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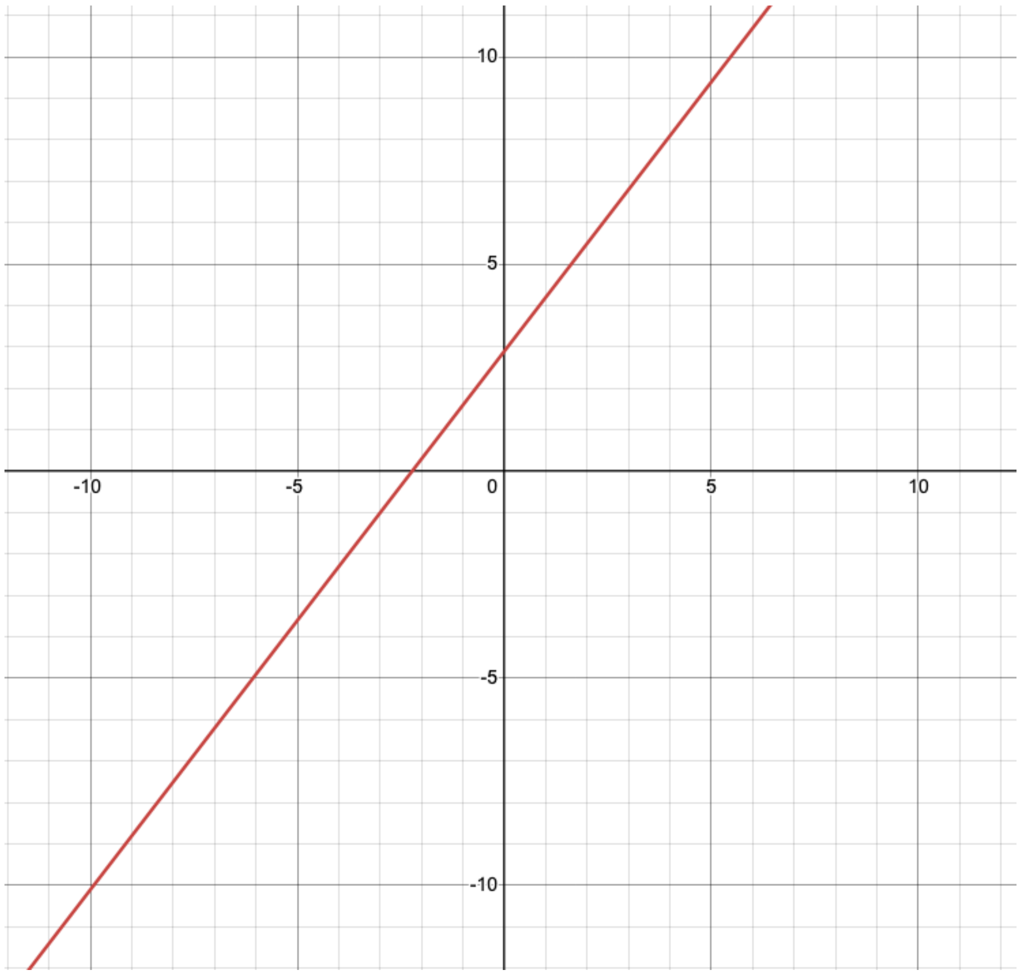
EN.605.645.82.SP26

module 3 – self check

Linear Regression:

1) We can draw the line $y = 1.3 + 2.9x$ as follows:



