PROBLEM I Assume $(N_t : t \geq 0)$ is a Possion process with parameter α. Let $P(t) := \mathbb{P}(2 \nmid N_t), Q(t) := \mathbb{P}(2 \mid N_t)$. Prove that $P(t) = \mathrm{e}^{-\alpha t}\sinh(\alpha t), Q(t) = \mathrm{e}^{-\alpha t}\cosh(\alpha t)$.

SOLUTION . $\square$

PROBLEM II Assume $(N_t : t \geq 0)$ is a Possion process with parameter α. Prove that $\lim_{t\to\infty} \frac{N_t}{t} = \alpha, a.s.$.

PROBLEM III Assume $(N_t : t \geq 0)$ is a Possion process with parameter $\alpha > 0$. Prove that $\frac{N_t - \alpha t}{\sqrt{\alpha t}} \xrightarrow{d} N(0, 1)$.

PROBLEM IV Assume $(X_t : t \geq 0), (Y_t : t \geq 0)$ are two independent Possion processes with parameter α, β respectively. Prove that $(X_t + Y_t : t \geq 0)$ is Possion process with parameter $\alpha + \beta$.

PROBLEM V Assume $(\xi_n : n \in \mathbb{N}^+)$ is a sequence of i.i.d. random variable ranging in $\mathbb{Z}^d$. Let $X_n = X_0 + \sum_{k=1}^{n} \xi_k$, and $X_0 \perp (\xi_n : n \in \mathbb{N}^+)$ ranging in $\mathbb{Z}^d$, too. Assume $(N_t : t \geq 0)$ is a Possion process with parameter $\alpha > 0$. Discuss $\frac{X_{N_t}}{t}$ when $t \to \infty$.