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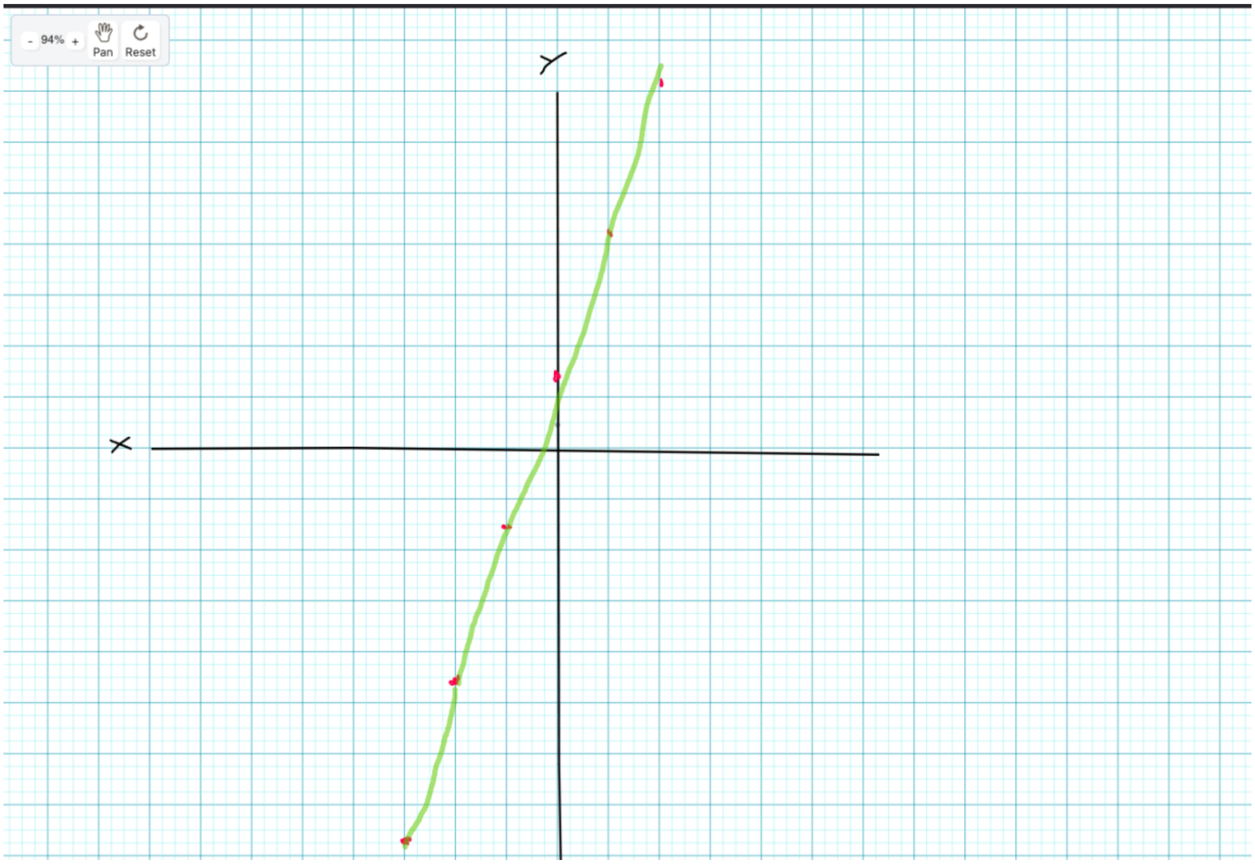
EN.605.645.82.SP26

module 3 – self check

Linear Regression:

1) We can draw the line $y = 1.3 + 2.9x$ as follows:



