Propositional Logic

CS161

Prof. Guy Van den Broeck

What are Logic-Based Systems?

- Structure: Reasoning 'engine'

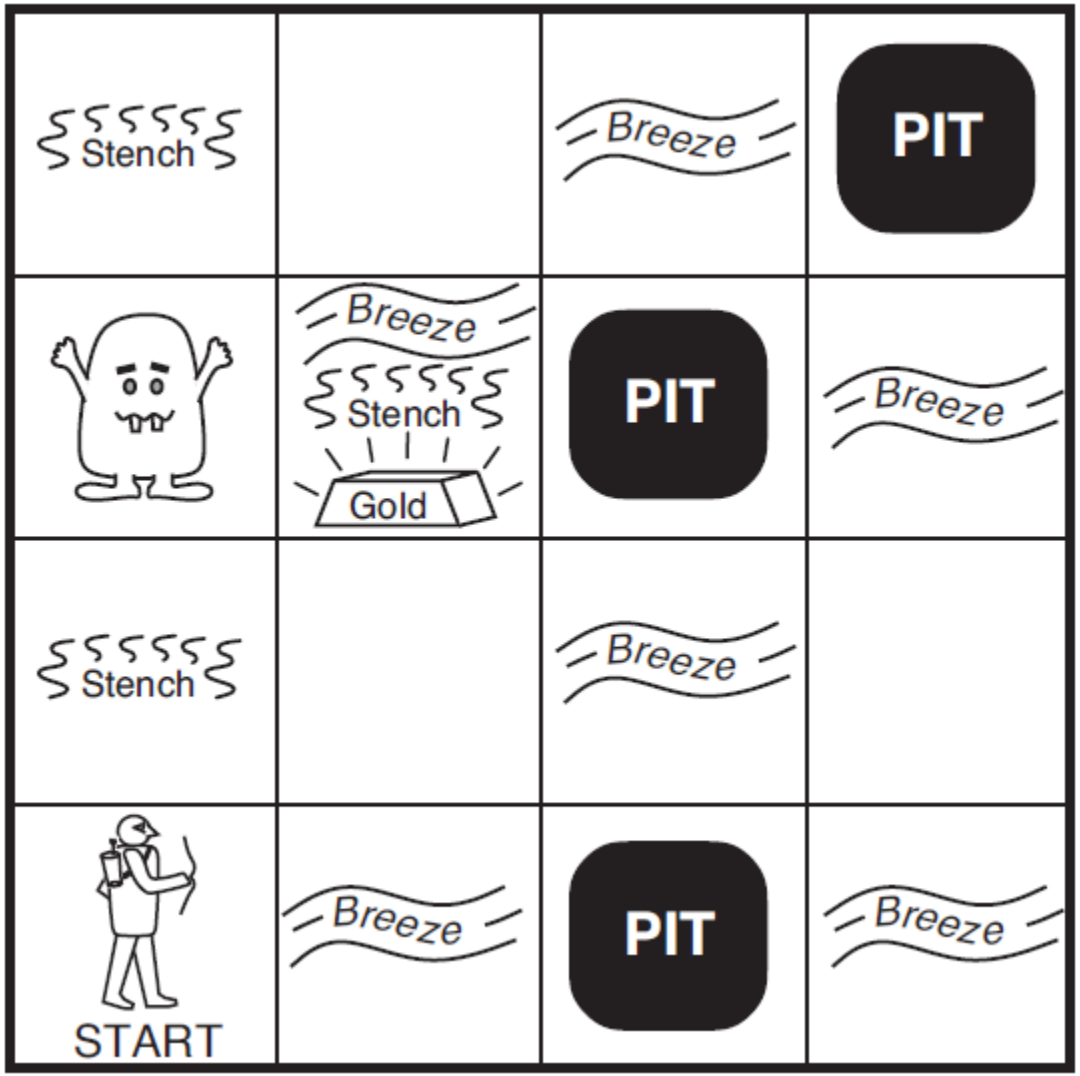
- Knowledge base: declarative sentences
- Dominant paradigm 1958-1988

  AI proved a new theorem in lattice theory!
- Current research on unifying logic, prob., neural

Wumpus World

- Grid world
- Pit causes breeze in adjacent cells
- Wumpus causes stench in adjacent cells
- Find the gold

