PROBLEM I Assume $(B_t : t \geq 0)$ is Brownian motion, prove that for $r > 0$, we have $(B_{t+r} - B_r : t \geq 0)$ is Brownian motion, too.

SOLUTION . $\square$

PROBLEM II Assume $(B_t : t \geq 0)$ is standrad Brownian motion start at 0. Prove that $\forall c > 0, (cB_{\frac{t}{c^2}} : t \geq 0)$ is standrad Brownian motion start at 0, too.

PROBLEM III Assume $(X_t : t \geq 0)$ and $(Y_t : t \geq 0)$ are two independent standrad Brownian motion, $a, b \in \mathbb{R}$ and $\sqrt{a^2 + b^2} > 0$. Prove that $(aX_t + bY_t : t \geq 0)$ is a Brownian motion with parameter $c = \sqrt{a^2 + b^2}$.

PROBLEM IV Assume $(B_t : t \geq 0)$ is standrad Brownian motion start at 0. Let $X_0 = 0$ and $X_t := tB_{\frac{1}{t}}$. Given

$$\limsup_{t \to \infty} \frac{B_t}{\sqrt{2t \log \log t}} = 1$$

Prove that $(X_t : t \geq 0)$ is standrad Brownian motion start at 0.

$$\left(\frac{111}{222} \middle| 222 \right)$$