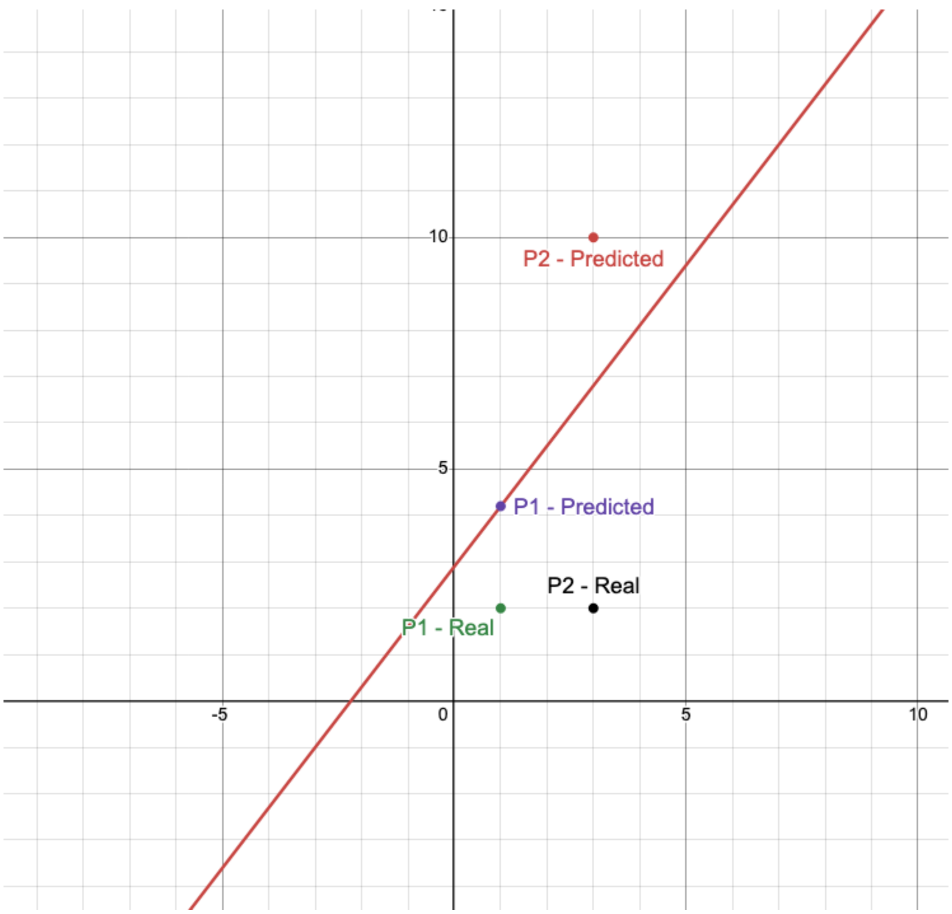

2) The data points in this problem are [1.0, 2.0] and [3.0, 1.0] which means the data points are expressed as:

$P1: xs = [1.0, 1.0], y = 2.0$

$P2: xs = [1.0, 3.0], y = 1.0$

The formula for y-hat is y-hat = Θx, where Θ is the vector [1.3, 2.9]. Therefore:
$P1: \hat{y}1 = 1.3 * 1.0 + 2.9 * 1.0 = 4.2$
$P2: \hat{y}1 = 1.3 * 1.0 + 2.9 * 3.0 = 10.0$

3) *switching to graphing calculator




4) Mean Squared Error is defined as: $J(\Theta) = \frac{1}{2n}\sum\left(\widehat{y_i} - y_i\right)^2$

