

# First-Order Logic

CS161

Prof. Guy Van den Broeck

# *Why are we learning about logic?*

- Specification  $\alpha$  and implementation  $\beta$
- Show that  $\beta$  entails  $\alpha$  using SAT solver
  - The implementation is “correct”
- Hardware verification
  - CPU computes addition correctly
- Software verification
  - Windows driver does not write to unallocated memory
- Cryptography
- Planning

# Why are we learning about logic?

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Posted by BeauHD on Monday May 30, 2016 @08:10PM from the red-pill-and-blue-pill dept.



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THE CONVERSATION

Academic rigour, journalistic flair

76 comments



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200 Terabytes. Thats about 400 PS4s.

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# Why are we learning about logic?

## Probabilistic Reasoning



Artificial Intelligence  
Volume 172, Issues 6–7, April 2008, Pages 772–799



### On probabilistic inference by weighted model counting ☆

Mark Chavira , Adnan Darwiche 

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<https://doi.org/10.1016/j.artint.2007.11.002>

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#### Abstract

A recent and effective approach to probabilistic inference calls for reducing the problem to one of weighted model counting (WMC) on a propositional knowledge base. Specifically, the approach calls for encoding the probabilistic model, typically a Bayesian network, as a propositional knowledge base in conjunctive normal form (CNF) with weights associated to each model according to the network parameters. Given this CNF, computing the probability of some evidence becomes a matter of summing the weights of all CNF models consistent with the evidence. A number of variations on this approach have appeared in the literature recently, that vary across three orthogonal dimensions. The first dimension concerns the specific encoding used to convert a Bayesian network into a CNF. The second dimension relates to whether weighted model counting is performed using a search algorithm on the CNF, or by compiling the CNF into a structure that renders WMC a polytime operation in the size of the compiled structure. The third dimension deals with the specific properties of network parameters (local structure) which are captured in the CNF encoding. In this paper, we discuss recent work in this area across the above three dimensions, and demonstrate empirically its practical importance in significantly expanding the reach of exact probabilistic inference. We restrict our discussion to exact inference and model counting, even though other proposals have been extended for approximate inference and approximate model counting.

## Deep Learning

### A SEMANTIC LOSS FUNCTION FOR DEEP LEARNING WITH SYMBOLIC KNOWLEDGE

Jingyi Xu, Zilu Zhang, Tal Friedman, Yitao Liang & Guy Van den Broeck

Computer Science Department

University of California, Los Angeles

Los Angeles, CA, USA

[jixu@cs.ucla.edu](mailto:jixu@cs.ucla.edu), [zhangzilu@pku.edu.cn](mailto:zhangzilu@pku.edu.cn), {[tal](mailto:tal@ucla.edu), [yliang](mailto:yliang@ucla.edu), [guyvdb](mailto:guyvdb@ucla.edu)}@cs.ucla.edu

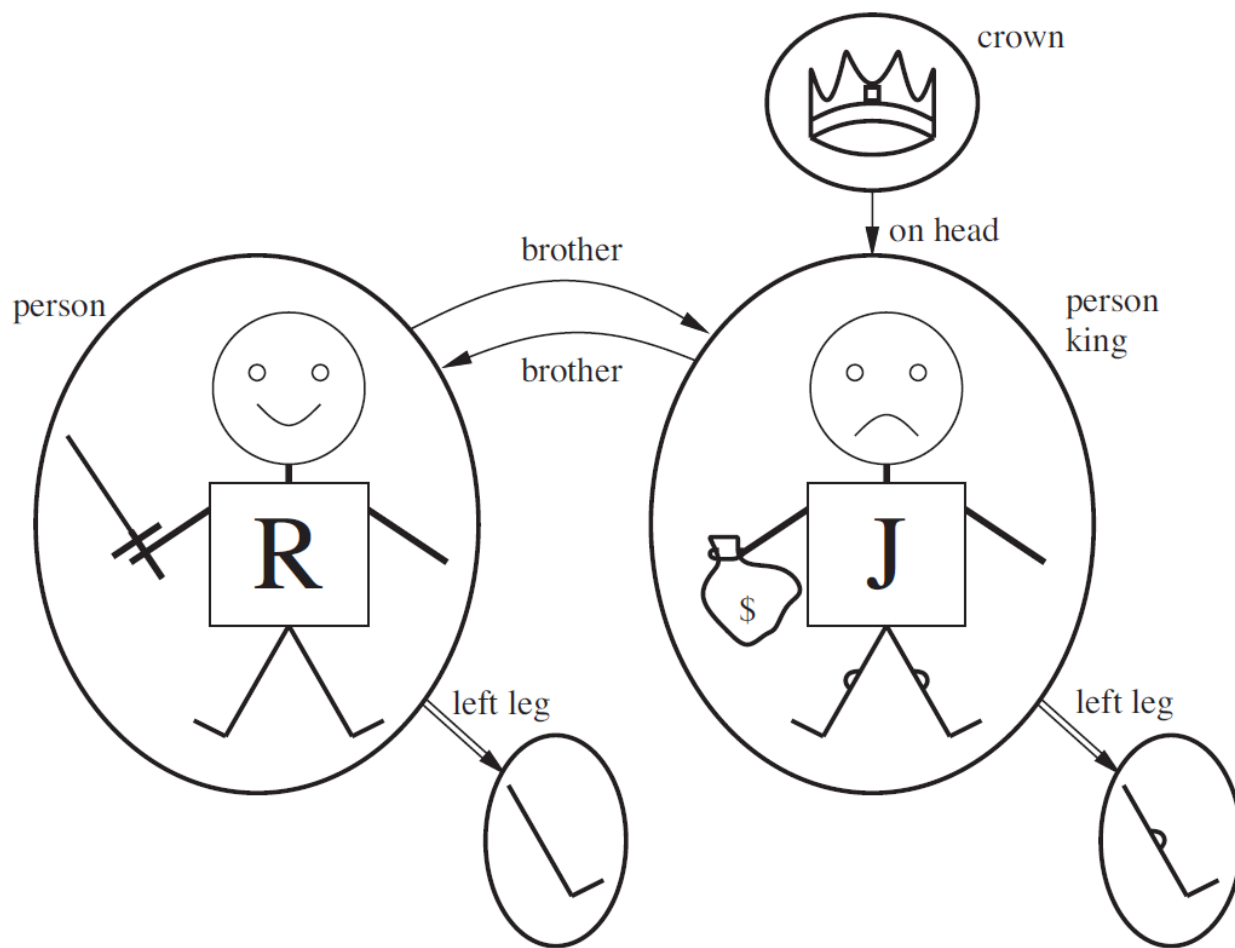
#### ABSTRACT

This paper develops a novel methodology for using symbolic knowledge in deep learning. From first principles, we derive a semantic loss function that bridges between neural output vectors and logical constraints. This loss function captures how close the neural network is to satisfying the constraints on its output. An experimental evaluation shows that our semantic loss function effectively guides the learner to achieve (near-)state-of-the-art results on semi-supervised multi-class classification. Moreover, it significantly increases the ability of the neural network to predict structured objects, such as rankings and paths. These discrete concepts are tremendously difficult to learn, and benefit from a tight integration of deep learning and symbolic reasoning methods.

# First-Order/Predicate Logic



# World

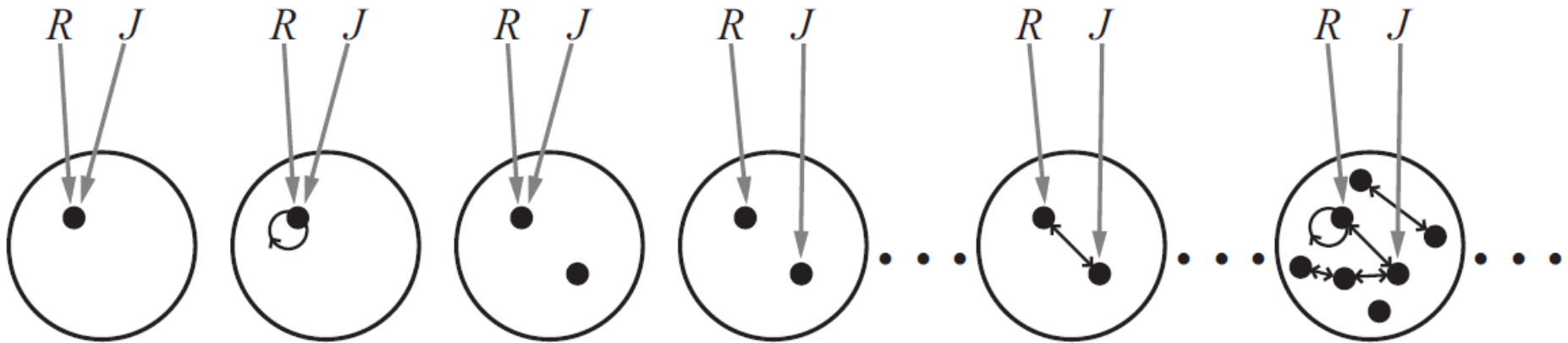


# Syntax and Semantics of First-Order Logic



# Models

- Two constants:  $R$ ,  $J$
- One binary relation





# Fun with First-Order Logic



# Fun with Quantifiers



# Inference Rules: Universal Instantiation

$$\forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$$



# Power of Propositionalization

- Problem: Infinitely many ground terms with function symbols (Father(Father(Father(...))))
- Theorem (Herbrand, 1930):  
If a sentence is entailed by a first-order KB, it is entailed by a finite subset of the propositional KB.
- Idea: Iteratively grow the propositional KB
- Theorem (Church/Turing 1936):  
Entailment in FOL is semi-decidable.  
Idea works if sentence is entailed, loops forever otherwise

