1. The Bregman distance generated by the kerneel $\phi(x) = \frac{1}{2t}\|x\|_2^2 - g(x)$ is

$$
\begin{aligned}
d(x,y) &= \frac{1}{2t}(\|x\|_2^2 - \|y\|_2^2) - g(x) + g(y) - \left(\frac{1}{t}y - \nabla g(y)\right)^T (x-y) \\
&= \frac{1}{2t}(\|x\|_2^2 + \|y\|_2^2) - g(x) + g(y) - \frac{1}{t}x^T y + \nabla g(y)^T (x-y) \\
&= \frac{1}{2t}\|x-y\|_2^2 - g(x) + g(y) - \nabla g(y)^T (x-y).
\end{aligned}
$$

Substitute y with x_k, the proximal point iteration boils down to

$$
\begin{aligned}
x_{k+1} &= \arg\min_x g(x) + h(x) + \frac{1}{2t}\|x-x_k\|_2^2 - g(x) + g(x_k) - \nabla g(x_k)^T (x-x_k) \\
&= \arg\min_x th(x) + \frac{1}{2}\|x-x_k\|_2^2 - t\nabla g(x_k)^T (x-x_k) \\
&= \arg\min_x th(x) + \frac{1}{2}\|x-x_k-t\nabla g(x_k)\|_2^2 - \frac{t^2}{2}\|\nabla g(x_k)\|_2^2 \\
&= \mathrm{prox}_{th}(x_k - t\nabla g(x_k)).
\end{aligned}
$$