GroupRepresentation 6

王胤雅

201911010205

201911010205@mail.bnu.edu.cn

2024 年 1 月 5 日

PROBLEM I H_1, H_2 are sub-modules of left module M over ring R, prove: $H_1 + H_2$ is direct sum iff $H_1 \cap H_2 = \{0\}$.

SOLUTION.

1. "$\Rightarrow$": Obviously.
2. "$\Leftarrow$": Only need to prove: $\forall h_1, g_1 \in H_1, h_2, g_2 \in H_2$, s.t. $h_1 + h_2 = g_1 + g_2$, then $h_1 = g_1, h_2 = g_2$: Since $h_1 + h_2 = g_1 + g_2$, then $h_1 - g_1 = g_2 - h_2 \in H_1 \cap H_2 = \{0\}$, so $h_1 - g_1 = h_2 - g_2 = 0$, so $h_1 = g_1, h_2 = g_2$.

□

PROBLEM II H_1, H_2 are sub-modules of left module M over ring R s.t. $H_1 \oplus H_2 = M$, prove: $M/H_1 \cong H_2$, $M/H_2 \cong H_1$.

SOLUTION. From the communitive of direct sum and the symmetry of H_1, H_2, so we only need to prove $M/H_1 \cong H_2$. Let $f : M \to H_2$, $x \mapsto x_2$, where $x = x - x_2 + x_2, x - x_2 \in H_1$. Since $H_1 \oplus H_2 = M$, then $\exists | x_2 \in H_2$ s.t. $x = x - x_2 + x_2, x - x_2 \in H_1$. So f is well-defined. And $x + y = (x - x_2 + x_2) + (y - y_2 + y_2) = (x + y) - (x_2 + y_2) + (x_2 + y_2)$, so f is homormophism. $rx = r((x - x_2) + x_2) = r(x - x_2) + rx_2$, where $r(x - x_2) \in H_1, rx_2 \in H_2$. So f is module homormophism from M to H_2, and $\ker f = H_1$. So $M/H_1 \cong H_2$. □

PROBLEM III $H_1, H_2, \cdots, H_n$ are sub-modules of left module M over ring R s.t. $H_1 \oplus H_2 \oplus \cdots \oplus H_n = M$, prove: $M/(H_2 \oplus \cdots \oplus H_n) \cong H_1$.

SOLUTION. Since $H_1' = H_1, H_2' = (H_2 \oplus \cdots \oplus H_n)$ are modules, so by PROBLEM II, $M/H_2' \cong H_1'$, so $M/(H_2 \oplus \cdots \oplus H_n) \cong H_1$. □