# Generalized Modus Ponens

*Can we do inference immediately?*

$\forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$

$\text{King}(\text{John})$

$\text{Greedy}(\text{John})$

*Can we do inference immediately?*

$\forall x \text{ King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$

$\text{King}(\text{John})$

$\text{Greedy}(\text{John})$

*When do sentences share information?*

# Substitution and Unification



# Most General Unifier (MGU)

Maximum amount of information  
shared between two sentences





# Unification Algorithm

Term-by-term

Depth-First

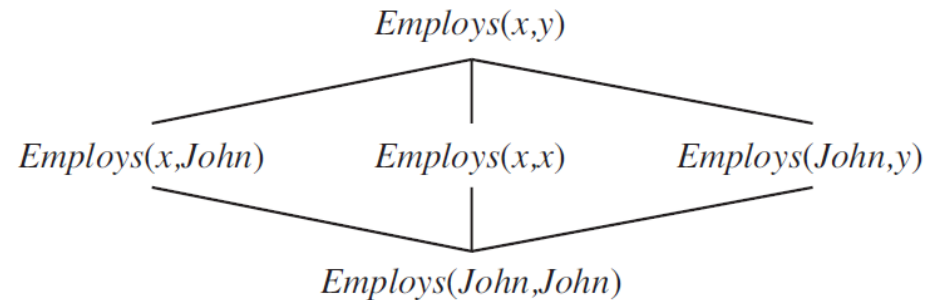
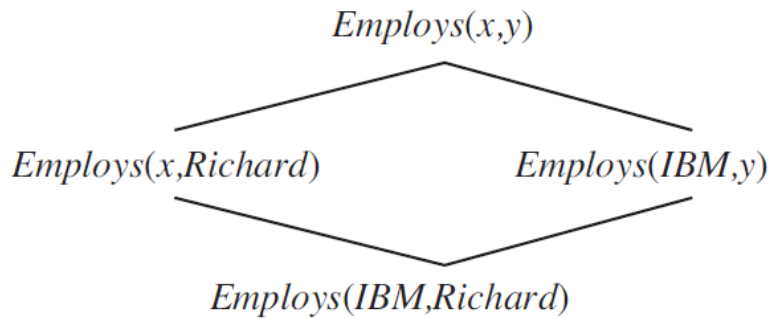
Keep track of substitutions used

Avoid infinite loops (occurs check)

See book for details.

# Subsumption Lattice

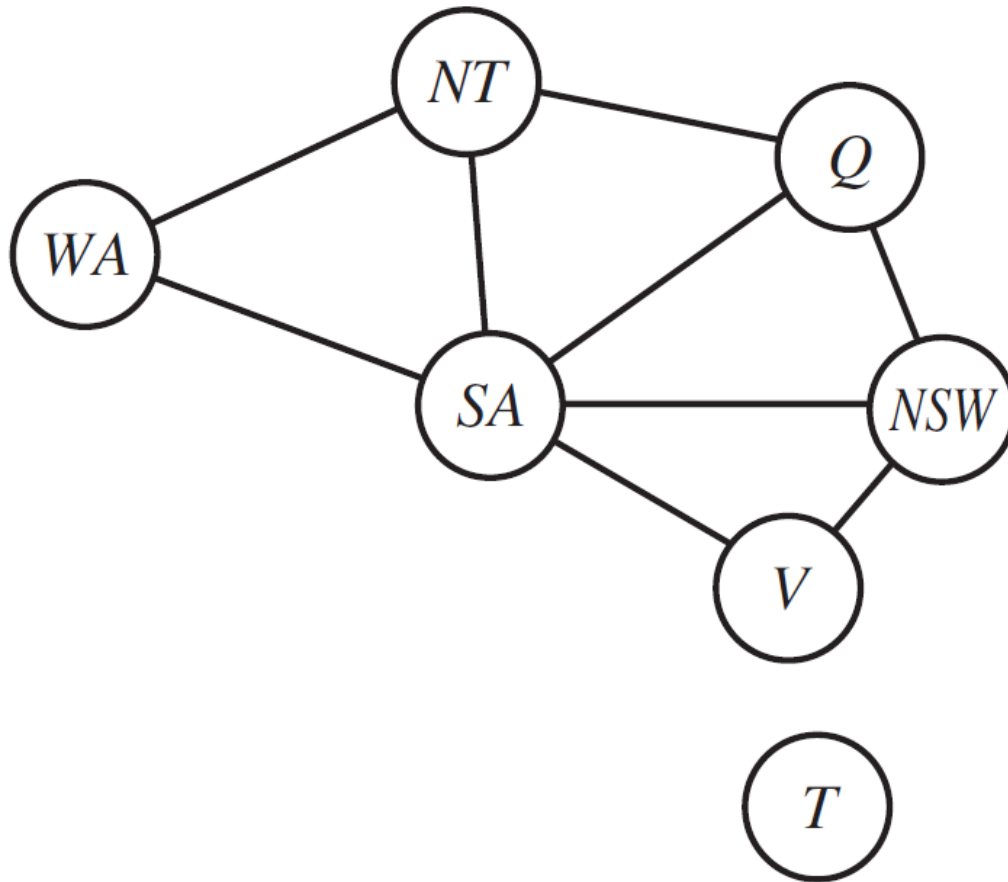
Sentence  $\alpha$  subsumes  $\beta$  iff  
there exists a substitution  $\theta$  such that  $\alpha\theta = \beta$



# Generalized Modus Ponens



# Complexity of Finding Premises


$$\begin{aligned} &Diff(wa, nt) \wedge Diff(wa, sa) \wedge \\ &Diff(nt, q) \wedge Diff(nt, sa) \wedge \\ &Diff(q, nsw) \wedge Diff(q, sa) \wedge \\ &Diff(nsw, v) \wedge Diff(nsw, sa) \wedge \\ &Diff(v, sa) \Rightarrow Colorable() \end{aligned}$$
$$\begin{aligned} &Diff(Red, Blue) \quad Diff(Red, Green) \\ &Diff(Green, Red) \quad Diff(Green, Blue) \\ &Diff(Blue, Red) \quad Diff(Blue, Green) \end{aligned}$$

# First-Order Resolution



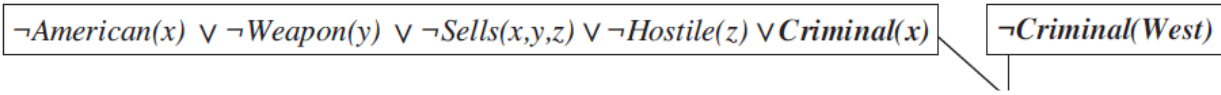
# Resolution

$\neg \textit{Criminal}(\textit{West})$

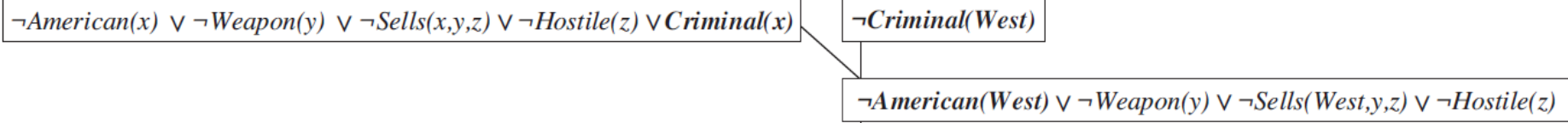
# Resolution

$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee Criminal(x)$

$\neg Criminal(West)$

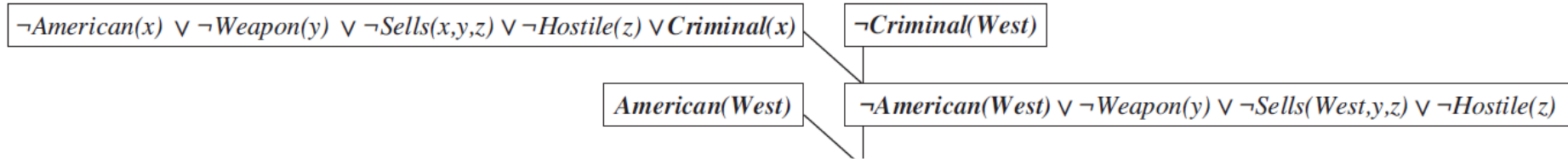


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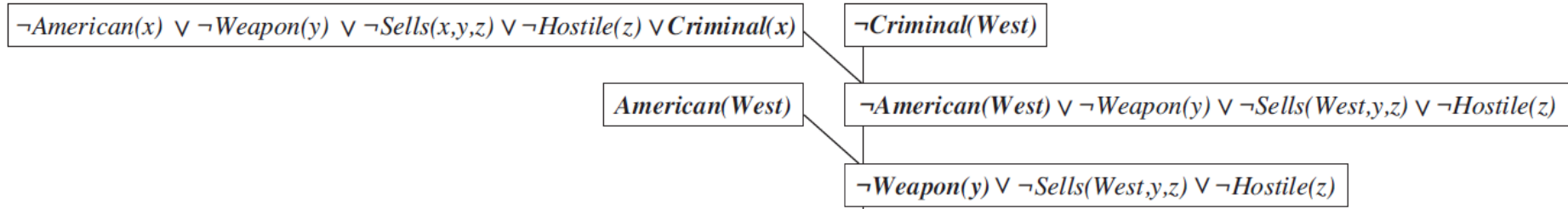




