# Title of project

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## 1 The detailed setup of the problem

Ising model is used to successfully explain the phase transition between paramagnetism and ferromagnetism in Statistical Physics. The phenomenon that a magnet disappears its magnetism after being heated over a critical temperature and obtains its magnetism through cooling down the same critical temperature, is called the phase transition between paraamagnetism and ferromagnetism.

This model assumes that magnet is made by small needles regularly arranged. Each needle only has two different directions (upwards and downwards). Generally we consider a magnet as a square matrix with period boundary, call it $A$. The direction of each needle in magnet is regarded as an element in $A$, and $\pm 1$ represent two direction. Thus, we can use a matrix $A \in M_n(\mathbb{R})$ to represent the state of a magnet. The macroeconomic appearance of magnet (that is whether to appear magnetism) is depended on directions of all needles. We use Hamiltonian to measure a microstate of magnet, i.e. $A$:

$$H(A) = -\sum_{\sigma_i \in A} \sum_{i \sim j} J\sigma_i\sigma_j - \sum_{\sigma_i \in A} H\sigma_j \tag{1}$$

, where $J$ represents the spin-spin interaction, $H$ represents the external field, $\sigma_i$ is individual spins on eacch of lattice sites. Since each needle will interact with sourrounding needles (these needles on it's left, right, up and down). The first sum represents the interactions between each small needle; latter one means the external field trying to align in one direction.

How does temperature influence magnetism? It turns out that each needle can convert its direction randomly influenced by temperature $T$. Assume that each needle convert in different time. Whether to accept the convert of a needle is deicided by the change of Hamiltonian before and after the convert, , and temperature at the same time, let's call the change as $\Delta H$.

If $\Delta H > 0$, then accept the convert. If $\Delta H > 0$, then we accept the convert with probabilty of $\mathrm{e}^{\frac{-\Delta H}{k_B T}}$ where $k_B$ is a constant. After the convert of the needle, a convert will happen afterwards. After enough long time, the process may reach a dynamic balance. Thus, the magnet will perform may different property.

Our aim is to find out how does the temperature influence the property of magnet and determine the critical temperature. In this article, we focus on these two problem:

1. Take $J = 1, k_B = 1, H = 0$, $N$ is the size of $A$. Fixing $T$, choosing an initial microstate and initial needle randomly, let $a$ convert, and judge whether to accept it, and then repaet the process as before. We will get a chain of microstates. Caculate internal energy $u$ in the process:

$$u = \frac{U}{N^2} U = \mathbb{E}(H) \tag{2}$$

   and specific heat $c$:

$$c = \frac{C}{N^2}, C = \frac{k_B}{T^2} \mathbb{D}(H) \tag{3}$$

   where $\mathbb{D}$ is the variance. And plot $u - T, c - T$ graph.

2. Take $J = 1, k_B = 1$, $H \neq 0$. Fixing $T, H$, choosing an initial microstate and initial needle randomly, let $a$ convert, and judge whether to accept it, and then repaet the process as before. We will get a chain of microstates. Caculate magnetization $m$ in the process:

$$m = \frac{M}{N^2}, M = \mathbb{E}(\sum_{\sigma_i \in A} \sigma_i). \tag{4}$$

   And plot $m - (T, H)$ graph.

## 2 The procedure you take to do the computation

1. First of all, set the initial paraments: Set up the grid to imitate the magnet. We write a function to generate the magnet. And we set the process time.

2. Secondly, we randomly choose a point to change its direction of magnet, and caculate the changed energy by the getDeltaEnergy function. And then, we write a use the WhetherAccept function to judge whether to accept the direction change of this point.

