Higuchi's Method applied to the detection of periodic components in time series and its application to seismograms

G. Gálvez Coyt[a], A. Muñoz Diosdado[a], J. A. Balderas López[a], J. L. del Río Correa[b], and F. Angulo Brown[c]
[a]*Department of Basic Sciences, UPIBI-IPN,*
Av. Acueducto S/N, Barrio la Laguna Ticomán, 07340, México, 07340, D. F. México.
[b]*Department of Physics, UAM-Iztapalapa,*
México, 09340, D. F. México.
[c]*Department of Physics, ESFM-IPN, Edificio 9 Zacatenco,*
México, 07738 D. F. México,
e-mail: ggalvezcoyt@gmail.com

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At the moment, the analysis of the named complex systems is based mainly on the study of the properties of the signals that is possible to measure. In general, these signals are obtained in a discreet way; therefore they constitute the so called time series in a natural way. Time series of complex systems present fractal and multifractal properties. The Higuchi's method is a method that if applied appropriately it can determine in a reliable way the fractal dimension D of the analyzed time series, this fractal dimension allows us to characterize the degree of correlation of the series. However, when analyzing some time series as heart interbeat, atmospheric pollutants (ozone), seismograms, etc., with the Higuchi's method, there are oscillations that have been observed at the right of the graph, that correspond to the considered big scales, which can cause a mistaken determination of the fractal dimension. In this work, an appropriate explanation is given to this type of behaviour, this fact enhances the understanding of the Higuchi's method, and we propose also its application as detector and alarm of possible earthquakes.

Keywords: Higuchi's method; time series; seismograms.

Por el momento, el análisis de los llamados sistemas complejos está basado principalmente en el estudio de las propiedades de las señales que son posible medir. En general, estas señales son obtenidas de una forma discreta, además, ellas constituyen de manera natural las llamadas series de tiempo. Las series de tiempo presentan propiedades fractales y multifractales. El método de Higuchi aplicado apropiadamente puede determinar muy bien la dimensión fractal D de la serie de tiempo analizada, Esta dimensión fractal permite caracterizar el grado de correlación de las series. Sin embargo, cuando se analizan series de interlatido cardiaco, contaminación por ozono, sismogramas, etc., con el método de Higuchi, aparecen oscilaciones en la parte derecha de la gráfica, lo cual corresponde a la parte de grandes escalas, esto puede causar un error en la determinación de la dimensión fractal. En este trabajo se da una explicación apropiada al tipo de comportamiento observado, esto redunda en el entendimiento de el método de Higuchi, además, proponemos la aplicación de este comportamiento como un detector y alarma de posibles sismos.

Descriptores: Método de Higuchi; series de tiempo; sismogramas.

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1. Introduction

Time series commonly appear in the experimentation when carrying out the measure of a physical quantity, sometimes the respective graph shows appreciable variations, in such a way that it seems random noise lacking of useful information. However, by means of diverse time series analysis techniques [1], it is possible to determine the correlation degree among the elements of the series, this degree of correlation can be expressed for example, in terms of some parameter that depends on the utilized method. For instance, the Higuchi's method [2,3] is characterized by D, parameter that allows to appropriately classify the series with their fractal characteristics.

All the time series analysis methods present advantages and disadvantages, in particular, the Higuchi's method presents the advantage of being a quite fast and reliable method for the determination of the correlation of the series compared with other methods, as spectral density and the Detrended Fluctuation Analysis method (DFA) [4]. However, it is not exempted of being applied in an erroneous way.

In the analysis of time series from different complex systems with the Higuchi's method, we have repeatedly noted the presence of oscillations in the right side of the log-log plot [5-8]. Normally, variations in this part of the plot are considered effects of noise and therefore devoid of useful information and they are removed from the calculations.