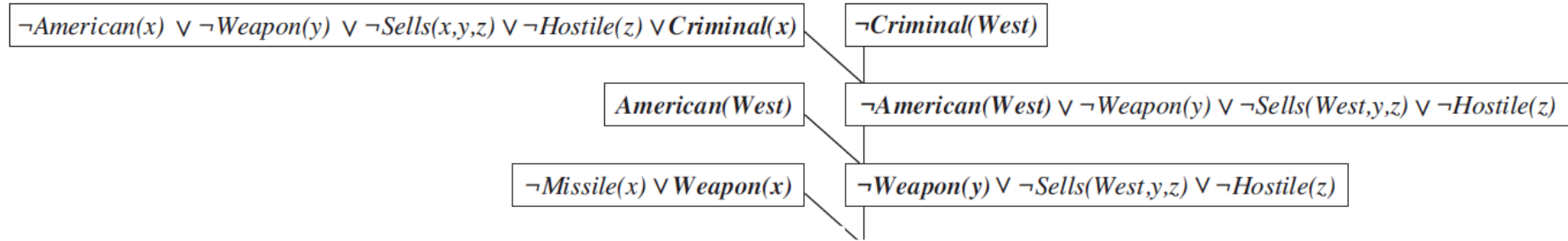
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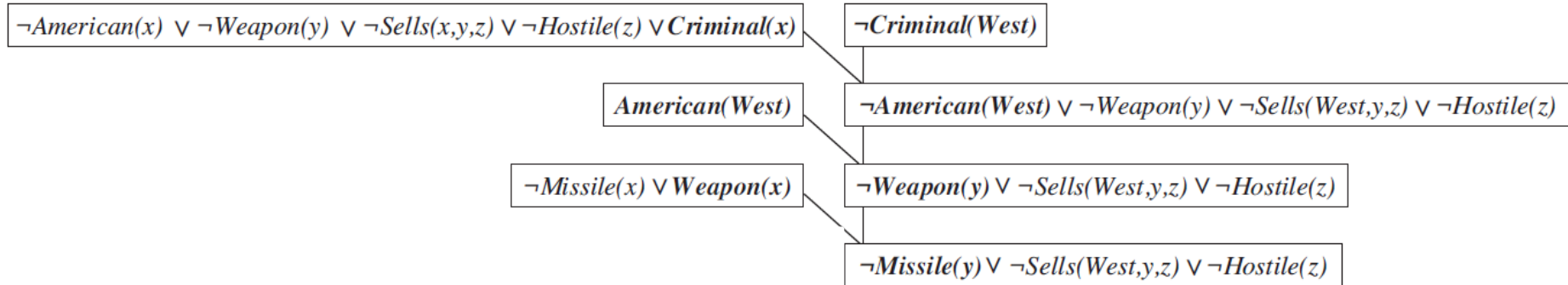
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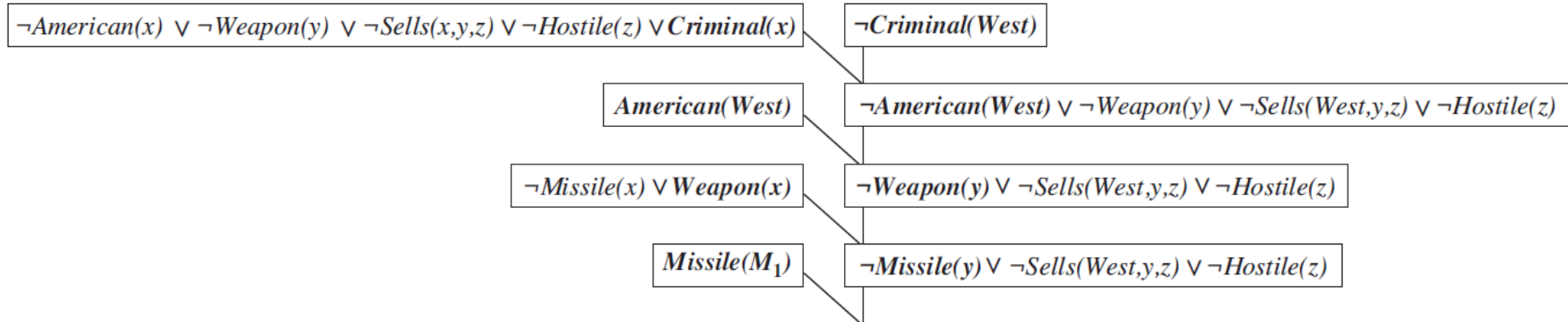
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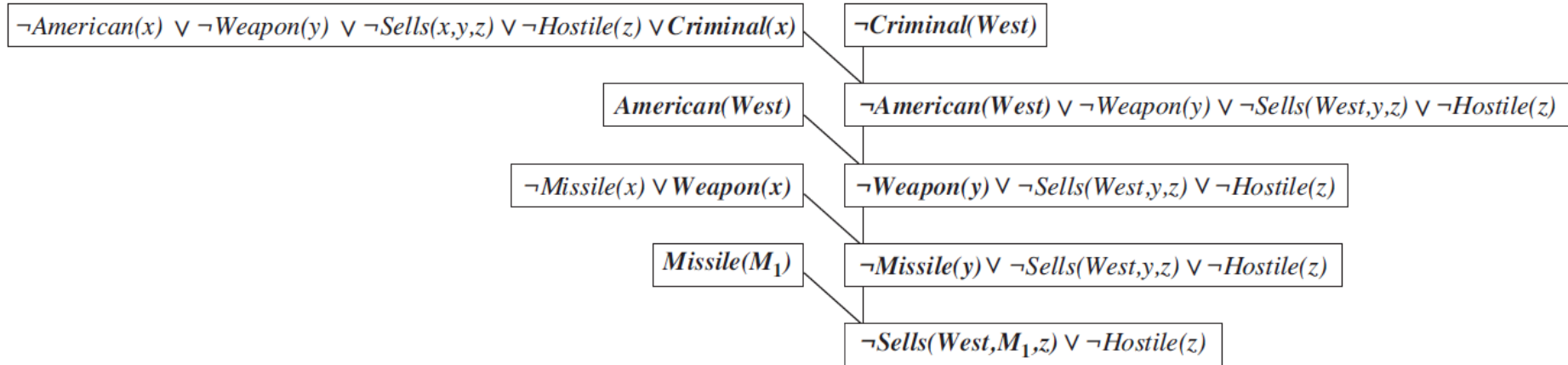
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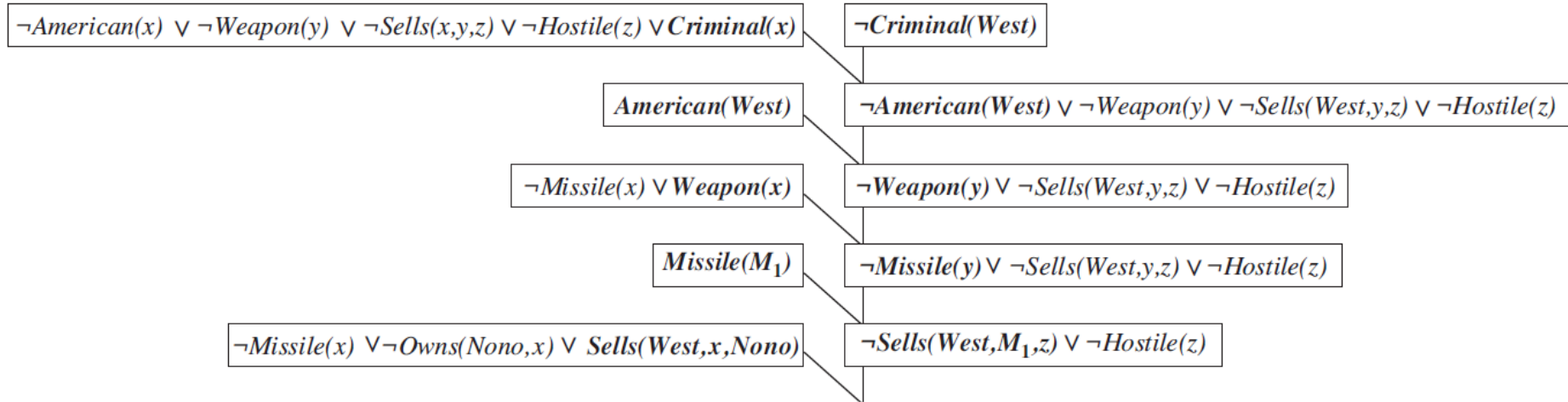
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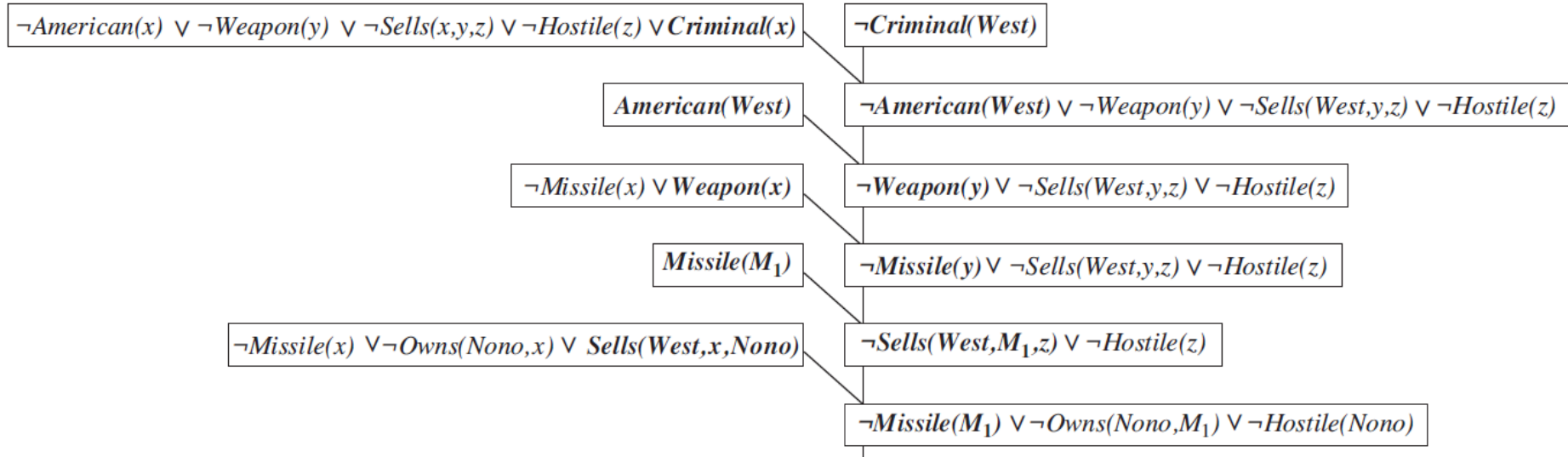
# Resolution