   $P1: error\ =\ (\ 4.2\ -\ 2\ )\text{^}2\ =\ 2.2\text{^}2\ =\ 4.84$

   $P2:\ error\ =\ (10\ -\ 1)\ =\ 9\text{^}2\ =\ 81$

   $MSE\ =\ 1/4\ *\ (4.84\ +\ 81)\ \ =\ 21.46$

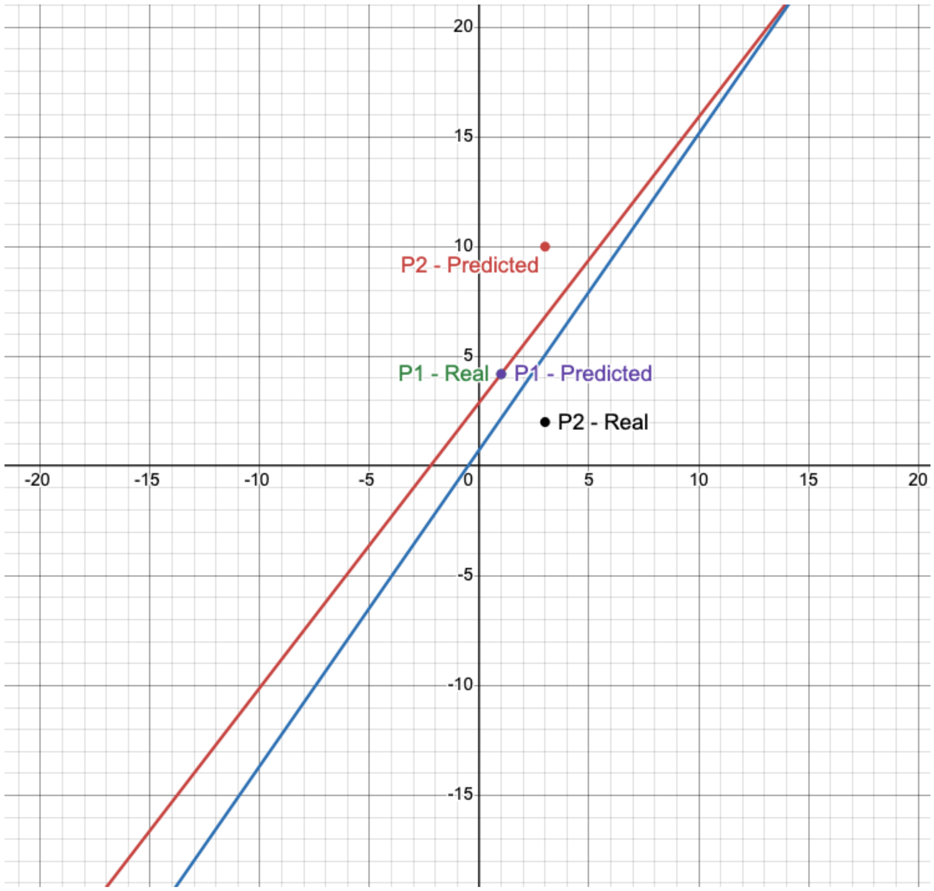
5) The update for thetas is: $new\ \Theta(\text{j}) =\ \Theta(\text{j}) -\ \alpha\ \left(\frac{dj}{d\Theta(j)}\right) =\ \Theta(\text{j}) -\ \alpha * \frac{1}{n} *$

   $\sum((\widehat{y_i} - y_i)\ x_{ij})$

   $$j1:\ 1.3 -\ 0.1\ * \frac{1}{2} * (2.2 * 1) + (9 * 1) = 1.3 - 0.1 * 5.6\ = 1.3 -\ 0.55 = 0.74$$

   $$j2: 2.9\ - 0.1 * \frac{1}{2} * (2.2\ *\ 1)\ +\ (9\ *\ 3)\ =\ 2.9\ -\ 0.1\ *\ 14.6\ =\ 2.9\ -\ 1.46\ =\ 1.44$$

6) The blue line indicates the newline produces using the updates thetas of [0.74, 1.44]




7) The new estimates can be generated using y-hat = $\Theta$x (with $\Theta_0$ *being the y − intercept and* $x_0$ *being* 1