3. Repeat the process above for times that we have set in the first step.

4. Last, we plot the object graphs and grams.

## 3 Analysis of the numerical results

1. In the first problem, we suppose that $J = 1, k_B = 1$, and $H = 0$. And we set the size of grid xxx and the program processing time xxx. Just like the picture xxx below, we find out the both specific temperature $C$- which is the variance of Hamiltonian energy - and inner energy $U$ vary with temperature in a particular patten. Here are the rules. As the temperature increases below the critical temperature, which is $T_c = \frac{2|J|}{k_B \ln(1+\sqrt{2})}$, the specific heat increases. And after temperature goes over $T_c$, as temperature increases, the specific heat decrease, which means the specific heat has a phrase transition when it reaches critical temperature. As for the inner energy, before the critical temperature, the increasing speed of inner energy slows down as temperature increases, and the increasing speed of inner energy speeds up as temperature increases, which showes the inner energy has a phrase transition when reaching critical temperature.

2. And in problem 2, we set the size of grid as xxx and the programing process time as xxx. We take $H, T$ xxx/ We observe that the magnetization $M$ change near the critical temperature, i. e. $T_c$.

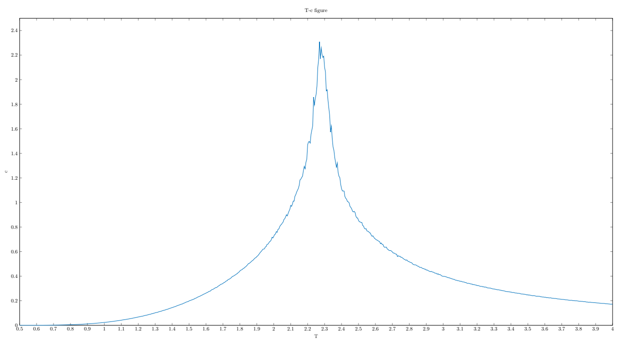


Figure 1: T-c

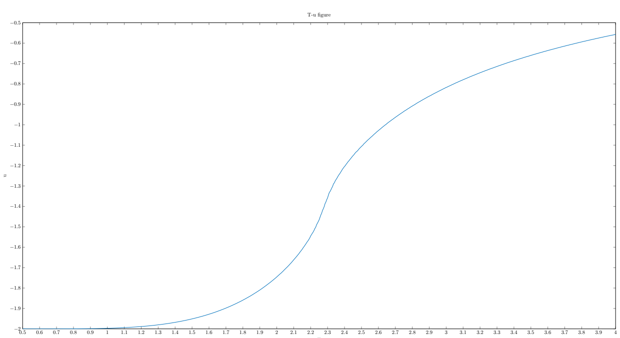


Figure 2: T-u

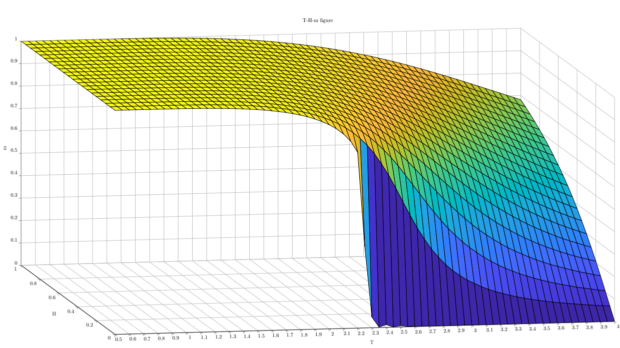


Figure 3: T-H-M

## 4 The issues you encounter and how you overcome

First of all, when we was running our program, our compute is not capable enough. But fortunately, we have a generous teacher, who borrowed us his compute to complete our program.

## 5 Possible discussion about the results and further thinking

Ising model can be applied to many other situations and fields. For example, opinion dynamics. Consider a group of people, each person can support one of two different candidates (like A and B). The candidate one support may change after exchanging ideas with sourroundings. What's more, the residents there may be influenced by advertisement of candidates. This is a typical reality situation for Ising model.

Besides, we can apply the Ising model to neuroscience. The activity of neuros in the brain can be modelled statistically. Each neuron at anytime is either active or inactive. Hopfield suggested in 1982 that a dynamic Ising model would provide a first approximate to a neural network which is capable of learning. And following the general approach of a lot scientists, we can model the process of neurons by the following model. Given a collection of neurons, we assume the energy as:

$$E = -1/2 \sum_{ij} J_{ij} SiSj - \sum_{i} h_i S_i \tag{5}$$

In this process, we can apply Ising model to simulate it.