Combination 3

王胤雅

201911010205

201911010205@mail.bnu.edu.cn

2023 年 10 月 19 日

PROBLEM I proof: $\sum_{i=0}^{\infty}(-1)^i\binom{n+i}{i}x^i = \frac{1}{(1+x)^{n+1}}$

SOLUTION.

1. $n=0$: obviously!
2. Assume n, we have the equation.
3. We go to $n+1$, so $\frac{1}{(1+x)^{n+2}} = \frac{1}{(1+x)^{n+1}}\frac{1}{1+x} = \sum_{i=0}^{\infty}(-1)^i\binom{n+i}{i}x^i\sum_{i=0}^{\infty}(-1)^i x^i = \sum_{i=0}^{\infty}(-1)^i\sum_{l=0}^{i}\binom{n+l}{l}x^i = \sum_{i=0}^{\infty}(-1)^i\binom{n+i+1}{i}x^i$

$\square$

PROBLEM II Power series expansion: $\frac{2+3x-6x^2}{1-2x}$.

SOLUTION. Let $t=2x$, so $\frac{2+3x-6x^2}{1-2x} = \frac{1}{2}\frac{4+3t-3t^2}{1-t} = \frac{2}{1-t}+\frac{3t}{2} = 2\sum_{i=0}^{\infty}t^i+\frac{3t}{2} = 2+\frac{7}{2}t+2\sum_{i=2}^{\infty}t^i = 2+7x+\sum_{i=2}^{\infty}2^{i+1}x^i$. $\square$