1. Find all (real) eigenvalues and a basis of each eigenspace for A.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

Is A diagonalizable? If so, find the invertible matrix S and diagonal matrix D such that $A = SDS^{-1}$.

2. Consider an n by n matrix $A \in \mathbb{R}^{n \times n}$.
(a) Write down the definition of the characteristic polynomial of A.
(b) Write down the definition of the algebraic multiplicity of an eigenvalue λ of A.
(c) Write down the definition of the geometric multiplicity of an eigenvalue λ of A.
(d) *True of false.* Given an eigenvalue λ of A, its geometric multiplicity must be greater than or equal to its algebraic multiplicity.
(e) **(Bonus, 1 point)** *True or false.* A can have at most n eigenvalues.
(f) **(Bonus, 2 point)** *True or false.* Assuming A is invertible, if λ is an eigenvalue of A, then $1/\lambda$ must be an eigenvalue of A^{-1}.