Consider the quadratic form

$$q(x_1, x_2, x_3) = 4x_1x_2 + 4x_1x_3 + x_2^2 - x_3^2.$$

Answer the following questions. To prevent calculation error in part (a) or (b) costing you points for all problems, consider answering part (c) and (d) not only specifically to this problem but also for general results.

(a) (1 point) Find the symmetric matrix A such that $q(x) = x^T Ax$ for $x \in \mathbb{R}^3$.

(b) (6 points) Find the orthogonal eigenvalue decomposition $A = SDS^T$ where S is an orthogonal matrix and D is diagonal.

(c) (2 points) Determine the definiteness of the quadratic form q.

(d) (1 point) What is the maximal value q takes on the set of all unit vectors? i.e. what is $\max_{\|x\|=1} q(x)$?