In this paper, we present an explanation to the observed oscillatory behavior in the graph corresponding to the Higuchi's method and its subsequent application to seismograms. The paper is organized as follows: in Sec. 2, we briefly describe the Higuchi's method; in Sec. 3, we present the Higuchi's analysis of periodic time series; in Sec. 4, we apply the Higuchi's analysis to time series of white noise with periodic components, and in Sec. 5, we apply the windowing Higuchi's method to the detection of periodic components in seismograms. Finally, we present some concluding remarks.

2. Higuchi's method

A time series can be expressed by $x(i)$ $i = 1, \ldots, N$, where each datum is taken at equally spaced time intervals, with a uniform time denoted by δ. Usually thought to be $\delta = 1$ because in principle this parameter does not alter the data analysis. The Higuchi's method is a method of analysis that is being increasingly used for the analysis of time series [2,3], it is a very efficient way to determine the fractal dimension D of a curve.

The following describes how to apply the Higuchi's method to a time series.

a) From the time series $x(i)$ the new series x_k^m (i) are obtained

$$x_k^m; x(m),\ x(m+k),\ x(m+2k),\ x(m+3k), \ldots, x\left(m + \left[\frac{N-m}{k}\right]k\right), \quad (m = 1,\ 2,\ 3, \ldots, k) \tag{1}$$

Where k and m are integer numbers, m and k represents the initial time interval width and [ ] denotes the integer part. Assuming that the series has only $N = 100$ elements, the following are the only three subsets that can be obtained for k = 3,

$$\begin{aligned} x_3^1 &: x(1),\ x(4),\ x(7),\ x(10), \cdots x(97),\ x(100) \\ x_3^2 &: x(2),\ x(5),\ x(8),\ x(11), \cdots x(98) \\ x_3^3 &: x(3),\ x(6),\ x(9),\ x(12), \cdots x(99) \end{aligned} \tag{2}$$

b) The length of the series x_k^m (i) is defined as:

$$L_m(k) = \left\{ \left(\sum_{i=1}^{\left[\frac{N-m}{k}\right]} |x(m+ik) - x(m+(i-1)k)| \right) \frac{N-1}{\left[\frac{N-m}{k}\right]k} \right\} \Big/ k \tag{3}$$

The term (N-1)/[($N-m$)/k]k represents the normalization factor for the length of the subset.

c) The length of the series $L(k)$ for $x(i)$ is obtained by averaging all the subseries lengths $L_m(k)$that have been obtained for a given k value.

d) If $L(k) \propto k^{-D}$, that is, if it behaves as a power law, we find that the exponent D is the fractal dimension of the series.

Applying the above relation implies the proper choice of a maximum value of k for which the relationship $L(k) \propto k^{-D}$ is approximately linear.

As an example of applying the Higuchi's method we show in Fig. 1(a) the time series corresponding to a Brownian motion generated synthetically with N = 4000 data. Moreover, Fig. 1(b) shows the Higuchi's analysis of Brownian motion (sometimes also known as Brownian noise) presented in Fig. 1(a). According to the analysis, D = 1.5075, a value very close to the expected theoretical value of 1.5.

3. Higuchi's analysis of periodic time series

The previous section described the process for applying the Higuchi's method, but it is important to note that it involves the proper choice of a maximum value of k (here denoted as K_{max}). In general, the choice of this value depends on the preference of the researcher, but it is usually restricted to the region where the plot $\log(L(k))$ vs. $\log(k)$ is approximately linear.