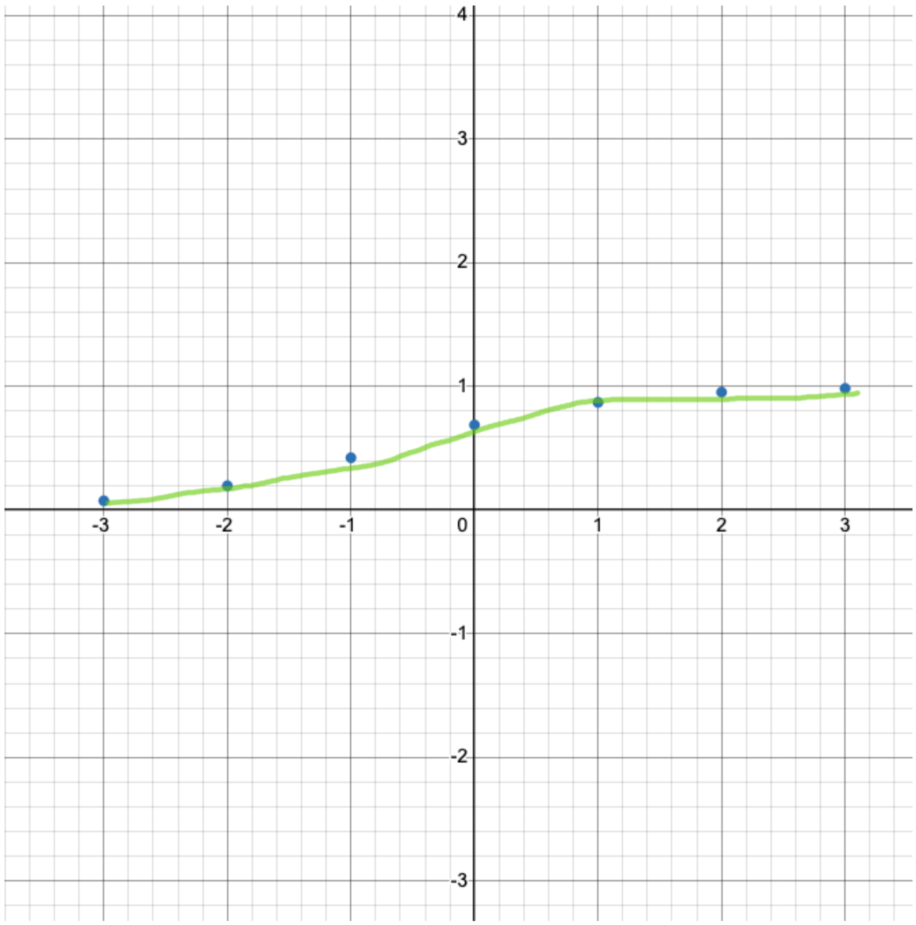
$$P1: \hat{y} = 0.74 * 1 + 1.44 * 1 = 2.18$$
$$P2: \hat{y} = 0.74 * 1 + 1.44 * 3 = 5.06$$

Logistic Regression:

$$\Theta = [0.8, 1.1], \mathrm{P1} = [1.0, 1.1], P2 = [1.0, 2.7], Y = [0,1]$$

1) We can use the formula $\hat{y} = 1/(1 + e\text{^} - (\Theta x))$ to compute the estimates:

| X | Z | Y |
|---|---|---|
| $\mathbf{-3}$ | $0.8 + 1.1 * -3 = -2.5$ | $1/(1 + e^{2.5}) = 0.076$ |
| $\mathbf{-2}$ | $0.8 + 1.1 * -2 = -1.4$ | $1/1 + e^{1.4} = 0.198$ |
| $\mathbf{-1}$ | $0.8 + 1.1 * -1 = -0.3$ | $1/1 + e^{0.3} = 0.425$ |
| $\mathbf{0}$ | $0.8 + 1.1 * 0 = 0.8$ | $1/1 + e^{-0.8} = 0.689$ |
| $\mathbf{1}$ | $0.8 + 1.1 * 1 = 1.9$ | $1/1 + e^{-1.9} = 0.870$ |
| $\mathbf{2}$ | $0.8 + 1.1 * 2 = 3$ | $1/1 + e^{-3} = 0.953$ |
| $\mathbf{3}$ | $0.8 + 1.1 * 3 = 4.1$ | $1/\ 1 + e^{-4.1} = 0.984$ |