# Resolution

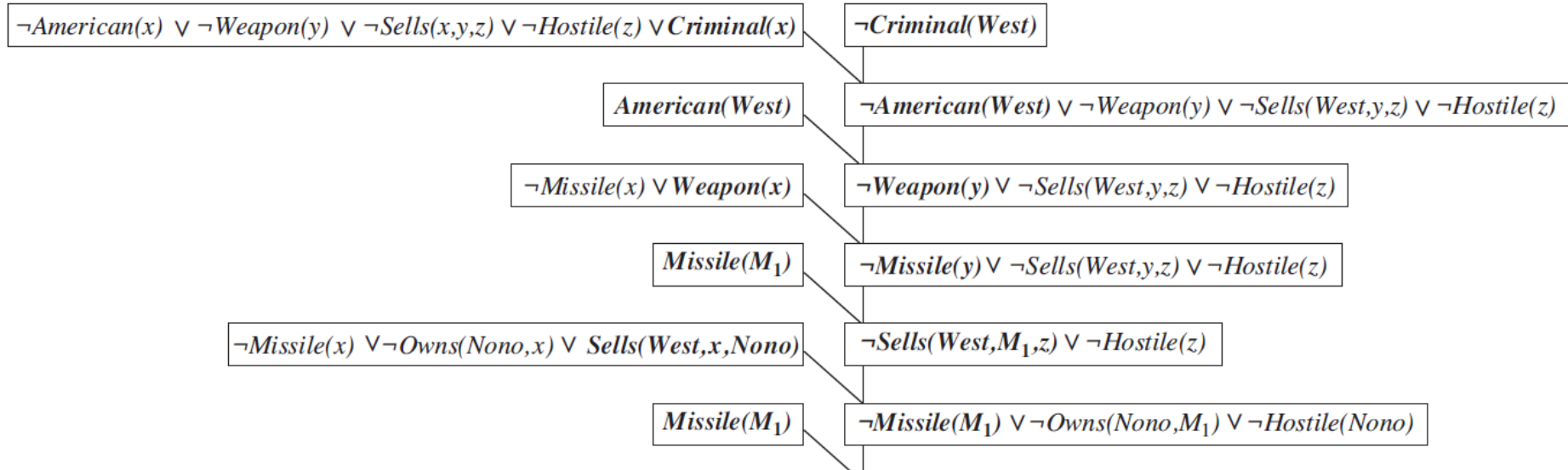


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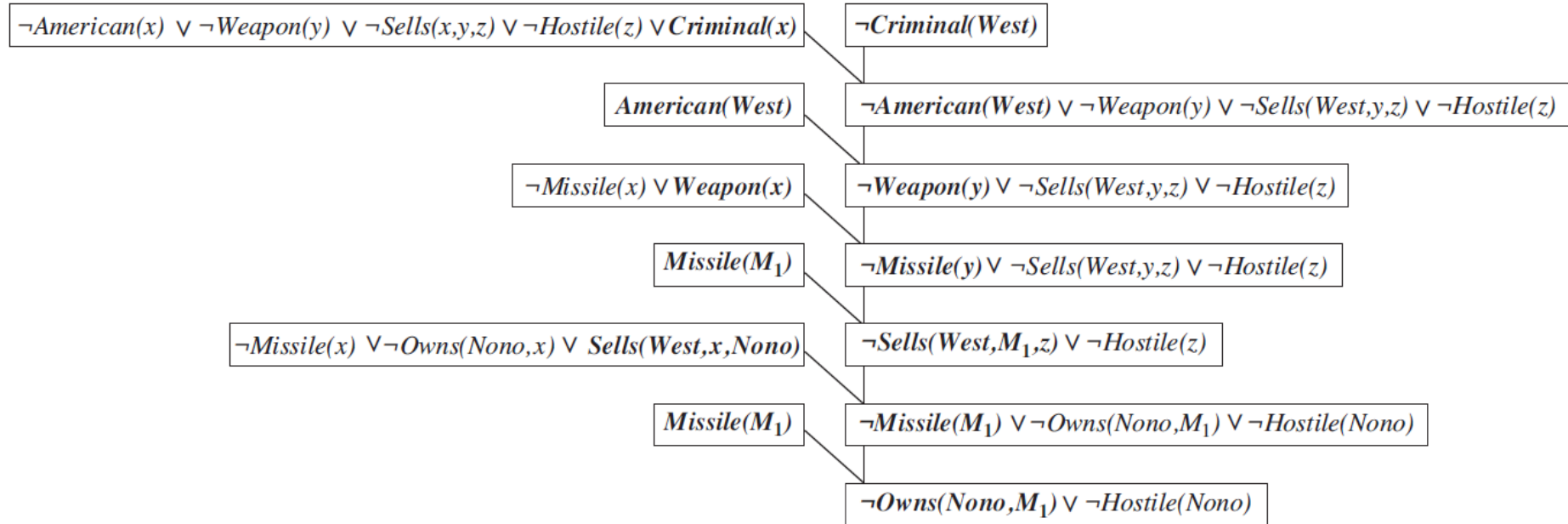




