1. Consider the basis $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$ of $\mathbb{R}^2$.

(a) Derive P such that $[x]_{\mathcal{B}} = Px$ for any $x \in \mathbb{R}^2$.
(b) Derive S such that $x = S[x]_{\mathcal{B}}$ for any $x \in \mathbb{R}^2$.

2. Derive the matrix A such that $T(x) = Ax$ corresponds to scaling $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ two times and scaling $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ three times, i.e.

$$T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = 2\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 5 \\ 2 \end{bmatrix}\right) = 3\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \end{bmatrix}.$$

You don't need to multiply out the matrix.

Bonus (2 points): What is the rank of A? *Hints:* Instead of computing $rref(A)$, you may want to assess $ker(T)$ and use rank-nullity theorem: $\dim(ker(T)) + rank(A) = 2$.