Laboratory 3

Due: at start of class, Wednesday, Oct. 8

15 pts (1) One of your classmates astutely asked whether a higher order polynomial approximation to a function is always better than a lower order polynomial. Answer this question by approximating the function

$$f(x) = \sin(6x)\cos(\sqrt{5}x) - x^2 e^{-x/5}$$

over the interval $x \in [-2, 2]$ using interpolating polynomials of order $N \geq 5$. Use two different types of interpolating polynomials, one that passes through $N+1$ points that are **uniformly spaced** over the interval and one that uses the roots of the $(N+1)^{th}$ order Chebyshev polynomial, i.e., the **Chebyshev optimal points** over the same interval. Plot both the interpolating polynomials and their errors over the entire interval for values of $N \geq 5$. Use Neville's Method to compute each value of the interpolating polynomials. Finally, discuss the merits of each approximation in terms of your results. For example, is the use of Chebyshev *optimal* points better? What is the behavior of both types of interpolating polynomials as N increases? Answer your classmates question.

15 pts (2) It is possible to represent a function that is not, in general, single-valued by introducing a parameter (say s) that represents the distance along a curve. A two-dimensional shape can then be represented with two separate functions of this parameter, say $x(s)$ and $y(s)$. If one has a set of discrete points along the two-dimensional curve, it is possible to determine the interpolating polynomials, $x(s_i)$ and $y(s_i)$, that pass through this set of points. Use this so-called *parametric interpolation* technique on the table of data points provided.

Be sure to plot $x(s)$, $y(s)$ as well as your interpolated shape. Note that you need to experiment a bit to choose an appropriate value of Δs with which to sample your functions. Report the value of Δs you use as well as the maximum and minimum values of $x(s)$ and $y(s)$.

20 pts (3) Repeat problem (2) with a natural cubic spline. In addition to showing similar plots and reporting the corresponding values, also list all 4 coefficients for each of the cubics which comprise the interpolants for both $x(s)$ and $y(s)$. How does your letter compare with that produced in problem (2)? Explain any differences.