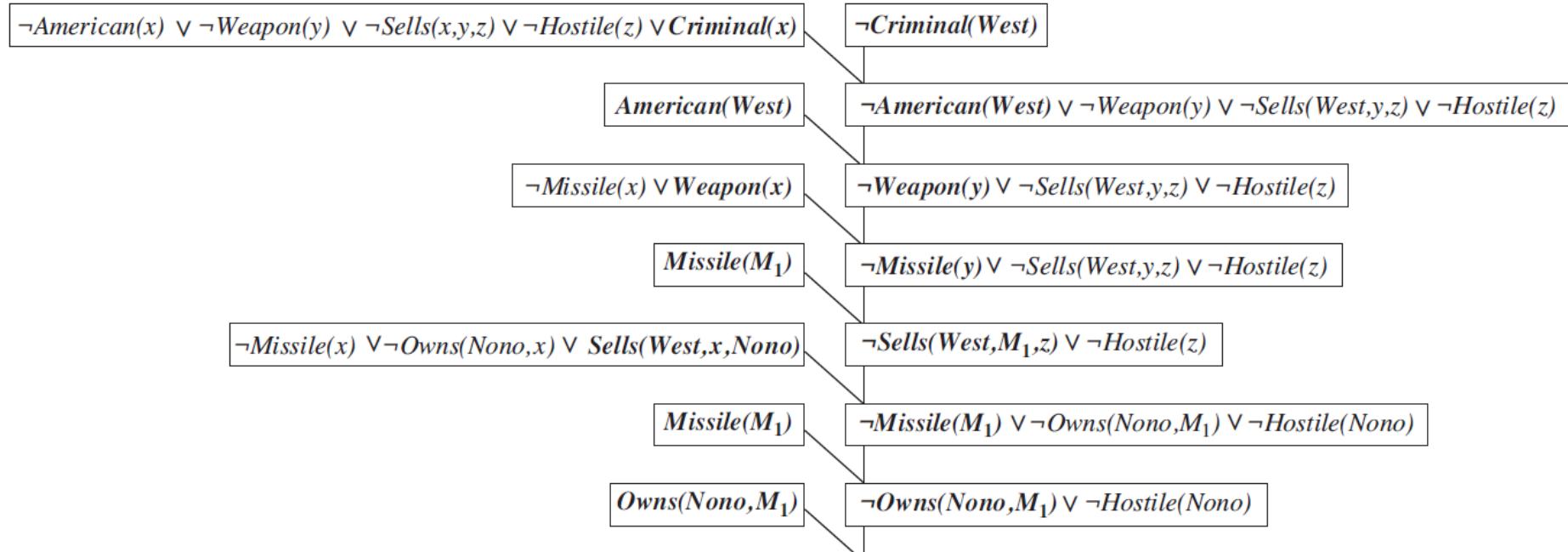
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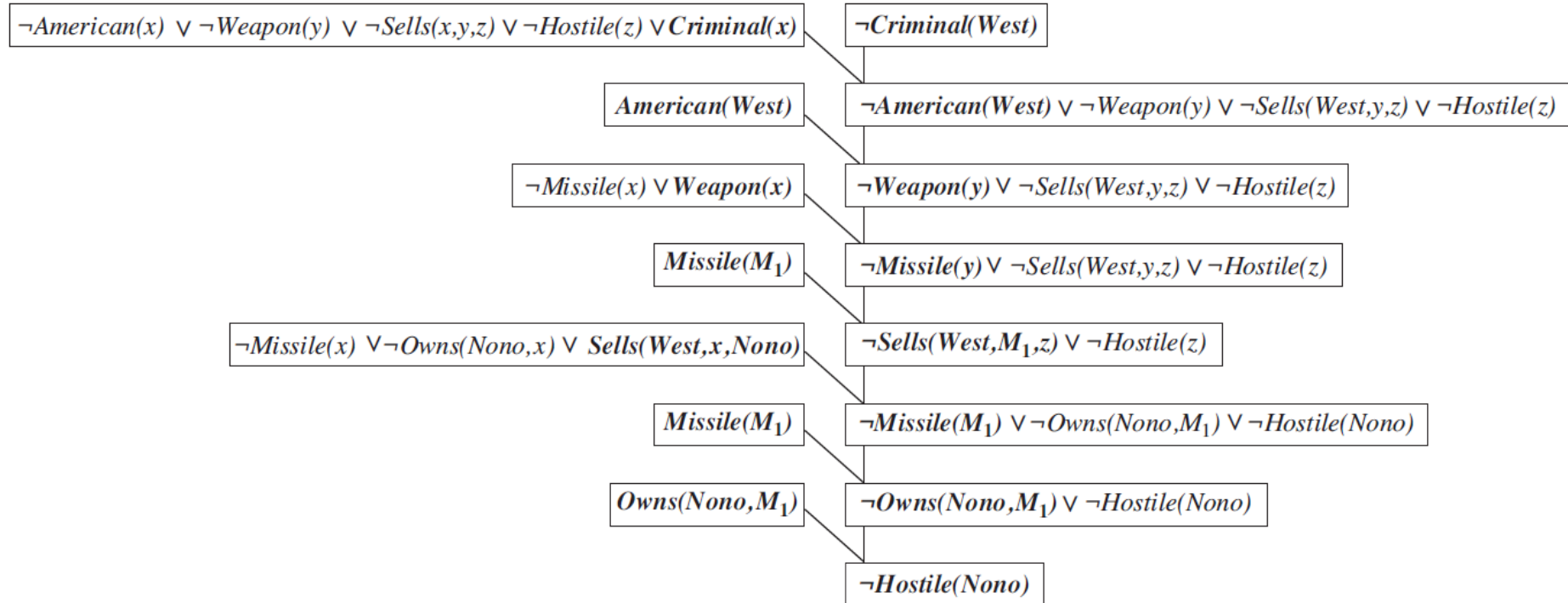
# Resolution



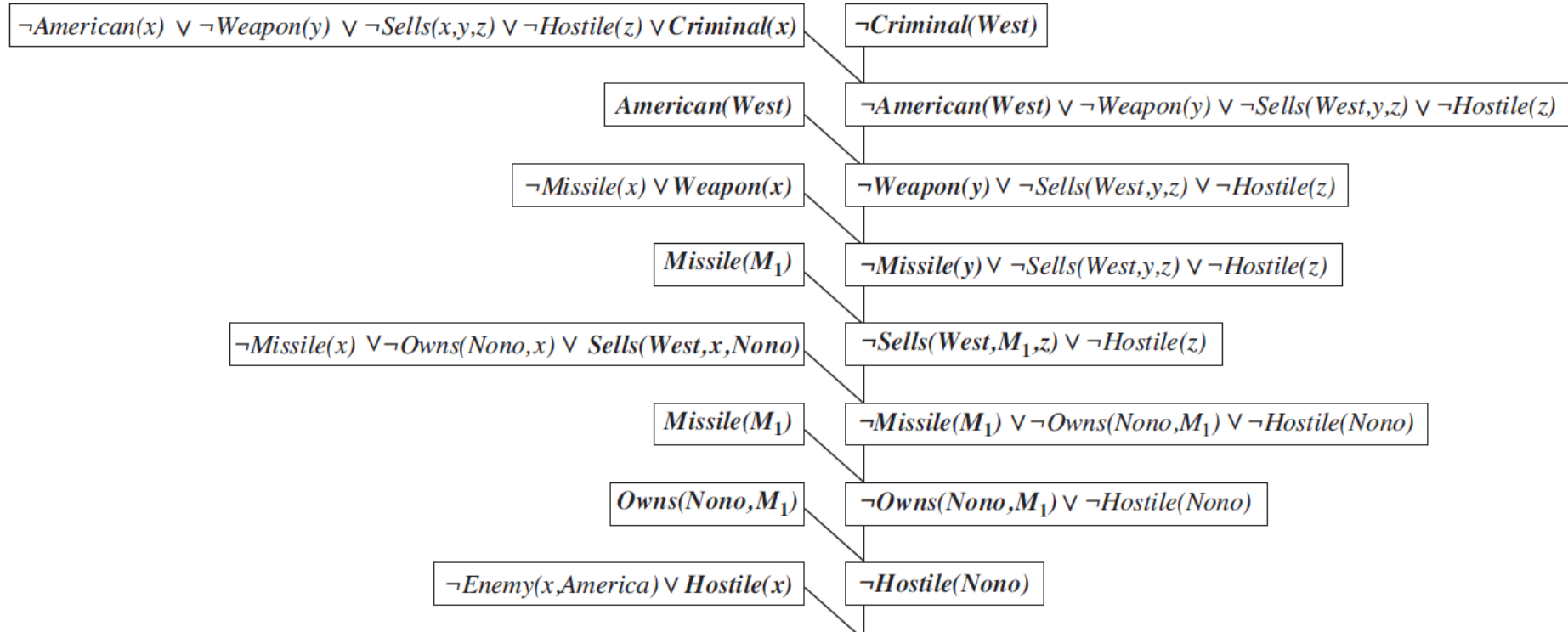
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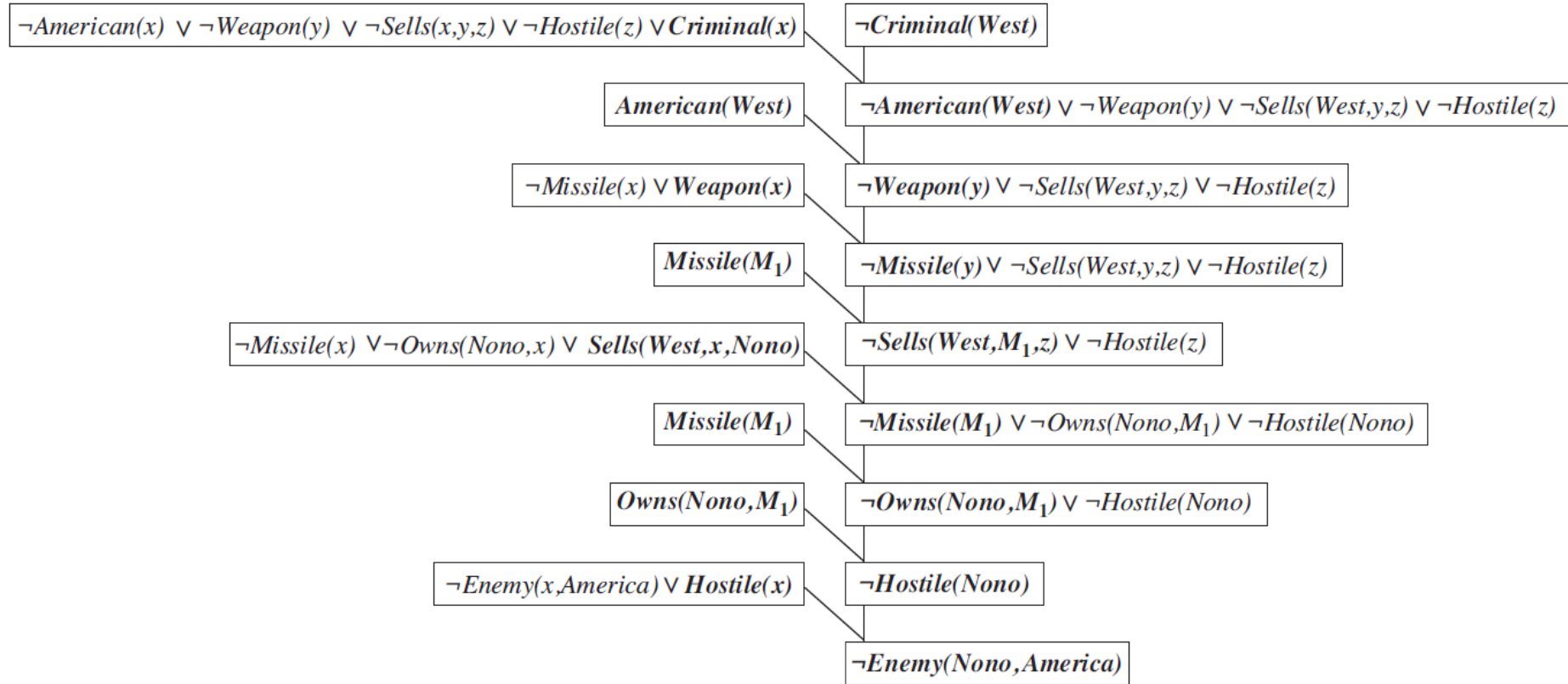
# Resolution