Normally the Higuchi's method is applied to time series for determining the dimension D, when the time series sample comes from regular functions of the form $y = f(x)$, the

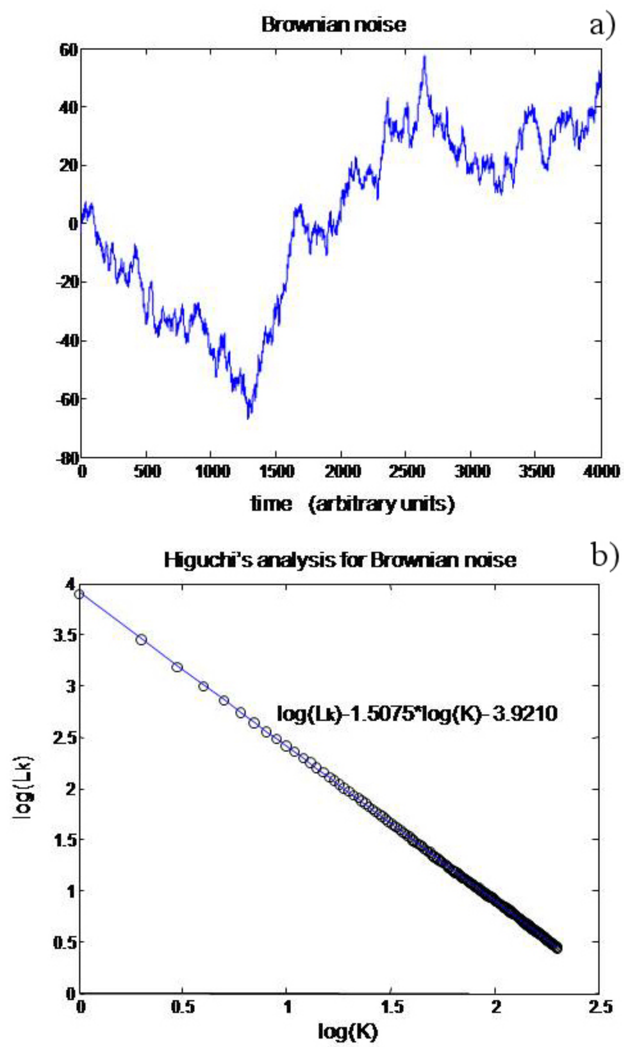

FIGURE 1. (a) Graph of a time series of Brownian noise, (b) Higuchi's analysis for Brownian motion (D = 1.5075).

topological dimension of the series will always be close to one, that is $D \approx 1$. For this reason it would seem unnecessary to apply the Higuchi's method to this type of series. However, when the functions involved in a time series contain periodic or approximately periodic functions, Higuchi's method may give useful information about the series.

Suppose we have time series y_1, y_2 and y_s=y_1+y_2, which are obtained from the sample at a frequency $f_m = 1000$ data/s for 10 seconds, of the respective functions $y_1 = \sin(5t)$, $y_2 = 2$ sin $(10t)$, then each of the series will have $N = 10000$ data.

Performing the respective Higuchi's analysis to each one of the series at a value of $K_{\max}$ = 4000, we obtain the result shown in Fig. 2 (a). Looking carefully we note that these plots show down peaks. With a further analysis we can see that the undulations in the Higuchi's method in the series y_s are an effect of the "sum" of the respective Higuchi's analysis of the component series y_1 and y_2. A detail of the area of the oscillations is shown in Fig. 2 (b).

