECEIVER AND GPSDO LOCKED SIGNAL

In the proposed architecture, the GPS signal is used as a frequency standard to correct a separate crystal oscillator on the SDRs as shown in Figure 1. GPS disciplined oscillator works by disciplining or steering a high-quality quartz or rubidium oscillator by locking the output to a GPS signal via a tracking loop (Figure 4). Figure 4 presents a GPS disciplined oscillator unit with a 1 PPS GPS antenna input, and outputs 10 MHz of signal frequency.

The idea of using a GPS receiver signal as a frequency standard is to provide an output signal called PPS. A PPS is a 1 Hz square wave signal, which depending on the model has its rising and falling slope synchronized to occur precisely at the moment that the GPS time advances to the next second. Naturally, this only works when the GPS system sees enough satellites to work out its position and the GPS time. GPS satellites carry atomic oscillators that are steered from stations to agree with the coordinated universal time scale maintained by the United States Naval Observatory [2]. UTC and the National Institute of Standard and Technology time scale are kept in close agreement and seldom differ from each other by more than $20ns$ [8]. The GPS receiver unit features are to provide a more reliable signal and can warn the GPSDO when the reference signal is not reliable [9]–[14].

The GPS receiver signal is fed into the GPSDO BG7TBL-2020-06-10 receiver (Figure 1). Unlike an ordinary consumer-grade navigation GPS receiver, in the GPSDO the GPS receiver has no display, no ability to navigate or store waypoints, and generally no way to communicate with the outside world at all. All access to it is through the control system. The basic function of a GPSDO is to receive signals from the GPS satellites and to use the information contained in these signals to control the frequency of a local oscillator. The satellite's signals can be trusted as a reference for two reasons [15] : First, they originate from atomic oscillators, and they must be accurate in order for the GPS to meet its specifications as a positioning and navigation system. Second, the maximum acceptable contribution from the satellites clocks to the positioning uncertainty is generally considered to be about one meter [2]. Since light travels at about $\sim 3\text{x}10^8 m/s$, one meter requirement is equivalent to a time error of about $3.3ns$. Thus, in order for the GPS system to meet its specification, the satellite clocks must be stable enough to keep time with uncertainty of less than $3.3ns$ during the period between corrections. This translates to a frequency stability specification nearly $6\text{x}10^{-14}$ [2]. The goal of the GPSDO design is to transfer the inherent accuracy and stability of the satellite signals generated by the local quartz oscillator. Therefore, that accurate and locked output signal is used as a common source to injection locking and feed multiple SDRs local oscillators (Figure 1).

The GPSDO output signal is fed into a function generator that divides it down to a frequency suitable to the SDR LO. In this research, the Keysight 33500B Series signal generator synthesizer has been used. The injection-locked range is constrained with the size of the injection current in a meaningful way when the signal is split into multiple outputs through the RF-splitter. Along with the 28.8 MHz selected frequency on the Keysight 33500B, 13.7 dBm of signal amplitude is selected with a simple sine waveform to model the injection waveform. A more universal measure of the injection size which accounts for power in all harmonics is the root-mean-square (rms) of

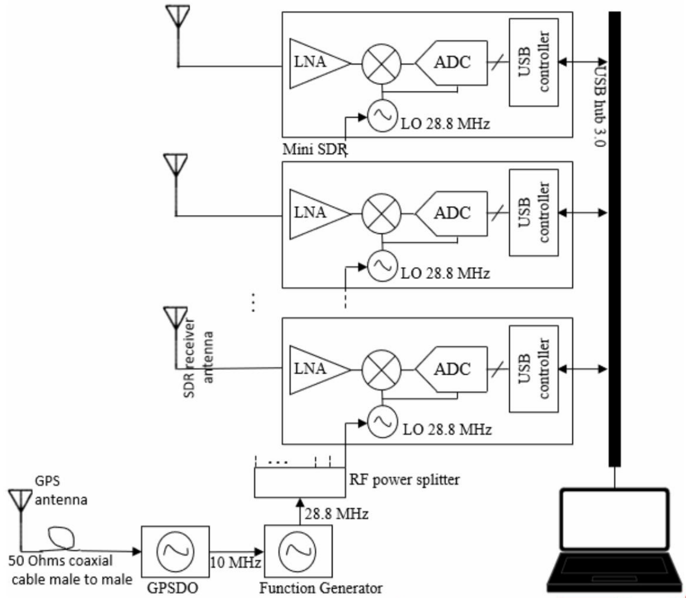

Fig. 1: Coherent SDRs system overall architecture.

