

1. Find the 3-volume of the 3-parallelepiped defined by the vectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

2. Write down the set of eigenvalues for the following linear transforms.

- (a) Reflection about the  $y$ -axis in  $\mathbb{R}^2$ . *Hint:* It's associated with the matrix  $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ .  
(b) Reflection about the  $xy$ -plane in  $\mathbb{R}^3$ .  
(c) Reflection about a plane  $V = \{x \in \mathbb{R}^3 : n^T x = 0\}$  in  $\mathbb{R}^3$ ; here  $n \in \mathbb{R}^3$  is the unit normal vector of the plane.  
(d) Orthogonal projection onto a plane  $V = \{x \in \mathbb{R}^3 : n^T x = 0\}$  in  $\mathbb{R}^3$ ; here  $n \in \mathbb{R}^3$  is the unit normal vector of the plane.

3. **(Bonus, 4 points)** Find the eigenvectors corresponding to the eigenvalues of the linear transformations in the above problem.