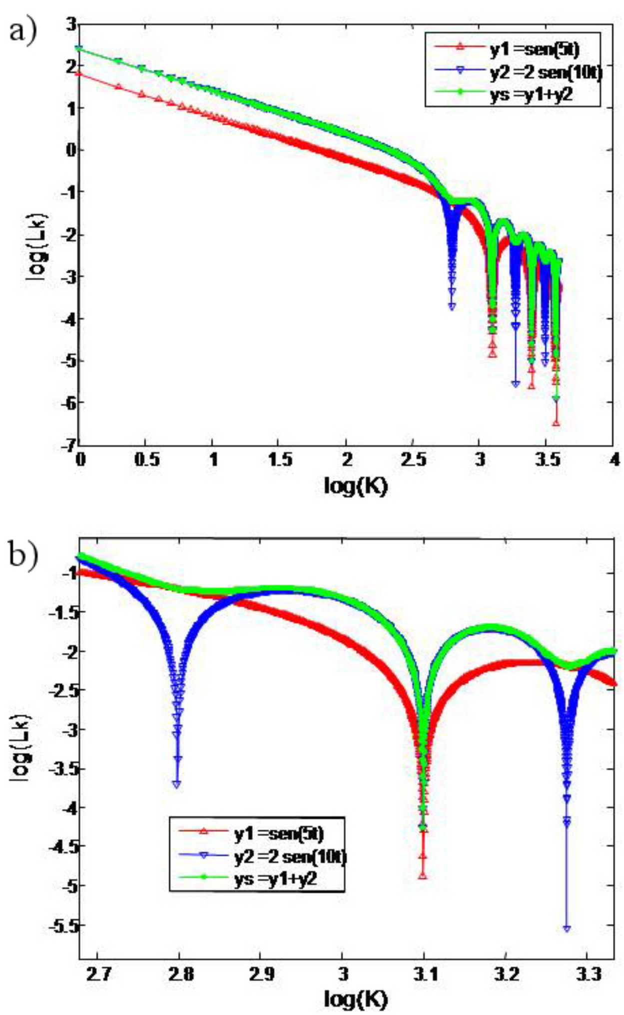

FIGURE 2. (a) Comparative Higuchi's analysis of individual series obtained for y_1 = sin (5t) (triangles), $y_2 = \sin(10t)$ (inverted triangles) and the corresponding sum $y_s = y_1 + y_2$ (points). (b) Detail of the graph in the region where waves and spikes appear.

The location of each peak is directly related to the frequency or period of the time series. To show this, consider that for a sampling frequency f_m, and a time τ, it will have $N_\tau = \tau f_m$ data. On the other hand, if the wave is periodic of the form y = sin $(\omega_0 t)$, we find that the period is T$= 2\pi/\omega_o$, so N_T can be calculated by the relationship

$$N_T = (2\pi/\omega_o)f_m, \tag{4}$$

In the case of the y_1 time series shown in Fig. 2(a) where ω_o= 5 and a frequency $f_m = 1000$ data/s, it must be N_T =(2π/10) 1000 = 628.3 data. Since the graphic analysis of the Higuchi's analysis is logarithmic, now we proceed to calculate $\log(N_T)=$ $\log$ (628.3) = 2.798, the next peak corresponds to log (1256.6) = 3.099, comparing with previous results observed in the respective graph the matching is complete.

The explanation for the appearance of these peaks on the graph of Higuchi analysis can be understood by looking at the definition of subsets $x_k^m(i)$ when k is very close or equal to the number of data for the period of the function $k \approx N_T$, the sub-series for each value of m will consist of values very similar to each other, i.e. they have the form: $x_0+\varepsilon_0$, $x_1+\varepsilon_1$, $x_2+\varepsilon_2$, $x_3+\varepsilon_3$, ..., x_n+ ε_n, where: ε_0, ε_1, ε_2, ε_3, ..., ε_n are very small values.

In the case of the equality, that is, $k = N_T$, virtually $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \varepsilon_3$ = ... = ε_n = 0, so the subsets are nearly constant. By using these sub-series in the calculation of the length $L_m(k)$ by the method of Higuchi, first, we have to evaluate the expression

$$\begin{aligned}
&\sum_{i=1}^{\left[\frac{N-m}{k}\right]} |x\,(m+ik) - x\,(m+(i-1)\,k)| \\
&= \sum_{i=1}^{\left[\frac{N-m}{k}\right]} |x_0+\varepsilon_i-(x_0+\varepsilon_{i-1})| \\
&= \sum_{i=1}^{\left[\frac{N-m}{k}\right]} |\varepsilon_i-\varepsilon_{i-1}| \approx \varepsilon_{mT},
\end{aligned} \tag{5}$$

where ε_{mT} is very small. Substituting in the definition of the length of the curve we have that

$$L_m\,(k) = \frac{N-1}{\left[\frac{N-m}{k}\right]k^2}\varepsilon_{mT} \cong L_{m\varepsilon}\,, \tag{6}$$

since, if ε_{mT} is small then $L_{m\varepsilon}$ will also be small. Moreover, the length $L(k)$ is the average of all lengths $L_m(k)$ that can be built from the original time series $x(i)$. Although each of the sub-series may have different length $L_m(k)$, in the case of periodic time series the values are very similar and therefore $L(k)$ will also be a very small value, so $L(k) \approx L_\varepsilon \approx$0.

