FUNCTIONAL4

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ProblemI. (X, d) is a distance space, $A \subset X$ is a self-sequence compact set. $\forall f \in C(A) := \{f \in \mathbb{R}^A : f\text{is continuous}\}$, $f(A) := \{f(x) : x \in A\}$. Proof: $f(A)$ is bounded and $\max f(A) = \sup f(A), \min f(A) = \inf f(A)$.

ProblemII. (X, d) is a distance space, $M \subset X$ is a self-sequence compact set. $\forall f \in C(M) := \{f \in \mathbb{R}^M : f \text{ is continuous}\}$. Proof: f is continuous uniformly.

ProblemIII. $M \subset C[a, b]$, M is bounded, proof: $S = \{\int_a^x f(t)\mathrm{d}t | f \in M\}$ is a sequence compact.

ProblemIV. $\varphi : \mathbb{R}^n \to \mathbb{R}$, $\forall x = (x_1, \cdots, x_n) \in \mathbb{R}^n, \varphi(x) = (\sum_{k=1}^n |x_k|^{1/2})^2$. Is $(\mathbb{R}^n, \varphi)$ a B^* space?

ProblemV. $|| \cdot || : \mathbb{C}^\infty \to \mathbb{R}$, $\forall x = (x_1, \cdots, x_n, \cdots)$, $||x|| = \sum_{n=1}^\infty 2^{-n} \min\{1, |x_n|\}$

1. Is $|| \cdot ||$ a norm on $\mathbb{C}^\infty$?
2. $\mathrm{d} : C^\infty \times C^\infty \to \mathbb{R}$, $\forall x, y \in C^\infty, \mathrm{d}(x, y) = ||x - y||$. Whether d is the distance on $\mathbb{C}^\infty$. If so, explain the meaning of $||x^{(n)} - x|| \to 0 (n \to \infty)$.