under Graduate Homework In Mathematics

FunctionalAnalysis 10

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PROBLEM I Let $f \in \mathcal{X}^*$, $f \neq 0$, let d $:= \inf\{\|x\| : f(x) = 1, x \in \mathcal{X}\}$, prove: $\|f\| = \frac{1}{d}$.

SOLUTION . kk □

PROBLEM II Let $f \in \mathcal{X}^*$, prove: $\forall \varepsilon > 0$, $\exists x_0 \in \mathcal{X}$, such that $f(x_{0)} = \|f\|$, and $\|x_0\| < 1 + \varepsilon$.

SOLUTION . kk □

PROBLEM III Let $T : \mathcal{X} \to \mathcal{Y}$ is linear, let $N(T) := \{x \in \mathcal{X} : Tx = 0\}$.

1. If $T \in \mathscr{L}(\mathcal{X}, \mathcal{Y})$, prove: $N(T)$ is closed subspace of $\mathcal{X}$.
2. Can $N(T)$ is closed subspace in $\mathcal{X}$ infer $T \in \mathscr{L}(\mathcal{X}, \mathcal{Y})$?
3. If f is a linear functional, prove: $f \in \mathcal{X}^* \iff N(f)$ is closed subspace in $\mathcal{X}$.

SOLUTION . kk □