To show the *linearized ADMM*

$$\begin{aligned} x_{k+1} &= \mathrm{prox}_{\sigma f}(x_k - \tau\sigma A^T(Ax_k - y_k + u_k)) \\ y_{k+1} &= \mathrm{prox}_{\frac{1}{\tau}g}(Ax_{k+1} + u_k) \\ u_{k+1} &= u_k + Ax_{k+1} - y_{k+1} \end{aligned}$$

is equivalent to *PDHG* applied to the dual problem,

$$\begin{aligned} z_{k+1} &= \mathrm{prox}_{\tau g^*}(z_k + \tau A\tilde{x}_k) \\ \tilde{x}_{k+1} &= \mathrm{prox}_{\sigma f}(\tilde{x}_k - \sigma A^T(2z_{k+1} - z_k)), \end{aligned}$$

first we eliminate the variable y in linearized ADMM by $u_{k+1} = u_k + Ax_{k+1} - \mathrm{prox}_{\frac{1}{\tau}g}(Ax_{k+1} + u_k)$. Using Moreau decomposition on $\frac{1}{\tau}g$, we can reformulate linearized ADMM as

$$\begin{aligned} x_{k+1} &= \mathrm{prox}_{\sigma f}(x_k - \tau\sigma A^T(Ax_k - y_k + u_k)) \\ u_{k+1} &= \frac{1}{\tau}\mathrm{prox}_{\tau g^*}(\tau(Ax_{k+1} + u_k)). \end{aligned}$$

From here consider $z_k = \tau u_k$, $\tilde{x}_k = x_k$ and start the iteration at u-update and renumber the iterates, we get PDHG applied on dual problem.