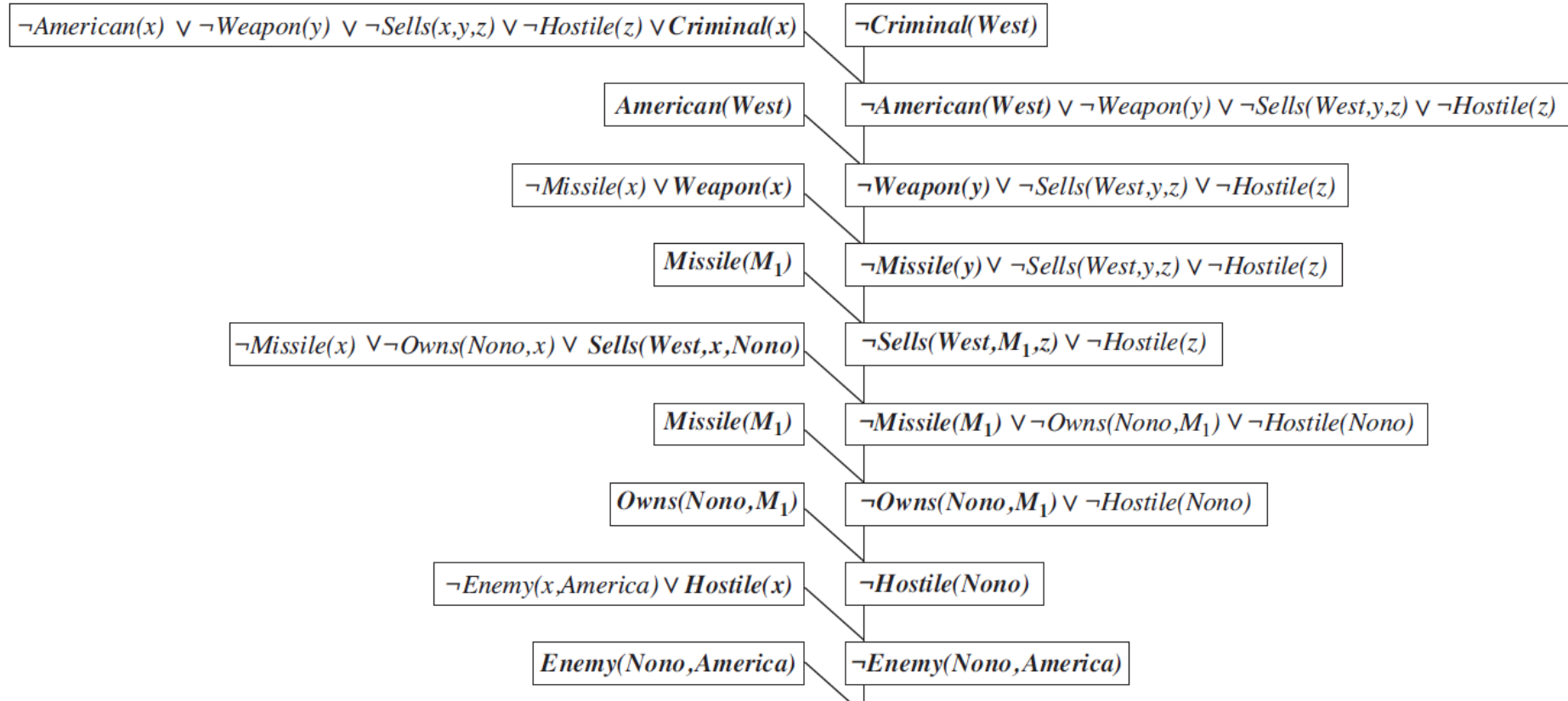
# Resolution



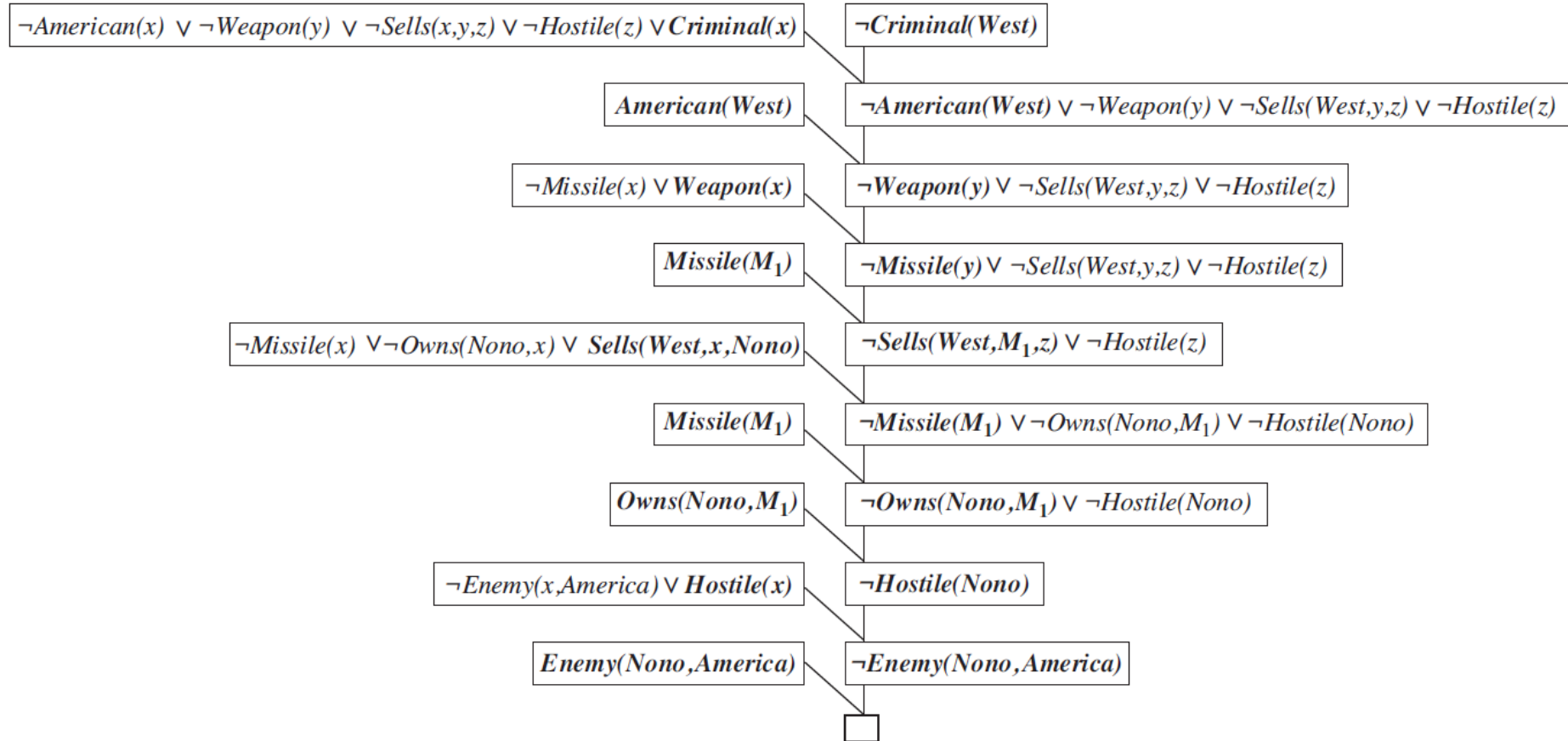
# Resolution



# Resolution

