# Combination 4

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2023 年 11 月 3 日

PROBLEM I Divide $n$ into positive integers that are less or equal $m$, let $p'(n,m)$ be the counting of different ways. Prove $p'(n,m) = p'(n,m-1) + p'(n-m,m)$.

SOLUTION . Since $p'(n,m)$ equal to the solution of equation $n = 1 \cdot k_1 + \cdots + m \cdot k_m$, where $0 \le k_i \le m, 1 \le i \le m$, then

1. When $k_m = 0$, the total solution of equation $n = 1 \cdot k_1 + \cdots + m \cdot k_m$, where $0 \le k_i \le m, 1 \le i \le m$ is $p'(n, m-1)$.
2. When $k_m \ge 1$, the total solution of equation $n = 1 \cdot k_1 + \cdots + m \cdot k_m$, where $0 \le k_i \le m, 1 \le i \le m$ equal to the total solution of equation $n - m = 1 \cdot k_1 + \cdots + m \cdot k_m$, where $0 \le k_i \le m, 1 \le i \le m$, which is $p'(n-m, m)$.

So $p'(n,m) = p'(n,m-1) + p'(n-m,m)$. $\square$

PROBLEM II Calculate the $p(9,5)$.

SOLUTION . By Ferrers picture, we can easily get $p(n,m) = p'(n,m) - p'(n,m-1)$. So by PROBLEM I $p(n,m) = p'(n-m,m)$, then $p(9,5) = p'(4,5) = p'(4,4) = p'(4,3) + p'(0,4) = p'(4,2) + p'(1,3) = p'(4,1) + p'(2,2) + p'(1,1) = p'(4,1) + p'(2,1) + p'(0,2) + p'(1,1) = 1 + 1 + 0 + 1 = 3$. $\square$

PROBLEM III Prove: when $m \equiv 0 \pmod 6$, $p(m,3) = \frac{m^2}{12}$.

SOLUTION . Let $m = 6k, k \in \mathbb{N}$, so it equals to prove $p(6k,3) = \frac{(6k)^2}{12} = 3k^2$ :

1. When $k = 0$, so $p(0,3) = 0$.
2. When $k$, there is $p(6k,3) = \frac{(6k)^2}{12} = 3k^2$. So same as PROBLEM II, $p(6k+6,3) = p'(6k+6,3) - p'(6k+6,2) = p'(6k+3,3) = p'(6k+3,2) + p'(6k,3) = p'(6k+3,2) + p'(6k,2) + p(6k,3)$. Since $p'(6k+3,2) = 3k+1+1 = 3k+2, p'(6k,2) = 3k+1$, $p(6k,3) = 3k^2$. So, $p(6k+6,3) = 3k^2 + 6k + 3 = 3(k+1)^2$.

$\square$