ℙℝOB𝕃EM I Use the method in the contexts of this section to judge whether these equations below have solutions.

1. $x^2 \equiv 429 \pmod{563}$

2. $x^2 \equiv 680 \pmod{769}$

3. $x^2 \equiv 503 \pmod{1013}$

where $503, 563, 796, 1013$ are prime.   ℙℝOB𝕃EM II Find out the expression of the prime with the quadratic residue -2; Find out the expression of the prime with the non quadratic residue -2; ℙℝOB𝕃EM III Assume $n \in \mathbb{N}_+$, $4n + 3, 8n + 7$ are prime, prove:

$$2^{4n+3} \equiv 1 \pmod{8n + 7}$$

Then prove $23 \mid (2^{11} - 1), 47 \mid (2^{23} - 1), 503 \mid (2^{251} - 1)$.   ℙℝOB𝕃EM IV Find out the expression of the prime with the quadratic residue ± 3; which prime has the non quadratic residue ± 3?   ℙℝOB𝕃EM V Find out the expression of the prime with the minimum non quadratic residue 3.