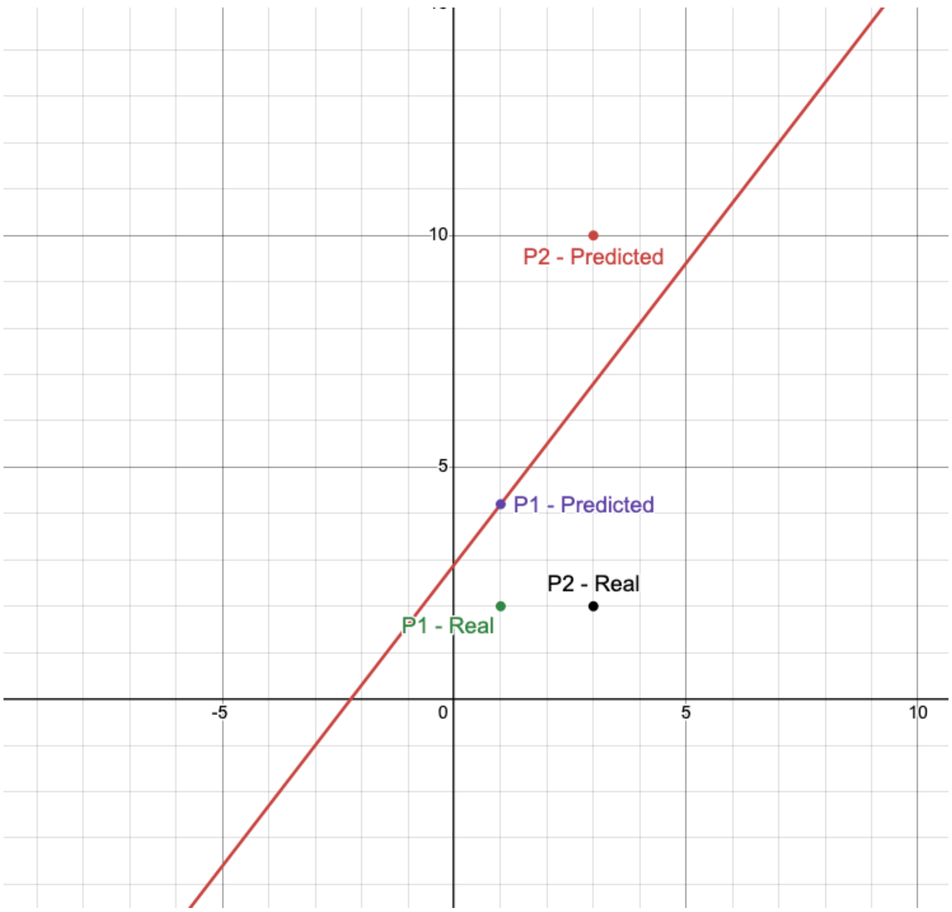
2) The data points in this problem are [1.0, 2.0] and [3.0, 1.0] which means the data points are expressed as:

$P1: xs = [1.0, 1.0], y = 2.0$

$P2: xs = [1.0, 3.0], y = 1.0$

The formula for y-hat is y-hat = Θx, where Θ is the vector [1.3, 2.9]. Therefore:
$P1: \hat{y}1 = 1.3 * 1.0 + 2.9 * 1.0 = 4.2$
$P2: \hat{y}1 = 1.3 * 1.0 + 2.9 * 3.0 = 10.0$

3)



4) Mean Squared Error is defined as: $J(\Theta) = \frac{1}{2n}\sum(\widehat{y_i} - y_i)^2$