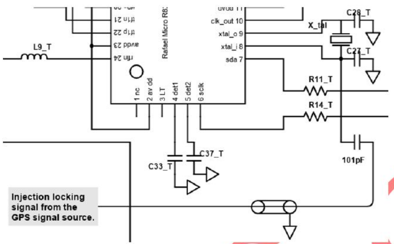

Fig. 2: Schematic illustrating the LO pin used for the injection-locking: RTL-SDR case

Fig. 3: Coherent system hardware assembled

the injection current (I_{rms})

$$I_{rms} \equiv \sqrt{\langle i_{inj}^2 \rangle} := \sqrt{\frac{1}{T_{inj}} \int_{T_{inj}} i_{inj}(t)^2 dt} \qquad (1)$$

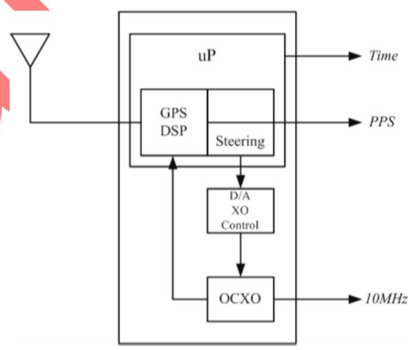

Fig. 4: Generic GPSDO scheme

where i_{inj} is a 2π periodic injection waveform that captures the shape and size of the injection current, $\omega_{inj} \equiv 2\pi/T_{inj}$ is the injection frequency, and T_{inj} the injection period. At first glance, the rms injection current might seem to represent the average power injected into the oscillator. From a different and more practical perspective, I_{rms} serves as a good measure of the average power consumption of the injection circuitry itself [16].

Then, the signal goes through a splitter and then leaked on the SDRs local oscillator pin using $101pF$ capacitors (Fig. 2). This technique is known as injection locking. Any drift or instability of the local oscillator leads to a tuning correction signal to the oscillator, which will stay locked to the PPS. This circuitry will produce an accurate reference with high quality coherent output signals.

IV. Principle of Injection Locking

Injection locking clocking is the physical phenomenon where an oscillator locks onto an external stimulus periodic signal and fundamentally synchronizes with the input when the frequency of the input signal is close enough to the oscillator native frequency or its sub-harmonics [6], [7], [17].

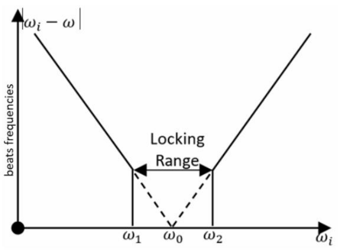

Fig. 5: Beat and injection locking phenomenon

Figure 5 illustrates the beat and injection locking phenomenon when an oscillator is driven by a single-frequency input signal fed by the GPS signal.
Let's assume the following definitions

$\omega_0 =$ free running frequency

$\omega_i =$ frequency of impressed signal

$\omega =$ instantaneous frequency of oscillation

$\Delta\omega_0 = |\omega_0 - \omega_i|$, " undisturbed" beat frequency

$\Delta\omega = |\omega - \omega_i|$, instantaneous beat frequency

Suppose a signal frequency ω_i is injected into an oscillator (Figure 5), which has a self-oscillation or free running frequency ω_0. When ω_i is quite different from ω_0, the "beats" of the two frequencies are observed [17]–[19]. The beat frequency is the difference in frequency of two waves. As ω_i approaches ω_0, the beats frequency ($|\omega_i - \omega_0|$) decreases. When ω_i enters some neighborhood very close to ω_0, the beats disappear and the oscillator starts to oscillate at ω_i. The frequency range in which injection locking happens is called the locking range ω_L [6], [7]. The oscillator frequency locks to the injected frequency with a phase shift of:

$$\psi = \arcsin\left(\frac{\omega_0 - \omega_i}{\omega_L}\right) \tag{2}$$

Disturbances to the local oscillator can cause fluctuation. The focus is on the local oscillator behavior when the disturbance is a periodic injection of current. The oscillator's total phase $\varphi(t)$ can be written as

