## Graduate Homework In Mathematics

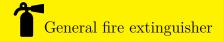
**GroupRepresentation 4** 

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2023年10月27日



ROBEM I K is a field, A is algebra on K,  $\emptyset \neq A_1 \subset A$ , we call  $A_1$  is a subalgebra of A, if  $A_1$  is a subring of A which contains 1 of A and  $A_1$  is a subspace of A on K and is also an algebra on K. Let  $Z(A) := \{c \in A : ca = ac, \forall a \in A\}$ . Prove: Z(A) is a subalgebra of A, we call Z(A) is the center of algebra A.

- SOUTION. 1. Z(A) is a subring of A which contains 1 of A: Since  $\forall a \in A$ , A is a ring, then 1a = a1. So  $1 \in Z(A)$ .  $\forall c_1, c_2 \in Z(A)$ ,  $\forall a \in A$ ,  $(c_1 c_2)a = (c_1 + (-c_2))a = c_1a + (-1)c_2a = ac_1 + (-1)ac_2 = ac_1 + a(-1)c_2 = a(c_1 + (-1)c_2) = a(c_1 c_2)$ , then  $c_1 c_2 \in Z(A)$ .  $(c_1c_2)a = c_1(c_2a) = c_1(ac_2) = (c_1a)c_2 = (ac_1)c_2 = a(c_1c_2)$ , then  $c_1c_2 \in A$ .
  - 2. Z(A) is a subspace of A on  $K: \forall k \in K$ ,  $\forall c, c_1, c_2 \in Z(A)$ ,  $\forall a \in A$ , since A is an algebra on K, then (kc)a = k(ca) = k(ac) = a(kc), then  $kc \in Z(A)$ . And by ??, we get  $c_1 + c_2 \in Z(A)$ .
- 3. By ??, ??, we get Z(A) is also an algebra on K. So Z(A) is a subalgebra of A.