$P1: error = (4.2 - 2)\text{^}2 = 2.2\text{^}2 = 4.84$

$P2: error = (10 - 1) = 9\text{^}2 = 81$

$MSE = 1/4 * (4.84 + 81) = 21.46$

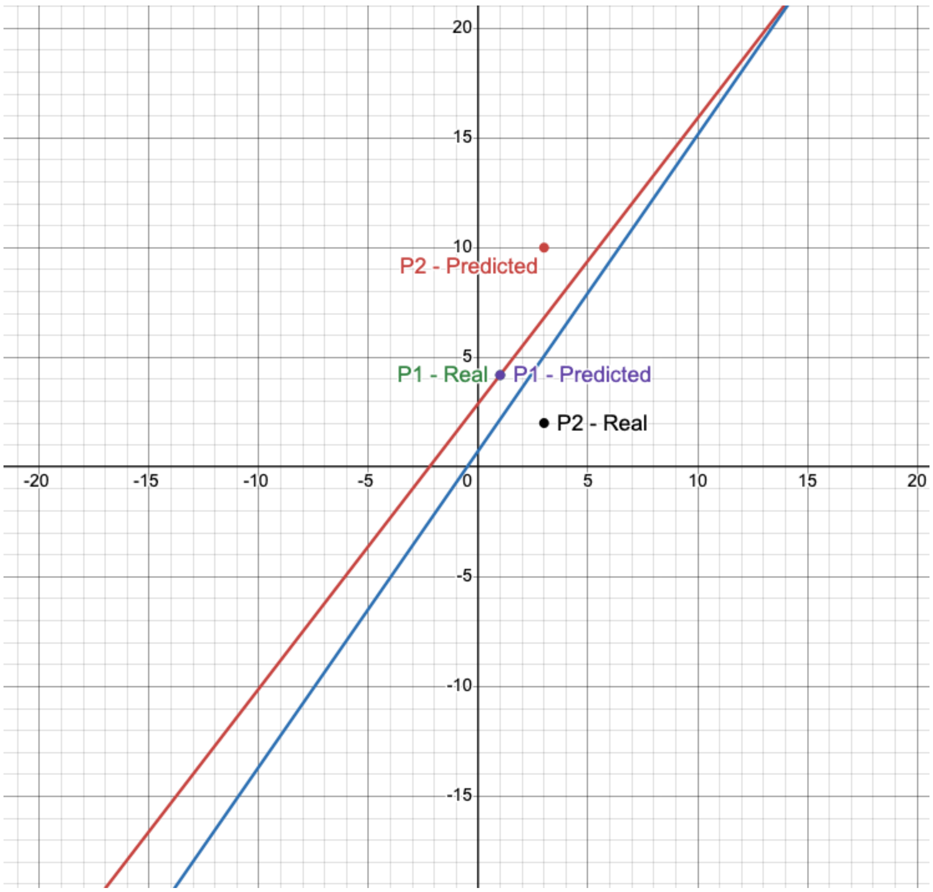
5) The update for thetas is: $new\ \Theta(\mathrm{j}) = \Theta(\mathrm{j}) - \alpha \left(\frac{dj}{d\Theta(j)}\right) = \Theta(\mathrm{j}) - \alpha * \frac{1}{n} *$

$\sum(\widehat{(y_i} - y_i)\, x_{ij})$

$$j1:\ 1.3 - 0.1 * \frac{1}{2} * (2.2 * 1) + (9 * 1) = 1.3 - 0.1 * 5.6 = 1.3 - 0.55 = 0.74$$

$$j2: 2.9 - 0.1 * \frac{1}{2} * (2.2 * 1) + (9 * 3) = 2.9 - 0.1 * 14.6 = 2.9 - 1.46 = 1.44$$

6) The blue line indicates the newline produces using the updates thetas of [0.74, 1.44]



7) The new estimates can be generated using y-hat = Θx (with Θ_0 $being\ the\ y-intercept\ and\ x_0\ being\ 1$