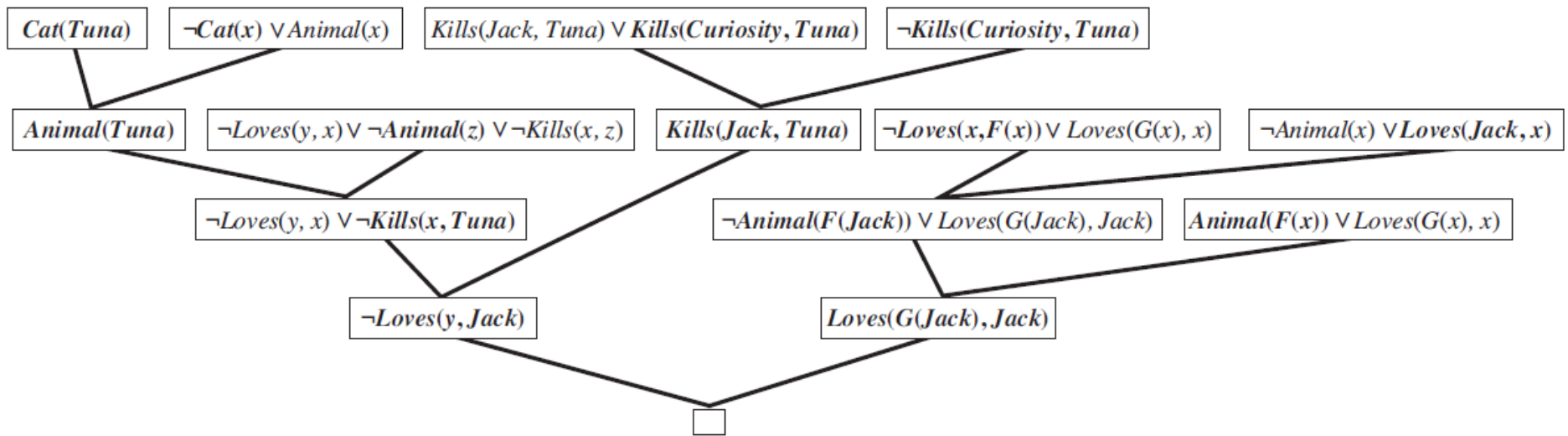


# Resolution





# Resolution



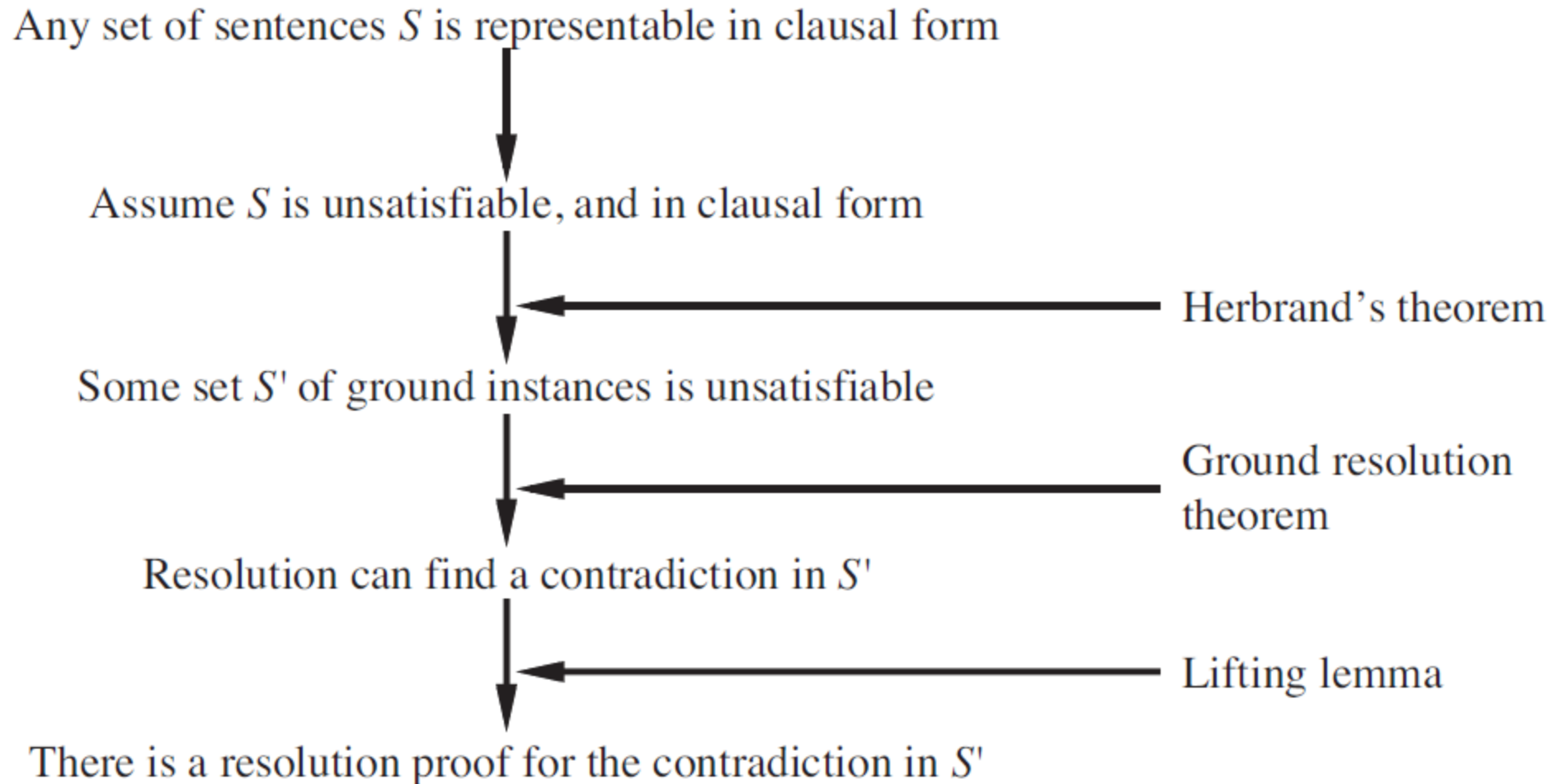
# What about $\exists$ ?



