Laboratory 2

Due: Wednesday, October 1

15 pts (1) As mentioned in class, it is possible to develop a cubically convergent fixed-point function often referred to as the "Cubic Newton's Method". Your task is develop the functional form of this cubicly convergent fixed-point iteration function $g(p_n)$ to solve the problem $f(p) = 0$ ("root finding") by writing

$$g(x) \;=\; x - \phi(x)f(x) - \psi(x)f^2(x)$$

and determining both $\phi(x)$ and $\psi(x)$. Specify the asymptotic order of convergence α, and write the asymptotic error constant λ. Write all expressions in terms of $f(p)$ and its derivatives and *simplify* your answers. Note: λ is hard. Note that $f^2(x)$ is the square of the function $f(x)$.

20 pts (2) Using the Secant method, Newton's method, Modified Newton's method and the Cubic Newton's Method you derived in the previous problem, find the root that is located between $x = 1$ and $x = 2$ for the functions listed below.
In each case iterate until you can demonstrate the convergence rate of each method (or you reach the limit of accuracy on your computer.) Report the root found and the number of iterations needed for each method. Include plots of error versus iteration number for each case.

(a) $f(x) = (x + \cos x)\, e^{-x^2} + x\cos x$

(b) $f(x) = \left[(x + \cos x)\, e^{-x^2} + x\cos x\right]^2$

(c) $f(x) = \left[(x + \cos x)\, e^{-x^2} + x\cos x\right]^3$

Comment on the observed convergence rates in these cases. Do your results agree with the analysis we did in class?

15 pts (3) The figure below shows a four bar linkage where θ is the input angle and the output angles θ_2 and θ_3 are to be determined. The relationships among the linkages can be expressed in terms of the two nonlinear equations

$$f_1(\theta_2, \theta_3) = r_2 cos\theta_2 + r_3 cos\theta_3 + r_4 cos\theta_4 - r_1 = 0$$

$$f_2(\theta_2, \theta_3) = r_2 sin\theta_2 + r_3 sin\theta_3 + r_4 sin\theta_4 = 0$$

Assume $r_1 = 43$, $r_2 = 23$, $r_3 = 33$, $r_4 = 9$, $\theta = 65°$, and solve for θ_2 and θ_3 using Newton's method for systems of nonlinear equations. Demonstrate the convergence rate of your solution. Report the number of iterations required to reach this level of convergence. Initial guesses for θ_2 and θ_3 could be estimated from creating a diagram with the appropriate dimensions and input angle.Be careful with degrees and radians!

Note: either invert the 2×2 Jacobian for this problem by hand or solve the linear system of equations using left division or a similar built-in command.

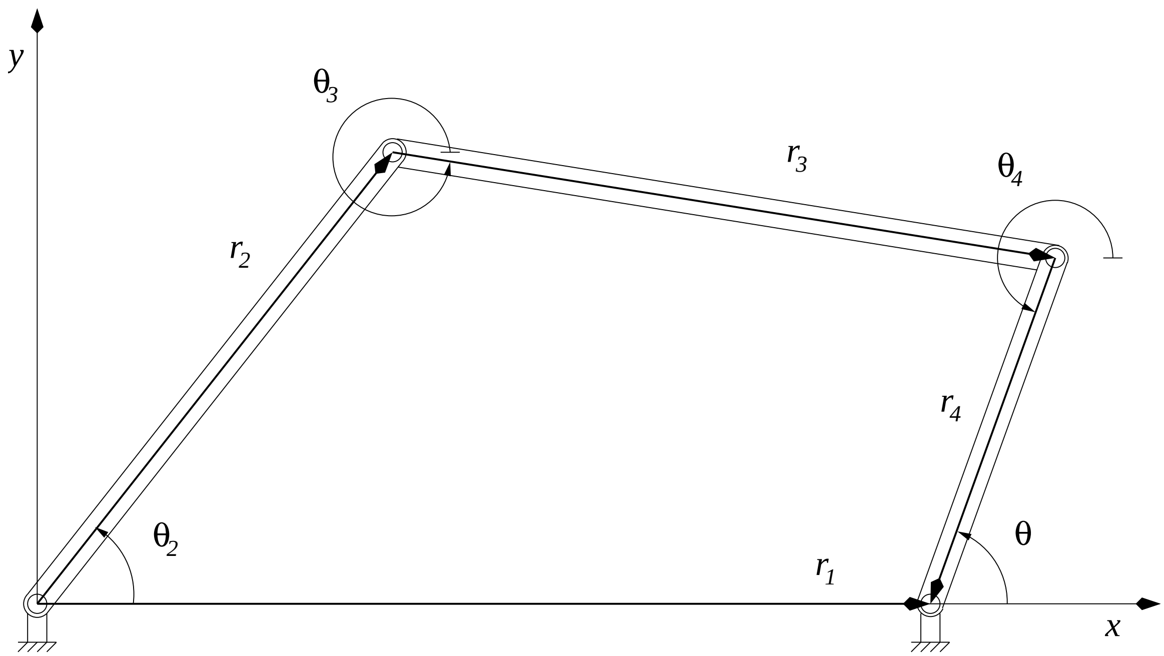

Figure 1: Four-bar linkage.