Furthermore, to determine D of the power law $L(k) \approx k^{-D}$, the logarithm is applied on both sides of the above equation, obtaining

$$\log\,(L\,(k)) = A - D\ \ \log(k). \tag{7}$$

On the left part, L_ε will be small when $k \approx N_T$ and therefore

$$\log(L_\varepsilon) \approx -\infty. \tag{8}$$

For numerical reasons and computer calculations it is not possible to reach the infinite value, but the graph of $L(k)$ vs. k will have a noticeable change or peak exactly at this value of k. Thus, the explanation of the peaks shown in the analysis of Higuchi is due to the periodic behavior of the time series.

4. Higuchi's analysis of time series of white noise with periodic components

In the previous section it has been shown how is the behavior of the Higuchi's method of time series formed by the sum of periodic functions. This section will show with an example that the behavior of the oscillations seen in the Higuchi's graph analysis is extended to any type of series, as long as it has in the composition one or more periodic components with appropriate frequency (preferably low) and whose amplitude is large enough.

To see what is described in the preceding paragraph, first, it is constructed a time series of white noise y_r (D = 2), also with a sampling rate given by f_m = 1000/s for 10 seconds. Now we properly add to the series y_r a periodic signal of the form $y_p = 3\sin 20t$, and we get the series $y_s = y_r + y_p$ whose graph is shown in Fig. 3 (a). At first glance a periodic behavior is not observed, however, the graph of the Higuchi's analysis shows oscillations in the large scales, revealing the existence of periodic behavior at low frequencies.

It is possible to determine the first minimum observed in the Higuchi's analysis of Fig 3 (b), using information from the periodic part of the series, according to data from the signal y_p and equation (4), the first minimum must be located at

$$N_T = \frac{2\,\pi}{\omega_o}\, f_m = \frac{2\pi}{20}\,(1000) = 314.1593.$$

Applying logarithm to the previous result, it is obtained, log (314.1593) =2.4971, value that is fairly close to the value 2.5040, which is obtained directly from the graph shown in Fig. 3.

Although it is possible to determine some frequencies contained in a time series with this idea, in the following section we show an application to the analysis of seismograms by using the fact that the earthquake frequencies modify the value of the Higuchi's fractal dimension and there is an increment of the quadratic error of the adjustment line in the analysis.

5. The windowing Higuchi's method applied to the detection of periodic components in seismograms

The seismogram is the graphical record of an earthquake; it shows the types of waves that are registered during an earthquake, which differ mainly in their transmission speed in the ground. There are at least 3 types of waves, the primary, secondary and surface waves. Often the primary waves (which are the first to arrive, hence the name) are of so small amplitude that they do not differ from background noise. However, the secondary waves have greater amplitude, especially in the case of large earthquakes, after the previous waves reach the surface. These waves are wider and therefore are destructive because they can cause damage to buildings and other structures. In Fig. 4a it is shown a seismogram for an event occurred on August 17, 2006 in Guerrero, México.

