Iterative 4

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PROBLEM I Using the notation of Section 7.1.2, prove that

$$q_{j+k}(t) = t^k p_j(t)$$

is orthogonal to the polynomials

$$p_1, p_2, \ldots, p_{j-k},$$

assuming that $k \le j$. Show that if q_{j+k} is orthogonalized against $p_1, p_2, \ldots, p_{j-k}$, the result would be orthogonal to all polynomials of degree $< j+k$. Derive a general **Look-Ahead non-Hermitian Lanczos procedure** based on this observation. PROBLEM II Consider the matrices

$$V_m = [v_1, v_2, \ldots, v_m], \qquad W_m = [w_1, w_2, \ldots, w_m],$$

obtained from the Lanczos biorthogonalization algorithm.

1. What are the matrix representations of the (oblique) projector onto $\mathcal{K}_m(A, v_1)$ orthogonal to the subspace $\mathcal{K}_m(A^T, w_1)$, and the projector onto $\mathcal{K}_m(A^T, w_1)$ orthogonal to the subspace $\mathcal{K}_m(A, v_1)$?

2. Express a general condition for the existence of an oblique projector onto a subspace K, orthogonal to another subspace L.

3. How can this condition be interpreted using the Lanczos vectors and the Lanczos algorithm?

PROBLEM III Show a three-term recurrence satisfied by the residual vectors r_j of the BCG algorithm. Include the first two iterates to start the recurrence.

Similarly, establish a three-term recurrence for the conjugate direction vectors p_j in BCG.