$$\varphi(t) = \omega_0 t + \phi(t) \tag{3}$$

where $\phi(t)$ is for the phase expression.
In injection locking, it is more useful to represent $\varphi(t)$ in the frame of reference of the injection signal:

$$\varphi(t) \equiv \omega_{inj} t + \theta(t) \tag{4}$$

where θ is the relative phase of the oscillator with respect to the injection. Equation (4) facilitates the treatment of the oscillator under a periodic injection by explicitly identifying the deviation from the injection, namely $\theta(t)$. The instantaneous frequency of oscillation, ω_{osc}, is defined as the time derivative of the total phase:

$$\omega_{osc}(t) := \frac{d\varphi}{dt} \tag{5}$$

Equation (5) being an instantaneous quantity, this oscillation frequency is generally a function of time.

An injection-locked oscillator oscillates at the injection frequency. This requires the total phase, $\varphi(t)$, to grow by exactly 2π every injection period. In light of (4), this means that the value of θ cannot change by a net amount over the same time frame:

$$\theta(t) = \theta(t + T_{inj}) \tag{6}$$

for all time t. Alternatively, this can be represented as

$$\frac{1}{T_{inj}} \int_{T_{inj}} \frac{d\theta}{dt} dt = 0 \tag{7}$$

In other words, an injection-locked oscillator has an average oscillation frequency of ω_{inj}, although the instantaneous oscillation frequency of ω_{inj}, the instantaneous oscillation frequency ω_{osc} may exhibit higher-order fluctuations within a single period.

Although the most general characterization of injection locking allows θ to exhibit intra-period variations, a more limiting notion that is often used is

$$\text{Injection Locked} \iff \frac{d\theta}{dt} = 0 \tag{8}$$

which corresponds to θ being constant in time. This definition of injection locking simplifies the mathematical treatment significantly while providing many useful insights.
Under the framework set forth by (8), a question that arises is what this constant value of θ is for an injection-locked oscillator? The phase difference between the oscillator and the injection is not arbitrary. θ varies with the injection frequency in a specific manner. To quantify this important relationship, let's define a function called the lock characteristic $\Omega(\theta)$, which is equal to the frequency difference $\Delta\omega := \omega_{inj} - \omega_0$ that results in an oscillation phase of θ relative to the injection when locked:

$$\Omega(\theta) := \Delta\omega \text{ for } \frac{d\theta}{dt} = 0 \tag{9}$$

Note that the lock characteristic depends on both the oscillator and the injection waveform $i_{inj,0}$.
It can be seen that the lock characteristic represents the range of possible frequency differences that are achievable under lock. Therefore, it can be used to calculate the upper and lower lock ranges, which are the maximum and the minimum values of $\Delta\omega$ that the oscillator can lock to, respectively [16]:

$$\omega_L^+ = \max_\theta \Omega(\theta) \text{ and } \omega_L^- = \min_\theta \Omega(\theta) \tag{10}$$

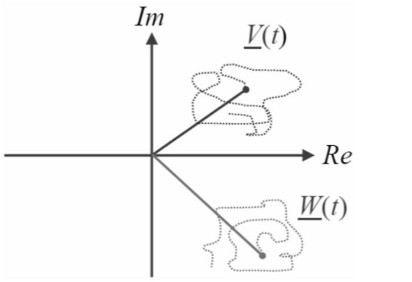

Fig. 6: Coherent signals oscillation illustrated

Recall that injection locking also happens when ω_i is close to the harmonic or sub-harmonic of ω_0, i.e., $n\omega_0$ or $\frac{1}{n}\omega_0$, with integer $n{>}1$. The former case can be used for frequency division, and the latter for frequency multiplication. The locking signal in the oscillator describes the case of synchronization where any transient disturbance vanishes in time and gives way to a steady state in which phase difference between oscillator and external signal is constant [17].

Experimentally, the locking range observed with the RTL-SDRs, i.e. the generic Nooelec NESDR Mini SDR & DVB-T USB, is between 28.73MHz and 28.89MHz. Typically, the locking range is $\sim 10\%$ of the oscillation frequency and the experiment has proven to be true [19]. Locking range is an important parameter for many reasons. First, it clearly controls the detuning range over which the oscillator will remain locked. Second, the locking range determines the time constant for the response of the oscillator to an external perturbation [19].