A simple model of a seismogram can be seen in a simple way as a signal consistent of several component signals: first, a background white noise of small amplitude; second, a signal with background noise mixed with a set of sine waves with an amplitude greater than the previous one, and third, a signal of rather larger extent compared with the previous white noise composed of damped sinusoidal signals, and

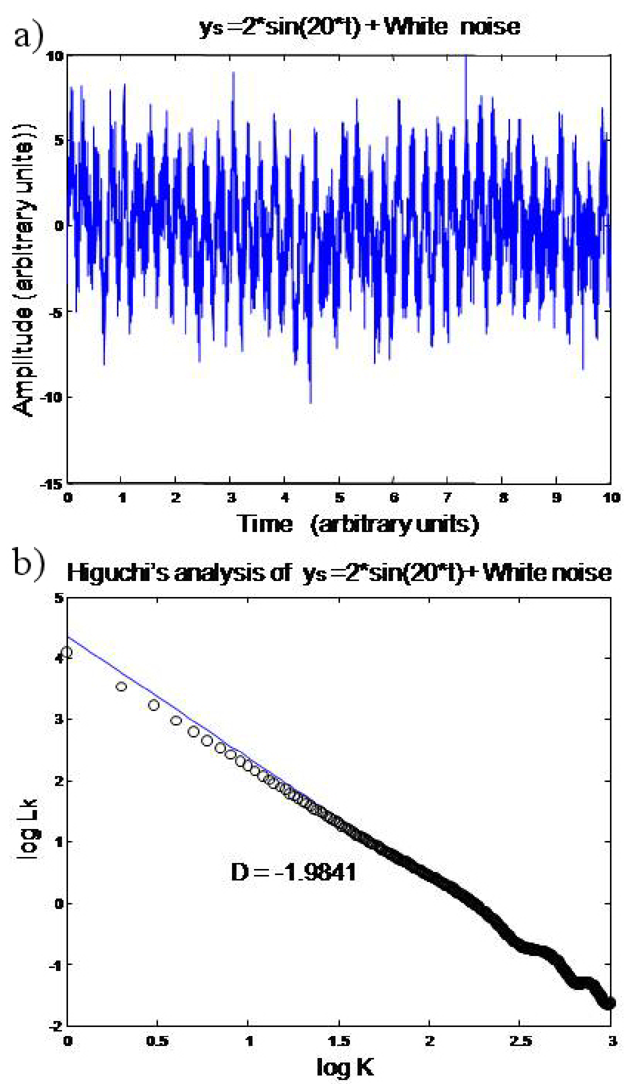

FIGURE 3. (a) Plot of the sum of the time series of white noise (D =2) with periodic series ($2\sin 20t$) (b) Higuchi's analysis of the series shown in part (a) and detection of the first minimum.

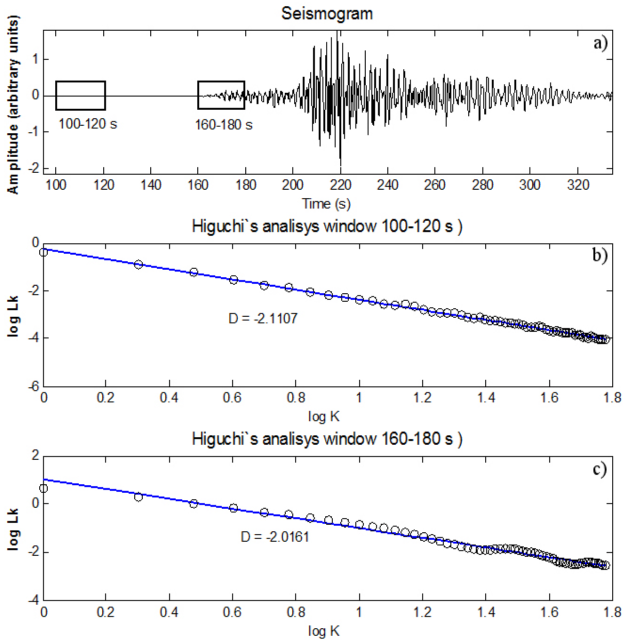

FIGURE 4. (a) Seismogram for an event occurred on August 17, 2006 in Guerrero, Mexico. (b) Higuchi's analysis for a window corresponding to 100-120 s. (c) Higuchi's analysis for a window corresponding to 160-180 s

and finally, it returns to the initial condition of background white noise signal. As mentioned in the previous section, the presence of periodic signals is reflected in the graph of the Higuchi's analysis, so the basic idea is to properly identify the moment in which the oscillations indicating the presence of primary and/or secondary waves appear. The appropriate method to carry out the series monitoring is by using the windowing, which consists of segmenting the signal in equal parts, long enough so that the method is applicable.