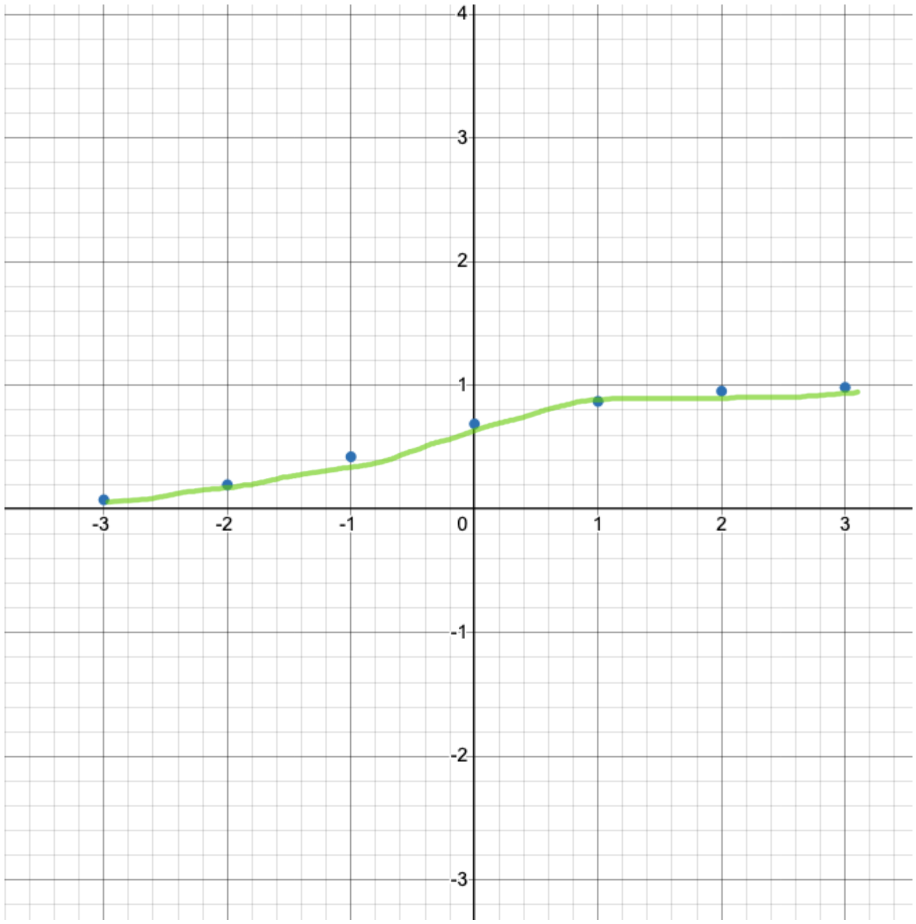
$$P1: \hat{y} = 0.74 * 1 + 1.44 * 1 = 2.18$$
$$P2: \hat{y} = 0.74 * 1 + 1.44 * 3 = 5.06$$

Logistic Regression:

$$\Theta = [0.8, 1.1], \mathrm{P1} = [1.0, 1.1], P2 = [1.0, 2.7], Y = [0,1]$$

1) We can use the formula $\hat{y} = 1/(1 + e\wedge - (\Theta x))$ to compute the estimates:

X	Z	Y
-3	$0.8 + 1.1 * -3 = -2.5$	$1/(1 + e^{2.5}) = 0.076$
-2	$0.8 + 1.1 * -2 = -1.4$	$1/1 + e^{1.4} = 0.198$
-1	$0.8 + 1.1 * -1 = -0.3$	$1/1 + e^{0.3} = 0.425$
0	$0.8 + 1.1 * 0 = 0.8$	$1/1 + e^{-0.8} = 0.689$
1	$0.8 + 1.1 * 1 = 1.9$	$1/1 + e^{-1.9} = 0.870$
2	$0.8 + 1.1 * 2 = 3$	$1/1 + e^{-3} = 0.953$
3	$0.8 + 1.1 * 3 = 4.1$	$1/1 + e^{-4.1} = 0.984$