Finally, an oscillator that is not injection locked, either because the injection is outside of the lock range or because steady state has not been reached, is said to be injection pulled [16].

V. Coherence of Received Signals

Two signals are coherent if they have a constant relative phase at all times and at a given frequency. Coherence is a statistic that can be used to measure the relation between two signals, which is different from correlation [6], [7], [20]–[22]. Consider two phasors, $\underline{V}(t)$ and $\underline{W}(t)$, which correspond to the time processes $v(t)$ and $w(t)$. They are also slowly time varying phases. At any point in time, they lie somewhere in the complex plane. When the motion of the two phasors is in locked steps or even they are out of phase by some degree, no matter the lag offset, the motions of the two signals are coherent. There is a function that describes that and is called the coherent function [23]:

$$\Gamma_{vw}(\tau) = \langle \underline{V}(t)\underline{W}^*(t-\tau)\rangle \tag{11}$$

The coherent function is not equivalent to the cross-correlation function. However, there is a relationship between the cross-correlation function and the coherency [23].

$$R_{vw}(\tau) = \langle v(t)w(t-\tau)\rangle = \frac{1}{2}\,\mathrm{Re}\{\Gamma_{vw}(\tau)e^{j\omega_0\tau}\} \tag{12}$$

where ω_0 is the center frequency that corresponds to these two phasors.

There is another related quantity which is the degree of coherence γ_{vw}. It is simply the coherency function normalized by the product of the standard deviation of phasors $\underline{V}(t)$ and $\underline{W}(t)$ [23]:

$$\gamma_{vw} = \frac{\langle \underline{V}(t)\underline{W}^*(t)\rangle}{\sqrt{\langle|\underline{V}(t)|^2\rangle\langle|\underline{W}(t)|^2\rangle}} \tag{13}$$

γ_{vw} measures how similarly the two signals oscillate, and not how similar their voltages or powers are. When the coherence function is equal to one, it means that two signals are perfectly related. The derived γ function is an important contribution to measure channels coherent status. The goal of this proposed feature engineering metric is to determine the status of a synchronous coherent systems. The feature is applicable to any system that claims coherent received signals. It is based on Equation (13) that measures the similarity between two oscillating signals.

$S\_u \longleftarrow$ initialize Signal U
$S\_w \longleftarrow$ initialize Signal W
$lagIndex \longleftarrow$ initialize lag index
$gamma\_uw\_t \longleftarrow$ initialize as an empty vector
Loop lagIndex:
  $S\_u\_offset \longleftarrow$ remove DC offset from S_u
  $S\_w\_offset \longleftarrow$ remove DC offset from S_w
  $gamma\_uw =$ S_u_offset * S_w_offset
  $SU\_Offset = (S\_u\_offset)\prime * S\_u\_offset$
  $SW\_Offset = (S\_w\_offset)\prime * S\_w\_offset$
  $denom = \sqrt{SU\_Offset * SW\_Offset}$
  $gamma\_uw\_t =$ [gamma_uw_t,gamma_uw /denom]
end Loop

(14)

Equation (13) can be decomposed algorithmically or in some set of initial conditions and be performed in a specific sequence to achieve the measurement goal as prescribed in (14).

Thermal noise is generated from different sources, and is always present and limits the sensitivity of any transceiver for either remote sensing or telecommunications.

Let's consider two phasors, $\underline{V}(t)$ and $\underline{W}(t)$,

$$\Gamma_{vw}(\tau) = \langle \underline{V}(t)\underline{W}^*(t-\tau)\rangle \tag{15}$$

Adding random white Gaussian noise $N_1(t)$ and $N_2(t)$ to each channel will result in the following:

$$\Gamma^n_{vw}(\tau) = \langle[\underline{V}(t)+\underline{N}_1(t)][\underline{W}^*(t-\tau)+\underline{N}^*_2(t-\tau)]\rangle$$
$$\Gamma^n_{vw}(\tau) = \Gamma_{vw}(\tau) + \sigma^2_{n_1n_2}(\tau)$$

