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HW 3

The work in this exercise is mine alone without un-cited help. No AI was used to answer these question .

1. Pseudocode:

Function AVL\_in\_range(a, b, node=T.root):

If node is NULL:

Return False

If a <= node.specific weight <= b:

Return True

Else if node. specific weight < a:

Return AVL\_in\_range(a, b, node.right)

Else if node. specific weight > b

Return AVL\_in\_range(a, b, node.right):

This algorithm is very similar to searching a binary search tree. If the node is Null, we have traversed the whole tree and should return false. If the node is not null and its specific weight is in the range [*a*,*b]* (inclusive) we return true. If the nodes specific weight is less than *a* we continue searching in the right subtree that contains values greater than *a*, as this follows from the BST property. Conversely, if the nodes specific weight is larger than *b* we search for smaller values in the left subtree. The runtime of this algorithm is O(height of tree), which in this case is O(logn), since each recursive call travels one level of the tree and performs constant time operations for each recursive call.

1. We can use optimal subproblems to define the bellman equation for this question. The minimum cost for week W, is the sum of the cost for all weeks up to W and the minimum cost of week W. We can use a dynamic programing approach to build up an array DP that stores the running tally of the minimum cost for each week. In the first 3 weeks the only option is to choose company A. That is by week three the minimum cost must be:

W(3) = S[0] \* r + S[1] \* r + S[2] \* r

Afterwards the cost for the week is either company A’s cost plus the previous weeks cost or Company B’s cost plus the cost for four weeks. Since we are building up subproblems from the bottom up we can “go back in time” to choose option B if the tally after 4 weeks is less than the cost of choosing B 4 weeks ago. Writing out the pseudocode:

function min\_cost(S, r, c):

n = S.length

DP = array of length n

DP[0] = S[0] \*r

for i=1 to n:

if i < 3:

DP[i] = DP[i-1] + S[i] \* r

else:

cA = DP[i-1] + S[i] \* r

cB = DP[i-4] + 4 \* c

DP[i] = min(cA, cB)

Return DP.last

From the pseudocode we can define the bellman equation for a given week i:

DP[i] = {

if i = 0

if 0 < i < 3

if i >= 3

}

1. (im using S instead of y for clarity)

The string segmentation with the maximum quality can approached like the rod cutting example in the textbook. We can use dynamic programing to solve the subproblem of finding the highest quality substrings of the input, noting that the highest quality substring of a string S of length n, can be viewed as finding the highest quality prefixes of S, and then removing the prefix and recursing. For example, for the string “meetateight”, the highest quality prefix is “meet”. We then recursively call our algorithm on “ateight” to get the highest prefix “at”. Instead of using recursion we can build up the prefix quality using bottom-up approach. For each position in S, denoted i, we find the best quality prefix from j to i and add that to the previous solved subproblem of finding the highest quality prefix of S from 0 to j. The Bellman equation for this process can be expressed as:

DP[i] = {

0 if i = 0

if i > 0

The DP[j] term is the solved subproblem of the best quality prefix string from S[0:j], and the Quality(S[j:i]) term finds the highest quality prefix over all prefixes of S up until the ith letter. The j term that maximizes quality is the location to cut the string for substring S[0:i]. The best cuts can be stored in an array and traversed backwards through starting at n to get the best prefixes of S. The prefixes can be joined to form the highest quality segmentation string of S.

The pseudocode for this algorithm would be:

Define segment\_str(S)

N = S.length

DP , cuts = array of length n

For i=1 to n+1:

max\_qual = -INF

for j=0 to i:

qual = DP[j] + quality(S[j:i])

if qual > max\_qual:

max\_qual = qual

cuts[i] = j

DP[i] = max\_qual

words = []

While n > 0:

c = cuts[n]

words.insert\_front(S[c:n])

n = c

return join(words, “ “)

The nested for-loop indicates that this program has runtime of ).

1. a) let K = 5 and have a set of containers of weights [5,4,3,2,1]. The greedy algorithm would use 4 trucks:

truck: containers

1: 5

2: 4

3: 3 + 2

4: 1

The optimal truck usage would be to stack container of weights 4 and 1 onto the same truck and use 3 trucks instead.

b) Let the minimum number of trucks for n containers using a greedy algorithm be denoted G. Let M be the minimum number of trucks needed using an optimal algorithm. We can show that G is at most double M, noting that as we pack a truck in the greedy algorithm there is some container that exceeds the weight of the truck and contributes to another truck being needed. At worst, these leftover containers contribute to half the total trucks (one new truck for each overflow container). The optimal algorithm would pack the trucks in a way that minimizes or even removes the trucks needed for the overflow containers. Thus, G/2 <= M or G <= 2M. nvim