

1. Use the determinant to find out for which values of the constant  $k$  the given matrix  $A$  is invertible:

$$A = \begin{bmatrix} 0 & 1 & k \\ 3 & 2k & 5 \\ 9 & 7 & 5 \end{bmatrix}$$

2. Consider an  $n \times n$  matrix  $A$  such that  $\det(A) = 3$ . Answer the following questions. You do not need to provide reasoning to your answers.

(a)  $\det(A^T) = ?$

(b)  $\det(A^T A) = ?$

- (c) Consider the QR-factorization  $A = QR$ .  $\det(Q) = ?$ ,  $\det(R) = ?$  *Hint:* Make observation to the value of  $\det(Q^T Q) = \det(I)$  to conclude about  $\det(Q)$ .

3. **(Bonus, 2 points)** We know that similar matrices have the same value of determinant. Can you give an example such that  $\det(A) = \det(B)$  for two matrices  $A, B \in \mathbb{R}^{n \times n}$  but  $A$  and  $B$  are not similar? You have to justify that the matrices you give are not similar.