PROBLEM I Let $X = \{X(n) : n \geq 0\}$ is Markorv chain defined on probability space $(\Omega, \mathscr{F}, \mathbb{P})$, with state space E and transition probability matrix $P = (p(i,j) : i, j \in E)$. Let $a, b \in E$, $\tau_0 = 0$, $\sigma_k = \inf\{n \geq \tau_{k-1} : X(n) = b\}, \tau_k = \inf\{n \geq \sigma_{k-1} : X(n) = a\}$. Prove: $\tau_n, \sigma_n, n \geq 1$ are all stopping time on $(\mathscr{F}_n : n \geq 0)$. PROBLEM II Let $(X(n) : n \geq 0)$ is a one-dimension simple random walk starting at 1. Let $e(n) = \{X(n \wedge \tau_1) : n \geq 0\}$, where $\tau_1 = \inf\{n \geq 0 : X(n) = 0\}$. Find the distribution of $\sup_{n \geq 0} e(n)$. PROBLEM III Prove:

1. When $0 < p \leq q$, the reflecting random walk with transition matrix Q_+^a is recurrent.
2. When $0 < q \leq p$, the reflecting random walk with transition matrix Q_-^a is recurrent.

PROBLEM IV Prove: collorary 4.4.3.