Or,

$$\Gamma_{vw}(\tau) = \Gamma^n_{vw}(\tau) - \sigma^2_{n_1n_2}(\tau) \tag{16}$$

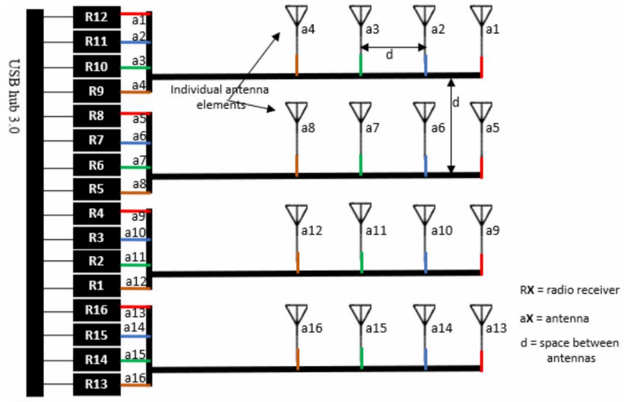

Fig. 7: Reconstructed image using all receive arrays

For Gaussian distribution, the mean vector is often assumed to be zero. Very often, the noise among channels is uncorrelated resulting in a diagonal covariance matrix. As is shown in equation (16), it is impossible to obtain 100 % degree of coherency between two signals because noise from different sources must be removed.

VI. Multichannel Receivers Synchronization

Although fed with the same clock source, SDRs ADCs do not sample at the same time. The generic RTL-SDR radio synchronization between several ADCs is difficult to achieve because the drift is substantial. Even if a common clock is used, there is no guarantee that the samples' streams are synchronized. Typically, a calibration is performed by recording a continuous-wave test signal, which is applied to each channel with a known phase offset. Then, by aligning the phases of the signals received on the individual channels, the constant phase errors of the channels are determined through cross-correlation technique. In reality the fields do not remain coherent for an infinite interval of time. Drifts may occur during the coherence time. From Figure 1, the uniform planar array (UPA) topology has been adopted to receive RF signals from antennas. UPA or 2D uniform linear array (ULA) is the simplest array type, which is planar array with equal inter-element spacing and a progressive phase shift across the array (Figure 7). In signal processing, cross-correlation is a measure of similarity of two series as a function of the displacement of one relative to the other. This is also known as a sliding dot product or sliding inner-product. The cross-correlation function computes the similarity between two signals for all possible lags. It is commonly used for searching a long signal for a shorter known feature. It has applications in pattern recognition, single particle analysis, cryptoanalysis, and neurophysiology. The cross-correlation is similar in nature to the convolution of two functions. In an auto-correlation, which is the cross-correlation of a signal with itself, there will always be a peak at a lag of zero, and its size will be the signal energy. The relationship between the cross-correlation function and the coherency is defined in equation (12). SDRs radios deliver IQ samples at its output that can be interpreted as a complex signal, exhibiting a phase and amplitude information. The delay value of τ in (12), at which the cross-correlation function has a peak, is the delay between the signals. However, in practice several challenges occur. The quality of cross-correlation, i.e. a unique peak, depends on many factors such as signal bandwidth, periodicity, multi-path propagation, noise, transmitted content and the fact that both signals go through different receivers. In theory, the correspondence between correlation and bandwidth is established by the Wiener-Khinchine-Theorem.

Let's assume a signal $v(t)$, a periodic power signal. The Fourier Series of $v(t)$ as a function of Fourier Coefficients is

$$v(t) = \Sigma_{m=-\infty}^{\infty} V_m e^{j2\pi m f_0 t} \quad \text{where } f_0 = \frac{1}{T} \tag{17}$$

Equation 17 is the summation of complex Fourier Coefficient amplitude, multiplied by the complex harmonic function (oscillating function).

The Fourier coefficient of the signal $v(t)$ is

$$V_m = \frac{1}{T}\int_0^T v(t) e^{-j2\pi m f_0 t} \tag{18}$$

Equation (18) uses the periodic signal $v(t)$ multiplied by the conjugate of the complex harmonic and integrated over one period, and divided by the period.

