Model-Free Reinforcement Learning

Learning Optimal Value Functions

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Today's Agenda

1. Model-Free Reinforcement Learning
2. Monte Carlo Methods
3. Temporal Difference Learning

Recap

Key Concepts

Model-Free Reinforcement Learning

So far we have seen:

- How to estimate an optimal value function and discover an optimal policy π^*
- We did this under the assumption that the environment $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, r \rangle$ was known

Plot Twist!

We will now consider situations where no complete knowledge is available:

- Set of possible states $\mathcal{S}$ ✓
- Set of possible actions $\mathcal{A}$ ✓
- Transition Function $\mathcal{P} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$ ✗
- Reward Function $\mathfrak{R} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ ✗

Model-Free Reinforcement Learning

When parts of of the MDP $\mathcal{M}$ are unknown we do not deal with Dynamic Programming methods anymore but with Reinforcement Learning algorithms!

- We need to overcome the lack of information of $\mathcal{M}$
- We can do this through experience
- Gathering experience means sampling states s, actions a and rewards r from the environment
- Recall the concept of trajectory τ $\langle s_t, a_t, r_t, s_{t+1} \rangle$ seen in Lecture 1!

Model-Free Reinforcement Learning

What does it mean in practice?

- We do not know the consequences of our actions
- We do not know the dynamics of the environment we are interacting with
- We are no longer computing value functions but rather learning them

The transition function $\mathcal{P}$ and the reward function $\Re$ are usually called the model of the environment

$\mathcal{P}$ and $\Re$ can be learned $\Rightarrow$ model-based Reinforcement Learning

Model-Free Reinforcement Learning

Why is model-free Reinforcement Learning so interesting?

- We learn without any prior knowledge of the environment
- Trajectories, and therefore experience, is sufficient for learning
- We "only" need a value function $Q(s, a)$, which is arguably easier to learn than the model

The effectiveness of model-free Reinforcement Learning algorithms highly depends from how much experience the agent is able to gather!

Monte Carlo (MC) Methods

Monte Carlo methods:

- Can be used for learning $V^\pi(s)$ as well as $Q^\pi(s, a)$
- Although in this lecture we only focus on learning $V^\pi(s)$
- The key idea is to learn through sampling returns

Assumption!

We assume that we are always dealing with episodic tasks i.e. episodes eventually terminate.

$$\langle (s_t, a_t, r_t, s_{t+1}) \rangle, t = 0, ..., T - 1$$

Monte Carlo (MC) Methods

Let us consider the notion of expected discounted return introduced in Lecture 1:

$$
\begin{aligned}
G_t &= r_t + \gamma r_{t+1}, \gamma^2 r_{t+2} + \dots \\
&= \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}.
\end{aligned}
$$

- The goal is to learn the state-value function of a given policy $V^\pi(s)$: Monte Carlo Prediction
- We do this with respect to the G_t that is obtained by following π

Monte Carlo (MC) Methods

Learning $V^\pi(s)$ involves the following steps:

- Before learning, each state has its own value $V(s_t)$
- We follow policy π until an episode terminates
- We compute the discounted return G_t that was obtained by π

Monte Carlo (MC) Update Rule

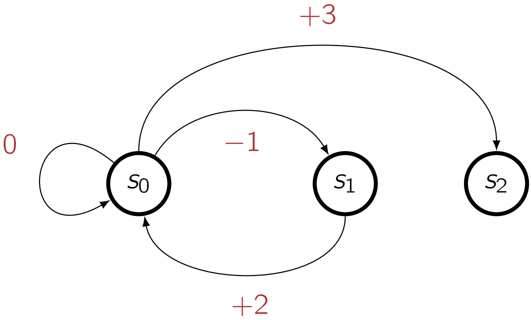
We change the value of each state $V(s_t)$ based on G_t

$$V(s_t) := V(s_t) + \alpha \left[G_t - V(s_t) \right].$$

where $\alpha \in [0, 1]$ is the learning rate parameter.

Monte Carlo (MC) Methods

Let us consider the following MDP:



- We have policy $\pi : s0 \rightarrow s1 \rightarrow s0 \rightarrow s2$
- π results in the sequence of rewards $-1, +2, +3$
- The starting value of each state is 0, $\gamma = 0.99$ and $\alpha = 0.5$

Monte Carlo (MC) Methods

We know that G_t for starting in $s0$ is

$$\begin{aligned} G_t &= \sum_{k=0}^{\infty} \gamma^k r_{t+k+1} \\ &= -1 + \gamma 2 + \gamma^2 3 \approx 3.92 \end{aligned}$$

Therefore

$$\begin{aligned} V(s_t) &:= V(s_t) + \alpha \left[G_t - V(s_t) \right] \\ V(0) &:= V(0) + \alpha \left[3.92 - V(0) \right] \approx 1.96 \end{aligned}$$

Monte Carlo (MC) Methods

More about Monte Carlo Methods:

- We have only considered the prediction case of learning $V^{\pi}(s)$
- Monte Carlo control algorithms learn an approximation of π^{*} through $Q^{\pi}(s, a)$
- They follow the ideas of Dynamic Programming seen in the previous lecture
- The key idea of sampling real returns remains!

