CS161: Homework #5
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Problem 1

(a) Neither.

$Smoke$	$Fire$	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
F	F	T
T	F	T
F	T	F
T	T	T

(b) Neither.

$Smoke$	$Fire$	$Heat$	$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

(c) Valid.

$Smoke$	$Fire$	$Heat$	$((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Problem 2

(a) Propositional Symbols:
Propositional symbols starting with uppercase letter: *Mythical, Mortal, Mammal, Horned, Magical* , denotes whether a unicorn is mythical/mortal/mammal/horned/magical.

Knowledge Base:

$$R1 : Mythical \Rightarrow \neg Mortal$$
$$R2 : \neg Mythical \Rightarrow (Mortal \wedge Mammal)$$
$$R3 : (\neg Mortal \vee Mammal) \Rightarrow Horned$$
$$R4 : Horned \Rightarrow Magical$$

(b)

$$R1 : \neg Mythical \vee \neg Mortal$$
$$R2 : Mythical \vee (Mortal \wedge Mammal) =$$
$$(Mythical \vee Mortal) \quad \text{(R5)}$$
$$\wedge (Mythical \vee Mammal) \quad \text{(R6)}$$
$$R3 : (Mortal \wedge \neg Mammal) \vee Horned =$$
$$(Mortal \vee Horned) \quad \text{(R7)}$$
$$\wedge (\neg Mammal \vee Horned) \quad \text{(R8)}$$
$$R4 : \neg Horned \vee Magical$$

Therefore, the knowledge base can be represented as:

$$KB =$$
$$(\neg Mythical \vee \neg Mortal)$$
$$\wedge (Mythical \vee Mortal)$$
$$\wedge (Mythical \vee Mammal)$$
$$\wedge (Mortal \vee Horned)$$
$$\wedge (\neg Mammal \vee Horned)$$
$$\wedge (\neg Horned \vee Magical)$$

(c) To prove horned, we add R9:¬ Horned.
Resolve R9 with R7, we have: R10: Mortal.
Resolve R9 with R8, we have: R11: ¬ Mammal.
Resolve R11 with R6, we have: R12: Mythical.
Resolve R12 with R1, we have: R13: Mortal.
Resolve R10 with R13, we have: EMPTY CLAUSE.
THUS, Horned is always true.

To prove Magical, we add R14: ¬ Magical
Resolve R14 with R4, we have R15: ¬ Horned.
So we go through the resolution for Horned, and we have EMPTY CLAUSE, so we can prove that Magical is always true.

To prove Mythical is not always true, we add R16: ¬ Mythical.
Resolve R16 with R5, we have R17: Mortal.

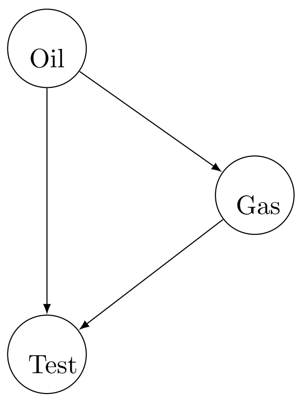
Resolve R16 with R6, we have R18: Mammal.
Resolve R8 with R18, we have R19: Horned.
Resolve R19 with R4, we have R20: Magical.
Resolve R6 with R8, we have R20: Mythical $\vee$ Horned.
Resolve R19 with R4, we have: Horned.
So this resolution goes forever, and it seems $\neg$ Mythical can be true. So we cannot prove Mythical is always true.

Thus, it can be proved that the unicorn is magical and horned. But it cannot be proved that the unicorn is mythical.

Problem 3

(a)

Oil T	Oil F
0.2	0.8



Oil	Gas	$P(Gas\|Oil)$
F	F	0.3
F	T	0.2
T	F	0.5
T	T	0

Oil	Gas	Test	P
F	F	F	0.9
F	T	F	0.7
T	F	F	0.1
T	T	F	0
F	F	T	0.1
F	T	T	0.3
T	F	T	0.9
T	T	T	0

(b) $Pr(oil|test = positive) = (Pr(test = positive|oil) * Pr(oil))/(Pr(test = positive))$

$= (Pr(test = positive|oil)*Pr(oil))/(Pr(test = positive|oil)Pr(oil)+Pr(test = positive|gas)Pr(gas)+ Pr(test = positive|gas, oil)Pr(gas, oil) + Pr(test = positive|\neg gas, \neg oil)Pr(\neg gas, \neg oil))$

$= (0.9 * 0.5)/(0.9 * 0.5 + 0.3 * 0.2 + 0 + 0.3 * 0.1)$

$= 0.8333$

Problem 4

(a) $Pr(A, B, C, D, E, F, G, H)$
$= Pr(A) * Pr(B) * Pr(C|A) * Pr(D|A, B) * Pr(E|B) * Pr(F|C, D) * Pr(G|F) * Pr(H|E, F)$

(b) $Pr(E, F, G, H)$
$= \sum_A \sum_B \sum_C \sum_D Pr(A) * Pr(B) * Pr(C|A) * Pr(D|A, B) * Pr(E|B) * Pr(F|C, D) * Pr(G|F) * Pr(H|E, F)$
$= Pr(G|F)Pr(H|E, F) \sum_A Pr(A) \sum_B (Pr(B)Pr(E|B)) \sum_C P(C|A) \sum_D Pr(F|C, D)Pr(D|A, B)$
(Assign factors)
$= f_1(G, F)f_2(E, F, H) \sum_A f_3(A) \sum_B f_4(B)f_5(B, E) \sum_C f_6(A, C) \sum_D f_7(C, D, F)f_8(A, B, D)$

(c) $Pr(a) * Pr(\neg b) * Pr(c|a) * Pr(d|a, \neg b) * Pr(\neg e|\neg b) * Pr(f|c, d) * Pr(\neg g|f) * Pr(h|\neg e, F)$
$= 0.2 * 0.3 * Pr(c|a) * 0.6 * 0.1 * Pr(f|c, d) * Pr(\neg g|f) * Pr(h|\neg e, F)$

(d) $Pr(\neg a, b) = 0.8 * 0.7 = 0.56$
$Pr(\neg e|a) = \frac{Pr(\neg e,a)}{Pr(a)} = \frac{Pr(a,\neg e,b)+Pr(a,\neg e,\neg b)}{Pr(a)} = \frac{0.2*0.7*0.9+0.2*0.3*0.1}{0.2} = 0.66$

(e) A variable X is independent of its non-descendants given its parents. So C is independent of nodes other than F given A, D is independent of nodes other than F given A and B. E is independent of nodes other than H given B. F is independent of nodes other than G, H given C, D. G, H are independent of all the nodes given F.

(f) A, B, C, F

(g) $Pr(D|A, B)\&Pr(E|B)$ corresponds to 2*2*2 and 2*2 matrices

A	B	D	E	$f_3(A, B, D, E)$
T	T	T	T	0.05
T	F	T	T	0.54
T	T	F	T	0.05
T	T	T	F	0.45
F	T	T	T	0.01
F	F	T	T	0.72
T	F	F	T	0.36
T	T	F	F	0.45
F	T	T	F	0.09
F	T	F	T	0.09
T	F	T	F	0.06
F	F	F	T	0.18
F	T	F	F	0.81
T	F	F	F	0.04
F	F	T	F	0.08
F	F	F	F	0.02

Problem 4 continued on next page. . .

(h)

A	B	E	$\sum_D f_3(A,B,D,E)$
F	F	F	0.1
T	F	F	$= 0.1$
F	T	F	0.9
T	T	F	0.9
F	F	T	0.9
F	T	T	0.1
T	F	T	0.9
T	T	T	0.1