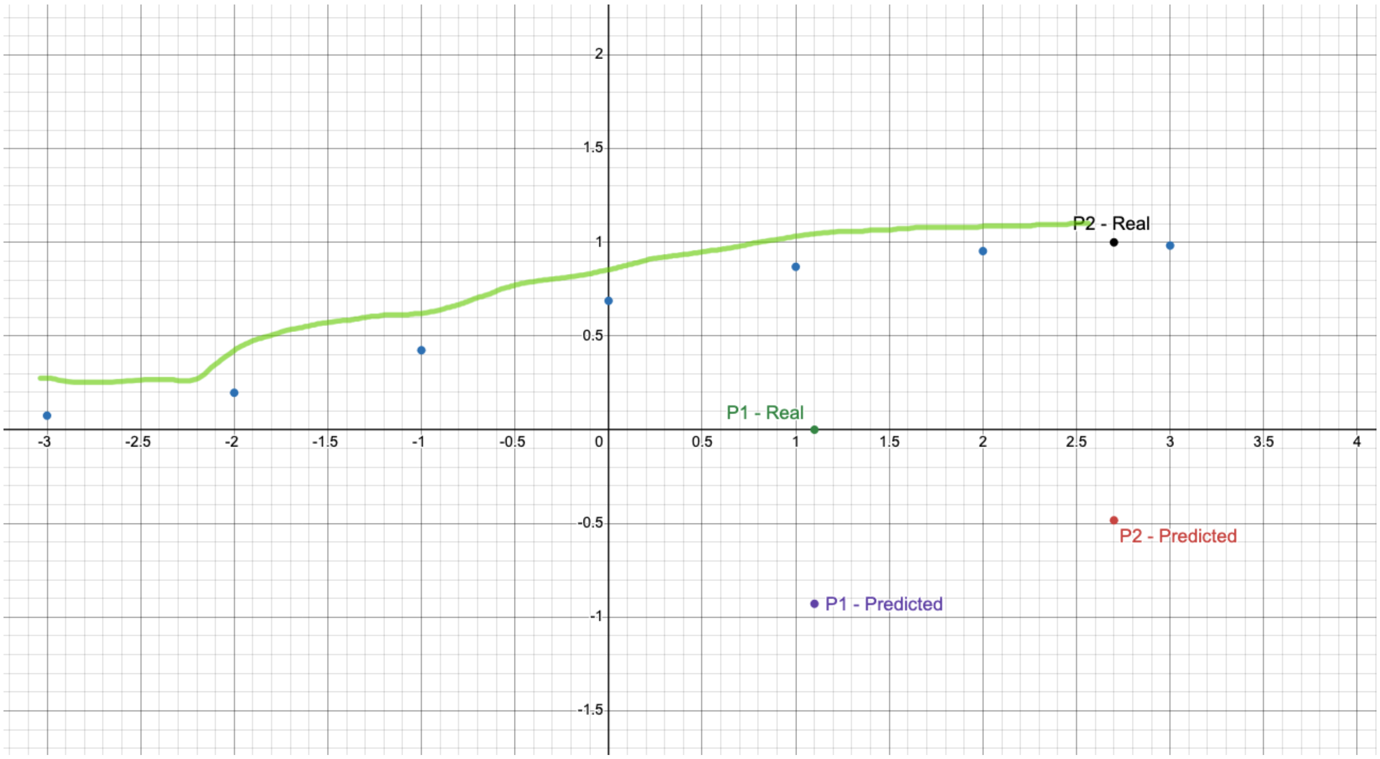

2) The error for these points:

$$P1: z = \Theta x = 0.8\,(1.0) + 1.1(1.1) = 2.01$$
$$\hat{y} = 1/(1 + e^{\wedge} - z) = 1/(1 + e^{\wedge} - 2.01) = 0.882$$
$$error = ylog(\hat{y}) + (1 - y)\log(1 - \hat{y})$$
$$= 0 * \log(0.882) + (1 - 0) * \log(1 - 0.882)$$
$$= \log(0.118) = -0.928$$

$$P2: z = \Theta x = 0.8\,(1.0) + 1.1(2.7) = 3.77$$
$$\hat{y} = 1/(1 + e^{\wedge} - z) = 1/(1 + e^{\wedge} - 3.77) = 0.977$$
$$error = ylog(\hat{y}) + (1 - y)log(1 - \hat{y})$$
$$= 1 * log(0.977) + (1 - 1) * log(1 - 0.977) = log(0.977)$$
$$= -0.010$$

$$Log\ loss = -\frac{1}{n}\sum ylog(\hat{y}) + (1 - y)log(1 - \hat{y}) = -\frac{1}{2} * (-0.977 + -0.010) =$$
$$0.469$$

3)



