Iterative 1

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Lemma 1.

PROBLEM I Let $A_1 = B^{-1}C$ and $A_2 = CB$ where C is a Hermitian matrix and B is a Hermitian Positive Definite matrix. Are A_1 and A_2 Hermitian in general? Show that A_1 and A_2 are Hermitian (self-adjoint) with respect to the B-inner product.   PROBLEM II Let a matrix A be such that $A^H = p(A)$, where p is a polynomial. Show that A is normal. Given a diagonal complex matrix D, show that there exists a polynomial of degree $< n$ such that $\overline{D} = p(\overline{D})$. Use this to show that a normal matrix satisfies $A^H = p(A)$ for a certain polynomial of p of degree $< n$. As an application, use this result to provide an alternative proof of 1.   PROBLEM III Let A be an M-matrix and u, v are two nonnegative vectors such that $V^T A^{-1} u < 1$. Show that $A - uv^T$ is an M-matrix.