Monte Carlo (MC) Methods

Pros & Cons of Monte Carlo Methods:

- Yield unbiased updates thanks to G_t ✓
- Scale well to function approximators ✓
- Learning can be very slow as one has to wait until the very end of an episode ✗
- There can be large variance in the value updates ✗

Temporal Difference (TD) Learning

"The simplest and most elegant idea idea of Reinforcement Learning ..."

Temporal Difference (TD) Learning

With TD-Learning methods we we do not have to wait until the end of an episode before updating a value estimate

- We only need to wait until the next step
- At $t + 1$ we immediately create a target for learning called the TD-target
- We do this by using the observed reward r_t and the estimate $V(s_{t+1})$

Temporal Difference (TD) Learning

Let us again consider the problem of estimating $V^{\pi}(s)$:

TD Prediction

We change the value of each state $V(s_t)$ with respect to $t+1$ only:

$$V(s_t) := V(s_t) + \alpha\big[r_t + \gamma V(s_{t+1}) - V(s_t)\big].$$

- It is clear that we only learn by *looking ahead* in the future one single step
- This is called TD(0) or *one-step TD*

Temporal Difference (TD) Learning

The key idea of TD-Learning it to learn through bootsrapping:

- We update the value of a state with respect to the value of its successor state only
- Ideally we would like to use $V^{\pi}(s_{t+1})$ for learning but it is unfortunately unknown

$$V(s_t) := V(s_t) + \alpha\big[r_t + \gamma V^{\pi}(s_{t+1}) - V(s_t)\big].$$

Therefore we replace with a guess instead:

$$V(s_t) := V(s_t) + \alpha\big[r_t + \gamma V(s_{t+1}) - V(s_t)\big].$$

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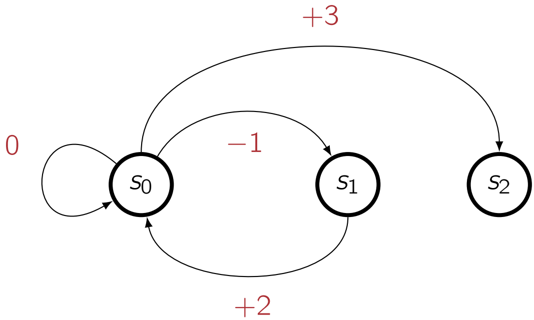
Temporal Difference (TD) Learning

Let us go back to the previous MDP:



- We again have policy $\pi : s0 \rightarrow s1 \rightarrow s0 \rightarrow s2$
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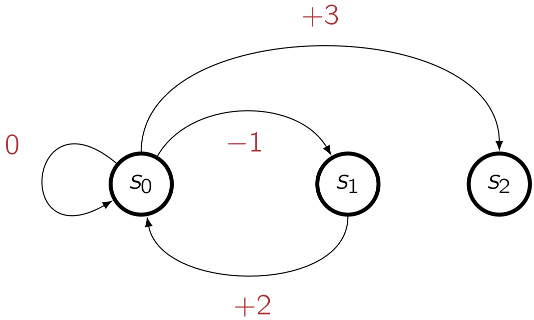
Temporal Difference (TD) Learning



Our first state transition is $s0 \rightarrow s1$

$$V(s_t) := V(s_t) + \alpha\big[r_t + \gamma V(s_{t+1}) - V(s_t)\big]$$
$$V(s0) := V(s0) + \alpha\big[r_t + \gamma V(s1) - V(s0)\big]$$
$$V(s0) := -0.5$$

Temporal Difference (TD) Learning

What is so cool about the TD Learning update rule?

$$V(s_t) := V(s_t) + \alpha\big[r_t + \gamma V(s_{t+1}) - V(s_t)\big]$$

We are learning by guessing!

- We want to learn $V(s_t)$ with respect to $V(s_{t+1})$
- But $V(s_{t+1})$ is as unknown as $V(s_t)$
- We use the information provided by the environment r_t immediately to construct the TD-error δ_t

$$V(s_t) := \underbrace{V(s_t) + \alpha\big[r_t + \gamma V(s_{t+1})}_{\delta_t} - V(s_t)\big]$$

Temporal Difference (TD) Learning

Why are the TD-errors δ_t so important?

