PROBLEM I Assume $A = \{a \in P \mid a \mid m\} = \{q_i \mid i = 1, \cdots, s\}$, where $P \subset \mathbb{N}$, $\forall p \in P$, p is prime, $s = |A|$. Prove: g is the primative root mod $m \iff g$ is q_i-tic non-residue mod m, $\forall i = 1, \cdots, s$.

PROBLEM II Prove:

1. 10 is the primative root mod $17, 257$.
2. The length of repetend of $\frac{1}{17}$ is 16, the length of repetend of $\frac{1}{257}$ is 256.

PROBLEM III Apply index table to solve the equation

$$x^{15} \equiv 14 \pmod{41}.$$

PROBLEM IV Assume $m > 2$ has primative root, prove $\forall g$, g is the primative root mod m, the index of -1 is $\frac{1}{2}\phi(m)$. PROBLEM V Assume g_1, g_2 are two primative root mod m, prove:

1. $\mathrm{ind}_{g_1} g \cdot \mathrm{ind}_g g_1 \equiv 1 \pmod{\phi(m)}$;
2. $\mathrm{ind}_g a \equiv \mathrm{ind}_g g_1 \cdot \mathrm{ind}_{g_1} a \pmod{\phi(m)}$