Figure 4b, shows the Higuchi's analysis corresponding to a 100-120 s seismogram window, the graph does not show significant variations, which indicates the absence of periodic components.

On the other hand, Fig. 4c shows the Higuchi's analysis for the segment corresponding to 160-180 s. This segment, as noted in the seismogram, includes the presence of interesting waves, which is reflected in the presence of oscillations in the graph of the Higuchi's analysis.

In the Fig. 5a the seismogram is divided into three main parts, the first is prior to the emergence of seismic waves, in the second, primary and secondary waves appear, and finally, the third part where the main amplitude occurs.

For the windowing analysis, the seismogram was divided into segments or windows of width = 200 data (as the sampling frequency is 10 data/s it corresponds to 20 s). On the other hand, the window has a displacement = 10 data (1 s) and the starting point is $a = 200$ and the end point is $b = 5700$ and $K_{\max} = 50$.

The graph corresponding to the total windowing is shown in Fig. 5b, in the first part of the seismogram it can be seen that the graph of D has a small fluctuation. In the second

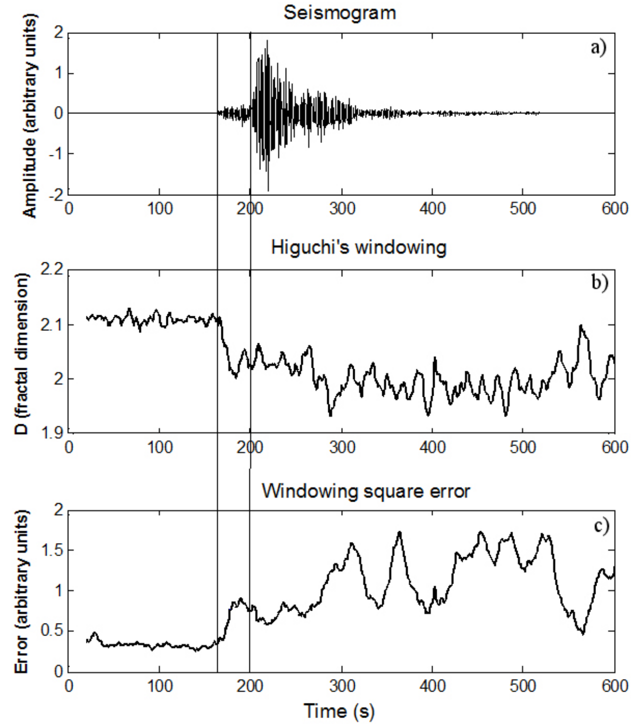

FIGURE 5. (a) Seismogram sectioned into three parts of interest. (b) Windowing Higuchi's method fractal dimension D. (c) Windowing square error.

part D has a very significant change when the seismogram changes, which was maintained for the entire duration of the quake, eventually returning to the initial condition.

Appreciable scale fluctuations observed in the behavior are easily explained by the presence of primary and secondary waves which have a periodic behavior and a change in magnitude. It is noteworthy that the change in D is seen immediately when they appear periodic signals.

From the same Higuchi's analysis it has been introduced the quadratic error as another indicator of the presence of primary and secondary waves, the result of the carried out windowing is shown in Fig. 5 (c). As in the case of fractal dimension, the quadratic error increases appreciably in the second part of the seismogram, when the periodic behavior of seismic waves appear.

