Laboratory 5

Due: 5 pm, Thursday, Oct. 30

20 pts (1) Apply Romberg integration to the following integrals until $R_{n-1,n-1}$ and $R_{n,n}$ agree to within 10^{-9}. Report the value of n and the number of function evaluations. Also, compare the result to that obtained from the trapezoidal rule for the same number n (note that you already calculate this value to get $R_{n,n}$) **and** the number n and number of function evaluations you would need to achieve the same level of accuracy using just the composite trapezoidal rule (i.e., $R_{n,0}$). Repeat using the limits of integration from 1 to 2. Explain your findings.

(a) $\int_0^1 x^2 e^{-x} dx$ (b) $\int_0^1 x^{1/3} dx$

10 pts (2) Approximate the integrals in problem 1a and 1b using Gaussian Quadrature with $n = 1$, 2, 3, 4, ... Report the number of function evaluations and compare the results with those obtained using Romberg integration in problem 1.

20 pts (3) Solve the initial-value problem

$$\frac{dy}{dt} = 2y\left(\frac{1}{t} - t\right)$$

on the interval $1 \leq t \leq 2$ subject to initial conditions $y(1) = e^{-1}$ using Euler's method with step-sizes $\Delta t = 2^{-n}$ for $n = 3, 4, 5, 6 \ldots$ and compare (graphically, i.e., plot) with the exact solution $y(t) = t^2 e^{-t^2}$. For each step-size compare your computed solution with its analytic counterpart. Repeat the same series of calculations using the Midpoint Rule, Modified Euler's method, a 2-step A-B/A-M Predictor Corrector Scheme (with a single correction) and the Runge-Kutta 4^{th} order method. Compare errors (plot error as a function of t). Plot the absolute value of the error at $y(2)$ for each method versus $1/\Delta t$ on a log-log scale. Do the trends that you observe agree with our knowledge of the order of accuracy (as Δt decreases) of these methods? Explain your findings.