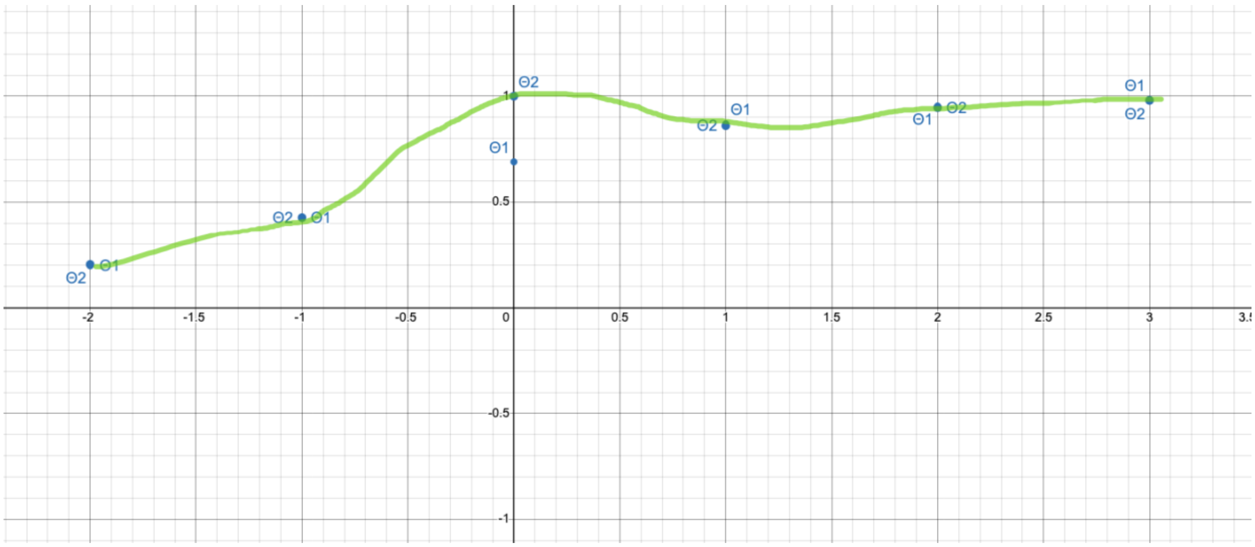

4) $new\ \Theta(\mathrm{j}) = \Theta(\mathrm{j}) - \alpha\left(\frac{dj}{d\Theta(j)}\right) = \Theta(\mathrm{j}) - \alpha * \frac{1}{n} * \sum(\widehat{(y_\iota} - y_i)\, x_{ij})$

$$P1\colon new\ \Theta(\mathrm{j}) = 0.8 - 0.1 * \frac{1}{2} * \ (0.882 - 0) * 1 + (0.977 - 1) * 1 = 0.757$$

$$P2\colon new\ \Theta(\mathrm{j}) = 1.1 - 0.1 * \frac{1}{2} * \ (0.882 - 0) * 1.1 + (0.977 - 1) * 2.7 = 1.055$$

| X | Z | Y |
|---|---|---|
| $-3$ | $0.757 + 1.055 * -3 = -2.408$ | $1/(1 + e^{2.408}) = 0.0826$ |
| $-2$ | $0.757 + 1.055 * -2 = -1.353$ | $1/1 + e^{1.353} = 0.205$ |
| $-1$ | $0.757 + 1.055 * -1 = -0.298$ | $1/1 + e^{0.298} = 0.426$ |
| $0$ | $0.757 + 1.055 * 0 = 0.757$ | $1/1 + e^{-0.757} = 1$ |
| $1$ | $0.757 + 1.055 * 1 = 1.812$ | $1/1 + e^{-1.812} = 0.860$ |
| $2$ | $0.757 + 1.055 * 2 = 2.867$ | $1/1 + e^{-2.867} = 0.946$ |

| | | |
|---|---|---|
| **3** | $0.757 + 1.055 * 3 = 3.922$ | $1/\,1 + e^{-3.922} = 0.980$ |




5) Updated yhats

$P1\colon z = \Theta x = 0.757\,(1.0) + 1.055(1.1) = 1.9175$

$\hat{y} = 1/(1 + e\hat{\ } - z) = 1/(1 + e\hat{\ } - 1.9175\,) = 0.872$

$P2\colon z = \Theta x = 0.757\,(1.0) + 1.055(2.7) = 3.606$

$$\widehat{y} = \frac{1}{1 + e^{-z}} = \frac{1}{1 + e^{-3.606}} = 0.974$$