## Graduate Homework In Mathematics

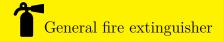
**GroupRepresentation 7** 

王胤雅

201911010205

201911010205@mail.bnu.edu.cn

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ROBEM I R is a ring with identity element. If every non zero element in R is inversible, we call R is division ring. Prove: if D is a division ring, then  $M_n(D)$  is a monocycle.

SOLTON. I is non zero two sides ideal of  $M_n(D)$ .  $\forall E_{ij}, 1 \leq i, j \leq n$ , let  $A = (a_{ij}) \neq 0$ ,  $a_{st} \neq 0$ . So  $E_{is}AE_{tj} = a_{st}E_{ij} \in I$ . So  $E_{ij} \in I$ . So  $M_n(D) \subset I \subset M_n(D)$ .

ROBEM II V is right module of division ring D,  $f: D \times V \to V$  is an action of D on V. Let  $\forall v \in V, d \in D$ , vd := f(d, v), we call V is right linear space of division ring D. All of module homomerphism from V to V is noted as  $\text{hom}_D(V, V)$ . Prove:  $\dim_D V = n$ , then  $\exists g : \text{hom}_D(V, V) \to M_n(D)$  is ring isomorphism.

SOUTHON.  $a_1, \dots, a_n \subset V$  is a set of V.  $\forall A \in \text{hom}_D(V, V)$ ,  $Aa_i = \sum_{k=1}^n d_{ki}a_k, d_{ki} \in D, \forall 1 \leq k, i \leq n$ . Let  $B = (d_{ij}) \in M_n(D)$ ,  $f : \text{hom}_D(V, V) \to M_n(D)$ ,  $A \mapsto B$ . Obviously f is well-defined and a bijection.

- 1. f preserves addition:  $A_1, A_2 \in \text{hom}_D(V, V), (A_1 + A_2)(a_i) = A_1 a_i + A_2 a_i = \sum_{k=1}^n d_{ki}^{(1)} a_k + \sum_{k=1}^n d_{ki}^{(2)} a_k = \sum_{k=1}^n (d_{ki}^{(1)} + d_{ki}^{(2)}) a_k$ . So  $f(A_1 + A_2) = f(A_1) + f(A_2)$ .
- 2. f preserves multiplication:  $A, B \in \text{hom}_D(V, V), (AB)(a_i) = A(\sum_{k=1}^n d_{ki}^{(2)} a_k) = \sum_{k=1}^n d_{ki}^{(2)} A a_k = \sum_{k=1}^n d_{ki}^{(2)} \sum_{j=1}^n d_{jk}^{(1)} a_j = \sum_{j=1}^n \sum_{k=1}^n d_{jk}^{(1)} d_{ki}^{(2)} a_j$ . So f(AB) = f(A)f(B).