Apr 1 2019

Sec 1.1 Intro to Linear Systems

Recall algebra, e.g. $x+5=3$. Generalize to two variables:

$$\begin{cases} x+y=5 \\ 3x-y=-1 \end{cases}.$$

Solving intuitively, $x=1, y=4$. The problem on Page 1:

$$\begin{cases} x+2y+3z=39 \\ x+3y+2z=34 \\ 3x+2y+z=26 \end{cases}. \tag{1}$$

Answer is $x=2.75, y=4.25, z=9.25$.

Some systems are not (uniquely) solvable.

$$\begin{cases} 2x+4y+6z=0 \\ 4x+5y+6z=3 \\ 7x+8y+9z=6 \end{cases}. \tag{2}$$

$$\begin{cases} x+2y+3z=0 \\ 4x+5y+6z=3 \\ 7x+8y+9z=0 \end{cases}. \tag{3}$$

Geometric interpretation: find points that lie on all three planes.

How do I picture a plane in $\mathbb{R}^3$?

$ax+by+cz=0$ defines a plane perpendicular to (a,b,c) passing origin. Translate it to get $ax+by+cz=d$.

Sec 1.2 Matrices, Vectors, and Gauss-Jordan Elimination

- matrix dimension; row, column, index notation
- identity, zero, square, upper/lower triangular, symmetric matrices
- vector, vector spaces $\mathbb{R}^n$ (column vectors!)
- solve (1) using extended matrix
- Gaussian reduction: three operations
- RREF: definition and solve (1)
- solve (2) with parameterization

Sec 1.3 On the Solutions of Linear Systems; Matrix Algebra

Geometric interpretation

Intersection of planes, either unique or infinitely many solutions.

Rank

Matrices from (1) and (2) have rank 3 and 2. Full rank matrix has identity in RREF.

"Degrees of freedom"

$$\begin{cases} x + z = -7 \\ x + 3z = 3 \\ x + 5z = 13 \end{cases} \qquad \begin{cases} x + y + z = 1 \\ y + 3z = 3 \end{cases} \qquad \begin{cases} x + y + 4z = 1 \\ x - y + z = 1 \\ 3x + y - z = 5 \\ x + 4y - 6z = 0 \end{cases}$$

First: $x = -12, z = 5$ but no constraint on y.

Quick check, doesn't prove solvability.

Matrix Algebra