An enlargement of the second region is shown in Fig. 6. Considered the beginning of the event in 170 s, it is observed that for 173 s the fractal dimension D and the quadratic error values have changed enough with regard to previous values indicating the presence of harmonic waves corresponding to the possible earthquake. Considering that the main oscillations of the earthquake begin approximately at 206 s, there are 33 s as warning time.

The windowing Higuchi's method can be adapted to continuous monitoring of seismograms along with other existing methods in order to detect earthquakes of magnitude more significant, in places enough away from the source, so the alarm can be raised with sufficient time to take preventive

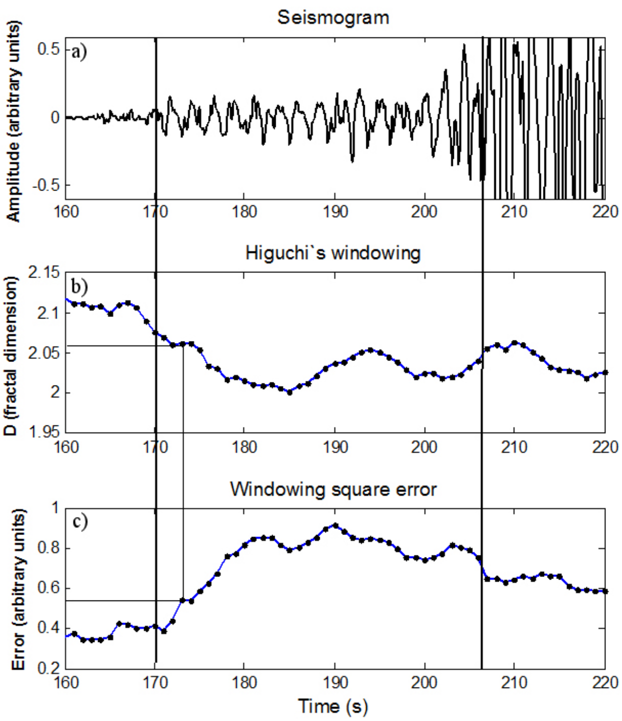

FIGURE 6. (a) Detail of the region of interest in the seismogram (b) Windowing Higuchi's method fractal dimension D. (c) Windowing square error.

action. For example, for the sampling frequency of the seismogram data used, it takes an average of 8 s for the presence of primary and secondary waves, on the other hand, if the earthquake comes from the seismic zones that affect Mexico City, *i.e.* the coasts of Guerrero, Oaxaca or Michoacan, there would be a time of at least 30 seconds to raise an alarm.

6. Conclusions

The oscillatory behavior in the graph of the Higuchi's method has been observed in real time series such as heart interbeat, series of air pollutants, seismograms, etc. This feature is explained with no doubt by the presence of one or more periodic components in the time series. Higuchi's method should be applied carefully considering the size K_{max} which must be small enough so that the calculation did not include a condition strictly periodic, if what is sought is the fractal dimension of the series.

On the other hand, if what you want in the series is to identify low-frequency periodic components, use a value of $K_{\max}$ large enough to be included in the plot of the method.

The presence of periodic components in a time series may be a possible explanation for the emergence of the so-called crossovers of the Higuchi plot analysis of the signal.

Higuchi's method can be applied to the detection of periodic components of high or low frequency and amplitude changes in time series. It is also possible to use it to detect the occurrence of sudden fluctuations in signals when they are continuously monitored.

The windowing Higuchi's method was applied to the seismogram time series of some considerable magnitude earthquakes and actually it was able to detect the presence of primary and secondary waves at the respective graphical display in the presence of waves, which correspond to the expected frequencies in the seismogram.

Mexico City is an area vulnerable to earthquakes generated on the coasts of Guerrero, Oaxaca or Michoacan, and because its distance, we would have at least 30 seconds to raise an alarm. This approach can be complementary to other methods of p-wave detection which work as seismic alarms [9,10].

Acknowledgements

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