The energy or pulse signals is defined as:

$$E = \int_{-\infty}^{\infty} |v(t)|^2 dt = \lim_{T\to\infty} \int_{-T}^{T} |v(t)|^2 dt < \infty \tag{19}$$

The Fourier Transform of the energy signal is

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t} dt = \mathcal{F}[\text{v(t)}] \quad \text{(V/Hz)} \tag{20}$$

The inverse Fourier Transform is

$$v(t) = \int_{-\infty}^{\infty} V(f) e^{j2\pi f t} df = \mathcal{F}^{-1}[\text{V(f)}] \tag{21}$$

The energy signals using Parseval's Theorem is

$$E \triangleq \int_{-\infty}^{\infty} |v(t)|^2 dt = \int_{-\infty}^{\infty} |V(f)|^2 df \quad (V^2 - sec) \tag{22}$$

The energy spectral density is defined as

$$G(f) : E = \int_{-\infty}^{\infty} G(f) df \Longrightarrow G(f) = |V(f)|^2 \quad (V^2/Hz^2) \tag{23}$$

The power signals using Parseval's Theorem is

$$P \triangleq \frac{1}{T}\int_0^T |v(t)|^2 dt = \Sigma_{m=-\infty}^{\infty} |V_m|^2 \tag{24}$$

Equation (24), Parseval's Theorem for energy signal, is the integral of energy squared in the time domain over one period and divided by the period length. It is also equal to the sum of all coefficients m of the magnitude-square of those coefficients. Therefore, the autocorrelation & Wiener-Khinchine theorem for power signals is

$$R(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{T}^{T} v(t)v^*(t-\tau)dt \tag{25}$$

$$R(0) = P = power = \int_{-\infty}^{\infty} S(f)df \tag{26}$$

The Wiener-Khinchine theorem as a function of auto-correlation is

$$S(f) = \mathcal{F}[R(\tau)] = \int_{-\infty}^{\infty} R(\tau)e^{-j2\pi f\tau}d\tau \tag{27}$$

Auto-correlation function and power spectral density are Fourier transform pairs

$$R(\tau) \overset{\mathcal{F}}{\longleftrightarrow} S(f) \tag{28}$$

In essence, (27) says that the power spectrum of a signal equals the power spectrum of its autocorrelation function, autocorrelation = correlation of a signal with a time-delayed version of itself. The perfect and desirable correlation is a single sharp peak. The power spectrum of such a sharp peak is a wideband, and following the theorem, is the power spectrum of the time signal.

Multiple SDRs simultaneously receiving signals at the same frequency will be perfectly aligned if they are fully synchronized. To handle the time and phase synchronization, a calibration step is required and is handled by the software [6]. The synchronization step added to the injection locking processes enables a fixed offset between channel signals.

VII. Experiment Results

To manifest the results of the methodology described in the previous sections, this section presents the contribution of the GPS signal for sufficient coherence of multiple SDR receiver systems using injection locking technique. The testbed utilizes GNU Radio standard packages, and the multiple channels receivers packages were used to build a program to capture data in real-time from the coherent and sampling synchronous system proposed [24], [25]. MATLAB is used for post processing to analyze the spectrum, cross-correlate channels, and plot the metric that illustrates the signals degree of coherency and their ADCs time sampling synchronization [6], [7]. The GPS signal for sufficient coherence of multiple SDR receivers system is implemented, as shown in Figure 8. A USRP B200 is used as a base station transmitter. The transmitter transmits a 1 KHz frequency modulated signal at -10 dBm of power. From Figure 1, two SDRs are configured to receive beamforming signals. The transmitter and receivers are tuned to the same frequency. The result shows a narrow main lobe beam, which demonstrates sufficient coherence between channels. Though the beam points in a particular direction, it also measures the degree of linear dependency of two antenna-signals by testing for similar frequency components. The narrower the beam, the higher the similarity of data received by both channels. Figure 10 presents a scatter matrix plot of 8-coherent channels. The scatter matrix plot provides a visual and statistical means to test the strength of a relationship between the eight channels. Each scatter plot in the matrix illustrates the relationship between a pair of two channels, allowing many relationships to be explored in one chart. It is effective in measuring the strength of relationships, i.e. the correlation between channels. Rather than points scatter around, the relationship between two-channels shows a compact structure, which induces a high degree of correlation between channels.

