Legendre Polynomials

Generating Function: Rodrigue's Formula

$$\mathcal{L}_n(x) \;=\; \frac{1}{2^n n!}\,\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

	Root(s)	Weight(s) for Gaussian Quadrature
$\mathcal{L}_0(x) \;=\; 1$		
$\mathcal{L}_1(x) \;=\; x$	$x=0$	2
$\mathcal{L}_2(x) \;=\; \frac{1}{2}(3x^2-1)$	$x=\pm\sqrt{\frac{1}{3}}$	1
$\mathcal{L}_3(x) \;=\; \frac{1}{2}(5x^3-3x)$	$x=0,\pm\sqrt{\frac{3}{5}}$	$\frac{8}{9},\frac{5}{9}$
$\mathcal{L}_4(x) \;=\; \frac{1}{8}(35x^4-30x^2+3)$	$x=\pm\frac{1}{35}\sqrt{525\pm 70\sqrt{30}}$	$\frac{1}{36}\left(18\mp\sqrt{30}\right)$
$\mathcal{L}_5(x) \;=\; \frac{1}{8}(63x^5-70x^3+15x)$	$x=0,\pm\frac{1}{21}\sqrt{245\mp 14\sqrt{70}}$	$\frac{128}{225},\frac{1}{900}\left(322\pm 13\sqrt{70}\right)$
$\vdots$	$\vdots$	$\vdots$

Orthogonality:

$$\int_{-1}^{1}\mathcal{L}_n(x)\mathcal{L}_m(x)dx \;=\; \delta_{nm}\,\frac{2}{2n+1}$$

Legendre Polynomials $\mathcal{L}_n(x)$ for n = 0, 1, 2, 3, 4, 5