- They allow us to start learning immediately
- They separate RL algorithms into two families of techniques: *off-policy* and *on-policy* techniques
- Each family comes with its own convergence properties

To know more about these families let us consider the problem of learning the state-action value function $Q^{\pi}(s, a)$

$$Q^{\pi}(s, a) = \mathbb{E}\left[\sum_{k=0}^{\infty} \gamma^k r_{t+k} \middle| s_t = s, a_t = a, \pi \right].$$

Temporal Difference (TD) Learning

The arguably most popular algorithm for learning $Q^\pi(s, a)$ is Q-Learning (Watkins & Dayan, 1992)

- Is an *off-policy* learning algorithm
- Able of converging to the optimal state-action value function $Q^*(s, a)$ with probability 1
- Works by keeping track of an estimate of the state-action value function $Q : \mathcal{S} \times \mathcal{A} \rightarrow \Re$

Q-Learning

The update rule of each visited state-action pair used by Q-Learning is:

$$Q(s_t, a_t) := Q(s_t, a_t) + \alpha \Big[r_t + \gamma \max_{a \in \mathcal{A}} Q(s_{t+1}, a_t) - Q(s_t, a_t) \Big].$$

Temporal Difference (TD) Learning

How does Q-Learning work?

$$Q(s_t, a_t) := \underbrace{Q(s_t, a_t) + \alpha \Big[r_t + \gamma \max_{a \in \mathcal{A}} Q(s_{t+1}, a_t)}_{\delta_t} - Q(s_t, a_t) \Big]$$

- We create the TD-error δ_t by using the $\max_{a \in \mathcal{A}}$ operator

Temporal Difference (TD) Learning

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- We create the TD-error δ_t by using the $\max_{a \in \mathcal{A}}$ operator
- We always update $Q(s_t, a_t)$ with respect to a greedy policy

Temporal Difference (TD) Learning

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- We create the TD-error δ_t by using the $\max_{a \in \mathcal{A}}$ operator
- We always update $Q(s_t, a_t)$ with respect to a greedy policy
- Even if the agent is exploring the environment, learning is done greedily $\rightarrow$ *off-policy* learning

Temporal Difference (TD) Learning

We can also learn the $Q^\pi(s, a)$ function in a way which is more similar to how we learned $V^\pi(s)$ beforehand: SARSA (Rummery & Niranjan, 1994)

- Is an *on-policy* learning algorithm
- Also works by keeping track of $Q : \mathcal{S} \times \mathcal{A} \rightarrow \Re$
- Has different convergence properties

SARSA

The update rule of each visited state-action pair used by SARSA is:

$$Q(s_t, a_t) := Q(s_t, a_t) + \alpha \Big[r_t + \gamma\, Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t) \Big].$$

Temporal Difference (TD) Learning

If we take a look at SARSA's TD-error ...

$$Q(s_t, a_t) := \underbrace{Q(s_t, a_t) + \alpha \Big[r_t + \gamma\, Q(s_{t+1}, a_{t+1})}_{\delta_t} - Q(s_t, a_t) \Big]$$

- We do not use the $\max$ operator anymore

Temporal Difference (TD) Learning

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- We do not use the max operator anymore
- We learn with respect to the next state visited by the agent s_{t+1} which might or might not (important!) correspond to a greedy policy

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- We do not use the max operator anymore
- We learn with respect to the next state visited by the agent s_{t+1} which might or might not (important!) correspond to a greedy policy
- As we are always taking into account the actions chosen by the agent $Q(s_{t+1}, a_{t+1})$ we learn $\rightarrow$ *on-policy*

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Temporal Difference (TD) Learning

Quiz Time!

Imagine an algorithm which jointly learns the state-value function $V^\pi(s)$ as follows

$$V(s) := V(s) + \alpha\big[r_t + \gamma V(s_{t+1}) - V(s_t)\big]$$

and the state-action value function $Q^\pi(s, a)$ as follows:

$$Q(s_t, a_t) := Q(s_t, a_t) + \alpha\big[r_t + \gamma V(s_{t+1}) - Q(s_t, a_t)\big].$$

- What is the TD-target of this algorithm?
- Is this an *off-policy* or *on-policy* learning algorithm?

Temporal Difference (TD) Learning

Quiz Time!

Now imagine an algorithm which jointly learns the state-value function $V^\pi(s)$ as follows

$$V(s) := V(s) + \alpha\left[r_t + \gamma \max_{a \in \mathcal{A}} Q(s_{t+1}, a_t) - V(s_t)\right]$$

and the state-action value function $Q^\pi(s, a)$ as follows:

$$Q(s_t, a_t) := Q(s_t, a_t) + \alpha\left[r_t + \gamma V(s_{t+1}) - Q(s_t, a_t)\right].$$

- What is the TD-target of this algorithm?
- Is this an *off-policy* or *on-policy* learning algorithm?

Temporal Difference (TD) Learning

Pros & Cons of TD-Learning methods:

- They do not yield unbiased estimates ✗
- But there is small variance in the updates ✓
- Learning starts faster ✓
- It can be complicated to combine them with non-linear function approximators ✗

Both approaches can be combined resulting in TD(λ) algorithms!

TD-Learning methods empirically work better but a formal proof about why this is the case is missing ...

Final Slide!

Lecture Takeaway

- What happens when parts of the Markov Decision Process are unknown
- Dynamic Programming $\rightarrow$ Reinforcement Learning
- Two families of model-free Reinforcement Learning algorithms
- We have seen the difference between *on-policy* and *off-policy* learning