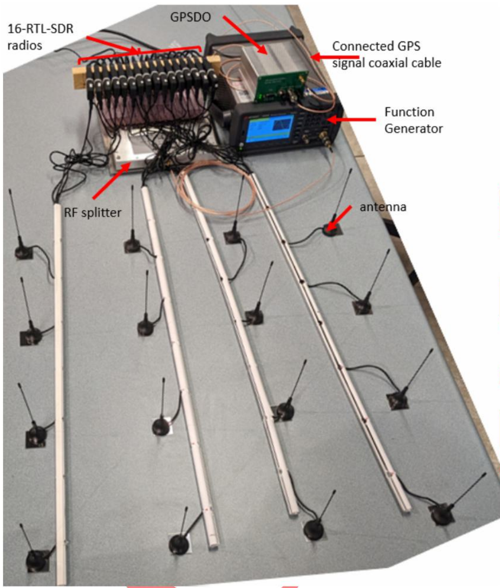

Fig. 8: SDRs coherent system using injection-locking with a GPS locked signal, and a uniform planar array topology setup

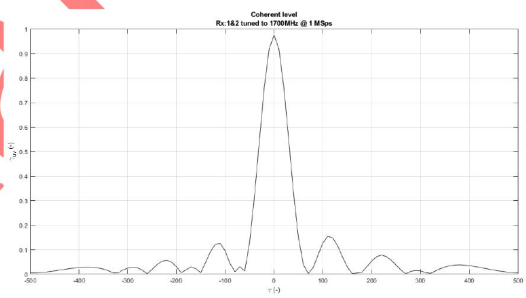

Fig. 9: Coherent degree beam measured using two SDRs.

VIII. Conclusion

This research demonstrates the benefits of using the highly stable GPS signal in operating the GPSDO, and experimentally illustrates the applicability of GPS signal as a common source to multiple SDR receivers. The implications to achieve sufficient coherence with multiple SDRs and its impact to a high degree of correlation channels and therefore to beamform received signals are demonstrated (Figures 9, 10).

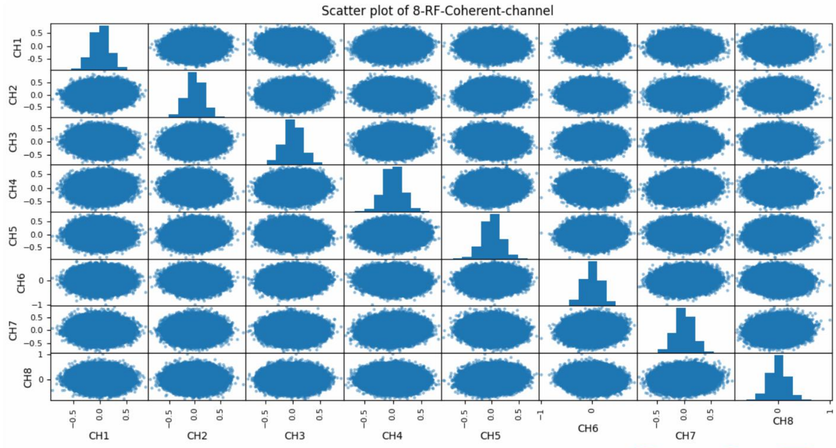

Fig. 10: Scatter matrix plot of 8-RF-Coherent Channels

For future work, Figure 3 will be upgraded and be used for radio frequency emitter geo-localization, applications to sweeping beamforming across an area, and applications of machine learning (ML) in wireless communication. The technology of wireless communications has drastically changed, e.g. the rapidly growing wave of wireless data is pushing against the boundary of wireless communication system's performance. ML has led to significant breakthroughs in all areas of science and technology. ML techniques continue to emerge, paving the way for new research directions and applications, and should have a transformative impact on wireless communication systems, which includes implementing more complex beamforming in 5G & 6G.

REFERENCES

[1] A. Silwal, S. Gautam, P. Poudel, M. Karki, B. Adhikari, N. Chapagain, R. Mishra, B. Ghimire, and Y. Migoya-Orue, "Global positioning system observations of ionospheric total electron content variations during the 15th january 2010 and 21st june 2020 solar eclipse," Radio Science, vol. 56, no. 5, pp. 1–20, 2021.

[2] M. A. Lombardi, "The use of gps disciplined oscillators as primary frequency standards for calibration and metrology laboratories," NCSLI Measure, vol. 3, no. 3, pp. 56–65, 2008.

[3] V. R. Ogrizovic, J. Marendic, S. Renovica, J. Gucevic et al., "Testing gps generated 1pps against a rubidium standard," Acta Imeko, vol. 2, no. 1, pp. 7–11, 2013.

[4] J. Elson and D. Estrin, "Time synchronization for wireless sensor networks," in Parallel and Distributed Processing Symposium, International, vol. 4. IEEE Computer Society, 2001, pp. 30 186b–30 186b.