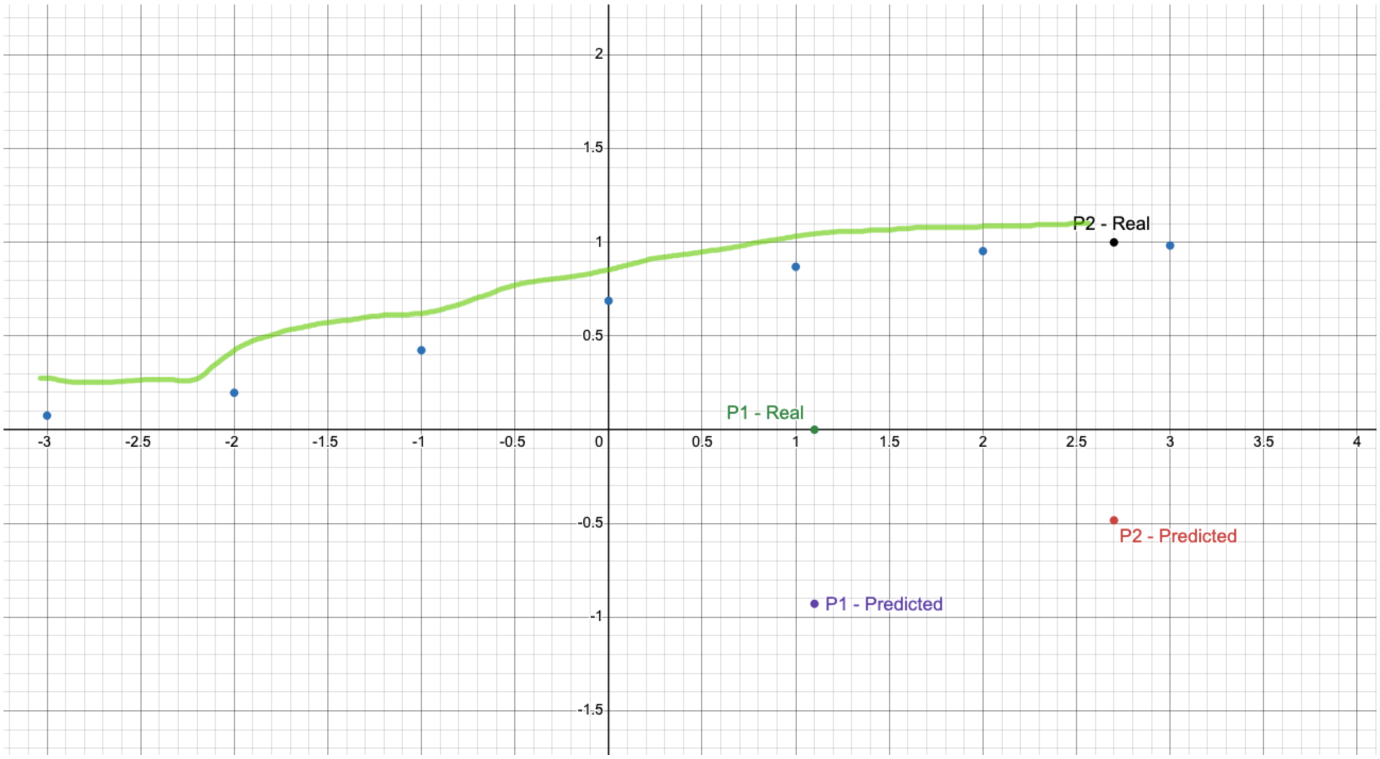
2) The error for these points:

$$P1: z = \Theta x = 0.8\,(1.0) + 1.1(1.1) = 2.01$$
$$\hat{y} = 1/(1 + e\wedge - z) = 1/(1 + e\wedge - 2.01) = 0.882$$
$$error = ylog(\hat{y}) + (1 - y)\log(1 - \hat{y})$$
$$= 0 * \log(0.882) + (1 - 0) * \log(1 - 0.882)$$
$$= \log(0.118) = -0.928$$

$$P2: z = \Theta x = 0.8\,(1.0) + 1.1(2.7) = 3.77$$
$$\hat{y} = 1/(1 + e\wedge - z) = 1/(1 + e\wedge - 3.77) = 0.977$$
$$error = ylog(\hat{y}) + (1 - y)log(1 - \hat{y})$$
$$= 1 * log(0.977) + (1 - 1) * log(1 - 0.977) = log(0.977)$$
$$= -0.010$$

$$Log\ loss = -\frac{1}{n}\sum ylog(\hat{y}) + (1 - y)log(1 - \hat{y}) = -\frac{1}{2} * (-0.977 + -0.010) =$$
$$0.469$$

3)



