under Graduate Homework In Mathematics

FunctionalAnalysis 9

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General fire extinguisher

ℝOBLEM I $(C[0,1], \|\cdot\|_1)$, let $f : C[0,1] \to \mathbb{R}$, $x \mapsto \int_0^1 sx(s)\,\mathrm{d}s$. Prove f is continous linear functional on $C[0,1]$, calculate $\|f\|$.

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ℝOBLEM II $T : (\mathbb{R}^n, l^1) \to (\mathbb{R}^n, l^1)$ is linear operation. Calculate $\|T\|$.

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ℝOBLEM III $f : C[a,b] \to \mathbb{R}$, $x \mapsto x(a) - x(b)$. Prove f is bounded linear functional, calculate $\|f\|$.

ℝOBLEM IV $f : \mathcal{X} \to \mathbb{R}$,$x \mapsto \int_0^1 \sqrt{t}x(t^2)\,\mathrm{d}t$.Calculate $\|f\|$

1. $\mathcal{X} = C[0,1]$.
2. $\mathcal{X} = L^2[0,1]$

ℝOBLEM V $\Phi : C[0,1] \to \mathbb{R}$, $\Phi(f) \mapsto \int_0^1 \phi(t)f(t)\,\mathrm{d}t$, where $\phi \in C[0,1]$ Calculate $\|\Phi\|$