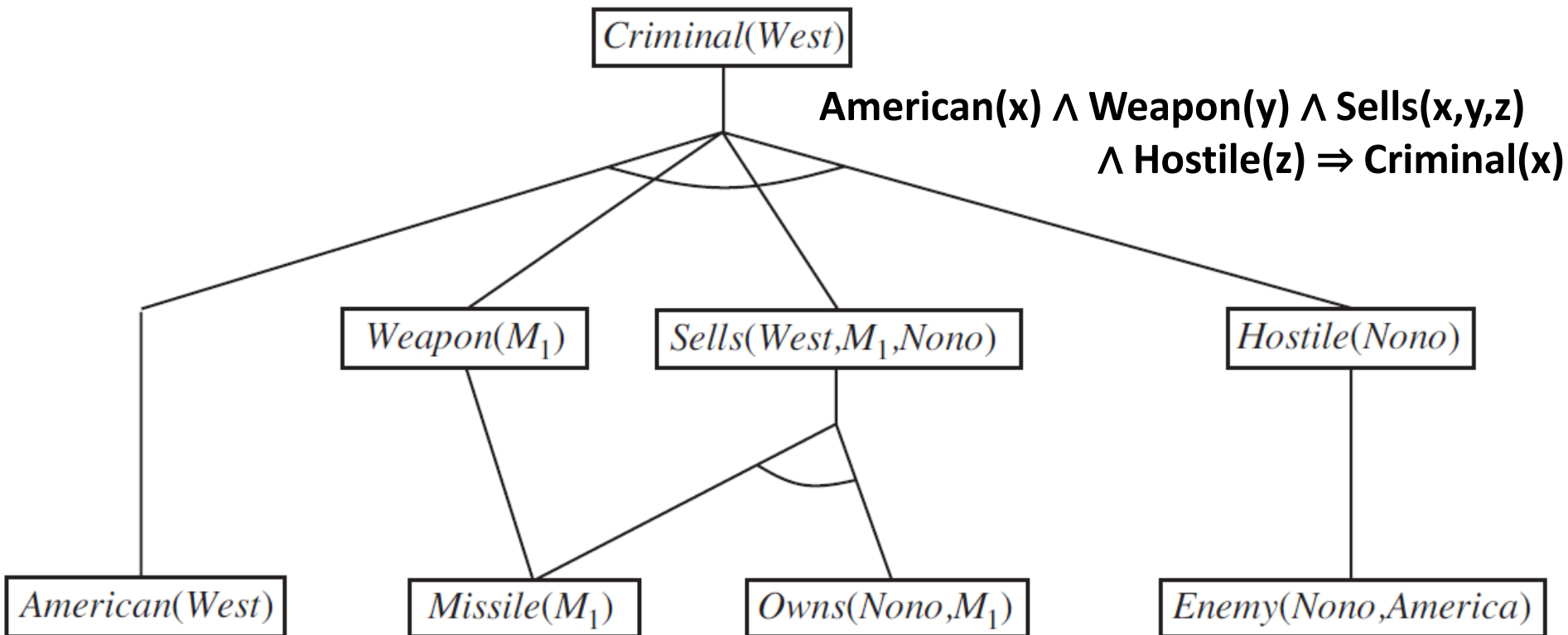
# Conversion to First-Order CNF



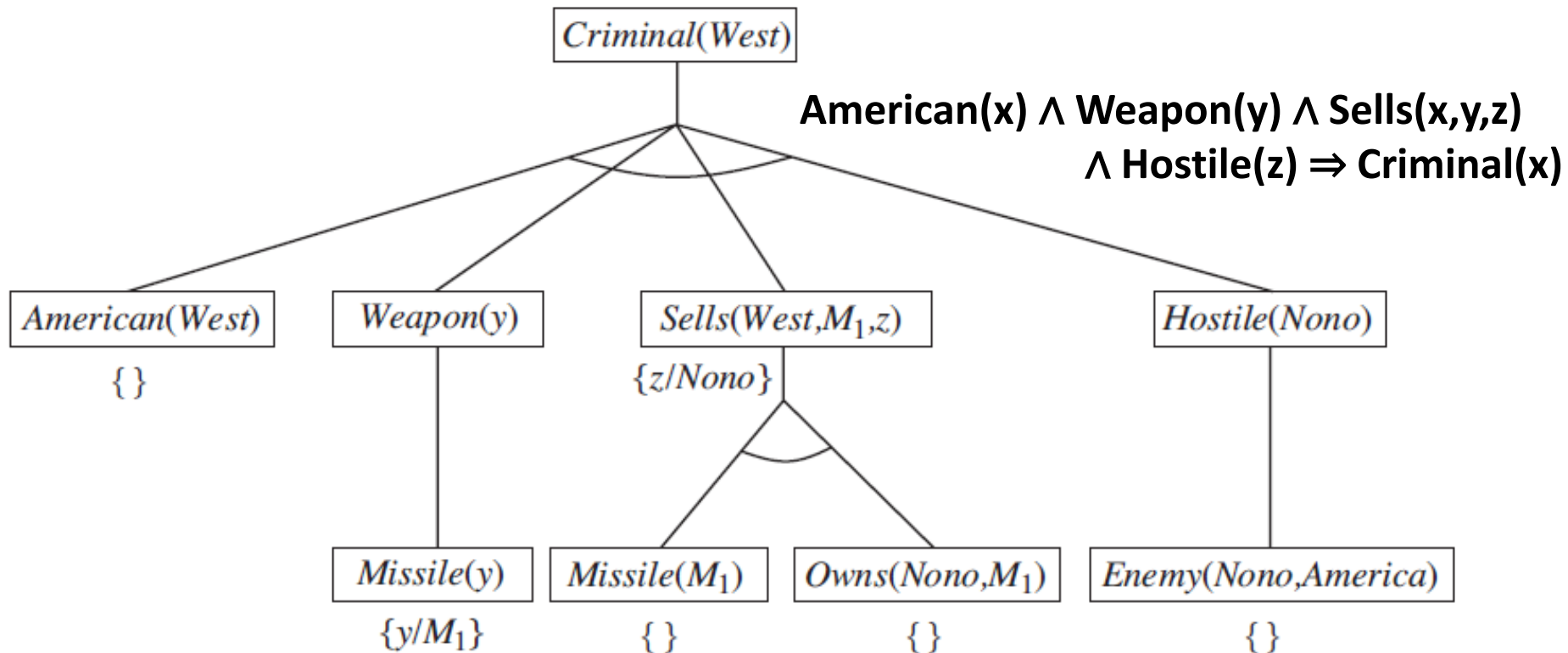
# Completeness of Resolution: Intuition



# Forward Chaining



# Backward Chaining



# Prolog

$\text{Link}(x,y) \Rightarrow \text{Path}(x,y)$

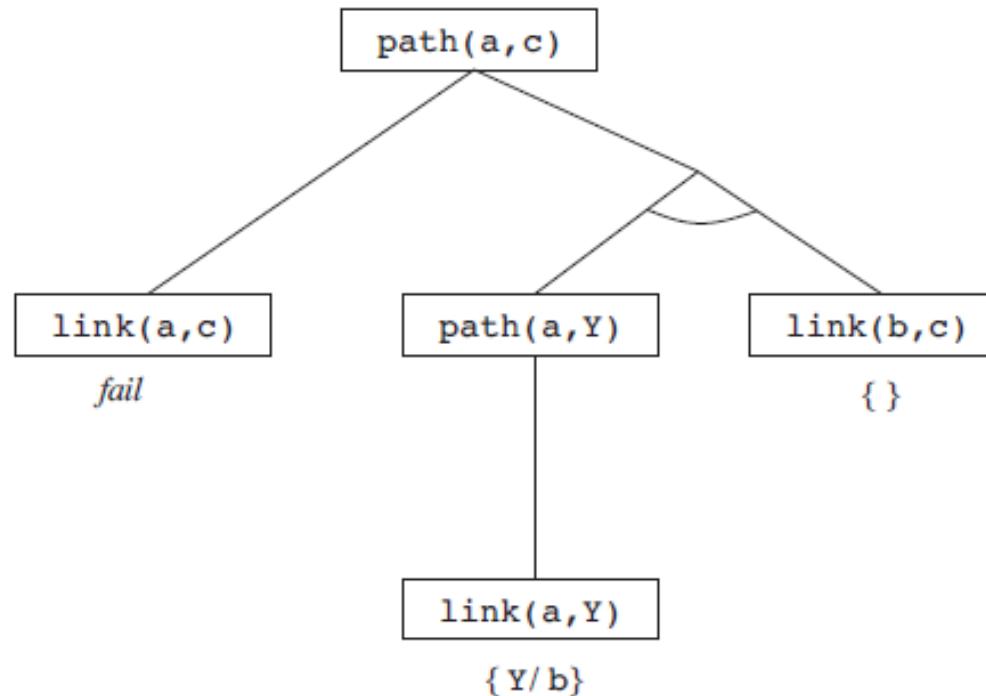
$\text{Link}(x,z) \wedge \text{Path}(z,y) \Rightarrow \text{Path}(x,y)$

$\text{Link}(a,b)$

$\text{Link}(a,d)$

$\text{Link}(b,c)$

...



# Prolog Programming

```
quicksort([X|Xs],Ys) :-  
    partition(Xs,X,Left,Right),  
    quicksort(Left,Ls),  
    quicksort(Right,Rs),  
    append(Ls,[X|Rs],Ys).  
quicksort([],[]).
```

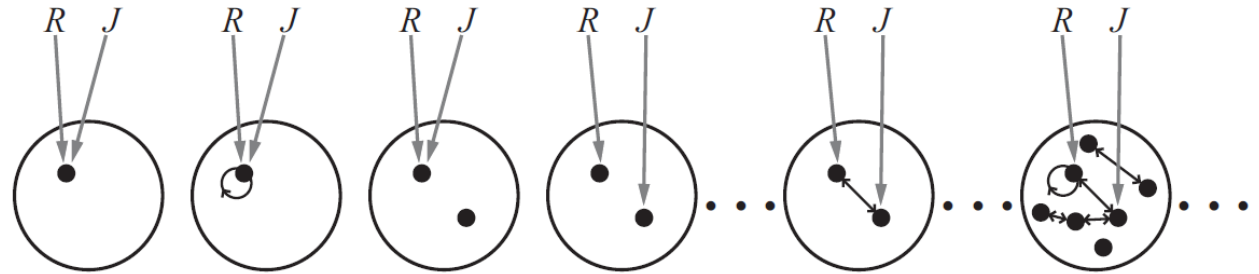
```
partition([X|Xs],Y,[X|Ls],Rs) :-  
    X <= Y, partition(Xs,Y,Ls,Rs).  
partition([X|Xs],Y,Ls,[X|Rs]) :-  
    X > Y, partition(Xs,Y,Ls,Rs).  
partition([],Y,[],[]).
```

```
append([],Ys,Ys).  
append([X|Xs],Ys,[X|Zs]) :-  
    append(Xs,Ys,Zs).
```

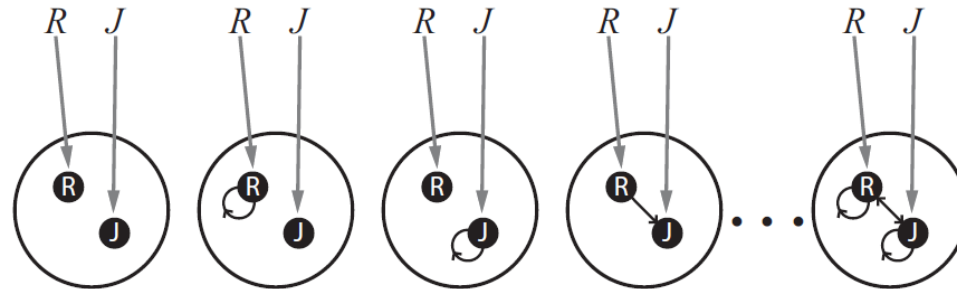


# Database Semantics (Prolog+SQL+...)

- Previous semantics:



- Now: unique names assumption



- Also: closed world and domain assumption

# Relational Databases

.... are just first-order logic!

SQL

or First-order logic

```
SELECT Scientist.X  
FROM Scientist, Coauthor  
WHERE Scientist.X = Coauthor.X
```

```
Q(x) =  
∃y Scientist(x) ∧ Coauthor(x,y)
```

Tables are relations

Tuples are positive literals (facts)

Select is exists, join is conjunction, etc.

# Why Relational Data?

- Our data is already relational!
  - Companies run relational databases
  - Scientific data is relational:
    - Large Hadron Collider generated 25PB in 2012
    - LSST Telescope will produce 30TB per night
- Big data is big business:
  - Oracle: \$7.1BN in sales
  - IBM: \$3.2BN in sales
  - Microsoft: \$2.6BN in sales