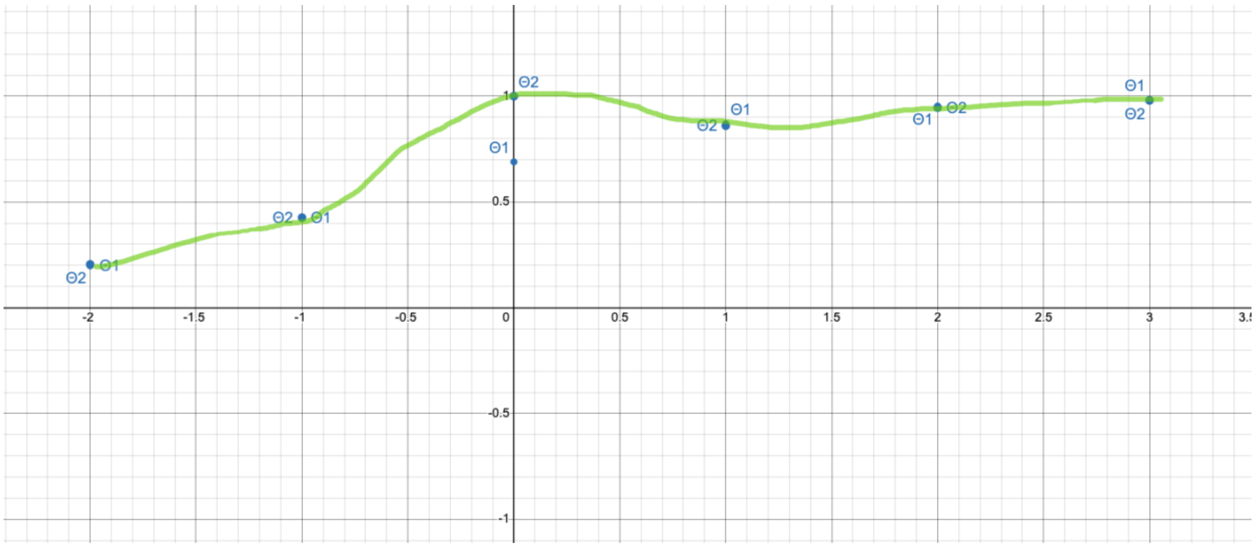
4) $new\ \Theta(j) = \Theta(j) - \alpha \left(\frac{dj}{d\Theta(j)}\right) = \Theta(j) - \alpha * \frac{1}{n} * \sum(\widehat{(y_\iota} - y_i)\ x_{ij})$

$$P1{:}\, new\ \Theta(j) = 0.8 - 0.1 * \frac{1}{2} * \ (0.882 - 0) * 1 + (0.977 - 1) * 1 = 0.757$$

$$P2{:}\, new\ \Theta(j) = 1.1 - 0.1 * \frac{1}{2} * \ (0.882 - 0) * 1.1 + (0.977 - 1) * 2.7 = 1.055$$

X	Z	Y
-3	$0.757 + 1.055 * -3 = -2.408$	$1/(1 + e^{2.408}) = 0.0826$
-2	$0.757 + 1.055 * -2 = -1.353$	$1/1 + e^{1.353} = 0.205$
-1	$0.757 + 1.055 * -1 = -0.298$	$1/1 + e^{0.298} = 0.426$
0	$0.757 + 1.055 * 0 = 0.757$	$1/1 + e^{-0.757} = 1$
1	$0.757 + 1.055 * 1 = 1.812$	$1/1 + e^{-1.812} = 0.860$
2	$0.757 + 1.055 * 2 = 2.867$	$1/1 + e^{-2.867} = 0.946$

3	$0.757 + 1.055 * 3 = 3.922$	$1/\ 1 + e^{-3.922} = 0.980$



5) Updated yhats

$P1: z = \Theta x = 0.757\,(1.0) + 1.055(1.1) = 1.9175$

$\hat{y} = 1/(1 + e^{\wedge} - z) = 1/(1 + e^{\wedge} - 1.9175\,) = 0.872$

$P2: z = \Theta x = 0.757\,(1.0) + 1.055(2.7) = 3.606$

$$\widehat{y} = \frac{1}{1 + e^{-z}} = \frac{1}{1 + e^{-3.606}} = 0.974$$