[5] A. Carta, N. Locci, C. Muscas, F. Pinna, and S. Sulis, "Gps and ieee 1588 synchronization for the measurement of synchrophasors in electric power systems," Computer Standards & Interfaces, vol. 33, no. 2, pp. 176–181, 2011.

[6] E. Some and A. J. Gasiewski, "Software defined radio injection-locking using a gps signal for multichannel coherent receivers," in 2023 IEEE Aerospace Conference. IEEE, 2023, pp. 1–10.

[7] E. Some, "Low-cost software defined radio injection-locking for coherent multichannel receivers," Ph.D. dissertation, University of Colorado at Boulder, 2023.

[8] R. E. Beehler and M. A. Lombardi, "Nist time and frequency services," NIST SPECIAL PUBLICATION;, no. 432, 1991.

[9] J. S. Sandenbergh, "Synchronising coherent networked radar using low-cost gps-disciplined oscillators," 2019.

[10] C.-L. Cheng, F.-R. Chang, and K.-Y. Tu, "Highly accurate real-time gps carrier phase-disciplined oscillator," IEEE Transactions on Instrumentation and Measurement, vol. 54, no. 2, pp. 819–824, 2005.

[11] W.-Q. Wang, "Phase noise suppression in gps-disciplined frequency synchronization systems," Fluctuation and Noise Letters, vol. 10, no. 03, pp. 303–313, 2011.

[12] E. Loria, S. Prager, I. Seker, R. Ahmed, B. Hawkins, and M. Lavalle, "Modeling the effects of oscillator phase noise and synchronization on multistatic sar tomography," IEEE Transactions on Geoscience and Remote Sensing, vol. 61, pp. 1–12, 2023.

[13] J. S. Sandenbergh and M. R. Inggs, "A common view gpsdo to synchronize netted radar," in 2007 IET International Conference on Radar Systems. IET, 2007, pp. 1–5.

[14] P. J. Beasley and M. A. Ritchie, "Multistatic radar synchronisation using cots gps disciplined oscillators," 2022.

[15] M. A. Lombardi et al., NIST time and frequency services. US Department of Commerce, Technology Administration, National Institute of . . . , 2003.

[16] B. Hong and A. Hajimiri, "A general theory of injection locking and pulling in electrical oscillators—Part I: Time-synchronous modeling and injection waveform design," IEEE Journal of Solid-State Circuits, vol. 54, no. 8, pp. 2109–2121, 2019.

[17] R. Adler, "A study of locking phenomena in oscillators," Proceedings of the IRE, vol. 34, no. 6, pp. 351–357, 1946.

[18] L. Zhang, A. Carpenter, B. Ciftcioglu, A. Garg, M. Huang, and H. Wu, "Injection-locked clocking: A low-power clock distribution scheme for high-performance microprocessors," IEEE Transactions on Very Large Scale Integration (VLSI) Systems, vol. 16, no. 9, pp. 1251–1256, 2008.

[19] M. S. Mehrjoo and J. F. Buckwalter, "A differential oscillator injection locking technique for an 8 ghz outphasing modulator with 22.7% modulation efficiency," IEEE Journal of Solid-State Circuits, vol. 51, no. 12, pp. 3093–3102, 2016.

[20] T.-J. Shan and T. Kailath, "Adaptive beamforming for coherent signals and interference," IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. 33, no. 3, pp. 527–536, 1985.

[21] K. Roberts, D. Beckett, D. Boertjes, J. Berthold, and C. Laperle, "100g and beyond with digital coherent signal processing," IEEE Communications Magazine, vol. 48, no. 7, pp. 62–69, 2010.

[22] M. J. Hinich, "A statistical theory of signal coherence," IEEE Journal of Oceanic Engineering, vol. 25, no. 2, pp. 256–261, 2000.

[23] A. J. Gasiewski, "Remote sensing signals and systems lecture," Spring 2020.

[24] P. Krysik, "Multi-rtl making multiple channels receiver from cheap rtl-sdr dongles," https://ptrkrysik.github.io/;, 2016.

[25] B. Bloessl, M. Segata, C. Sommer, and F. Dressler, "Performance assessment of ieee 802.11 p with an open source sdr-based prototype," IEEE transactions on mobile computing, vol. 17, no. 5, pp. 1162–1175, 2017.