Wumpus World



Wumpus World: Discussion

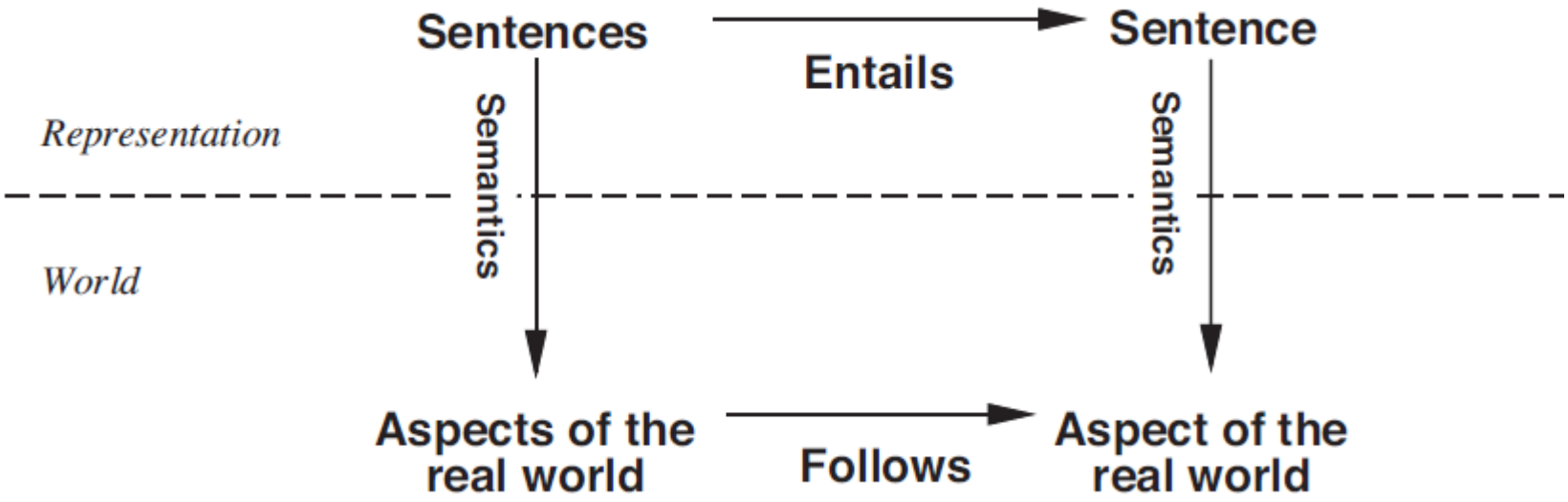
- Overcome ignorance about the world by reasoning
- Conclusions guaranteed to be correct
- Inferred new knowledge
  - from observations
  - from lack of observations
  - at different point in time

Syntax: What am I allowed to write?

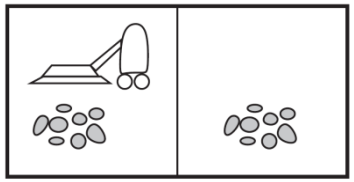
- x+5=y or x5y+=?
- Variables/propositions: X, Y, Z, Rain, Sun
- Grammar of sentences:
  - Variables are sentences (called atoms)
  - If α is a sentence, so is $\neg\alpha$
  - If α and β are sentences, so are

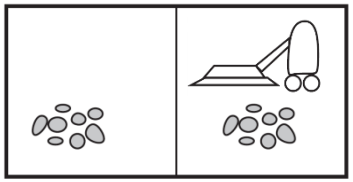
    $(\alpha \vee \beta), \alpha \wedge \beta, \alpha \Rightarrow \beta, \alpha \Leftrightarrow \beta$, … (not all required)
- Negated atoms are literals

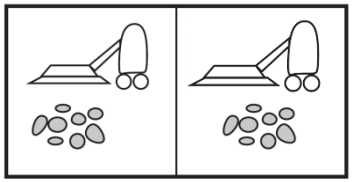
Semantics

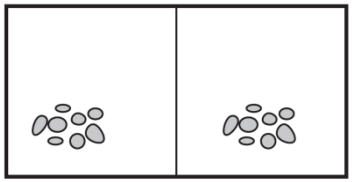

Sentences
Entails
Sentence
Representation
Semantics
Semantics
World
Aspects of the real world
Follows
Aspect of the real world

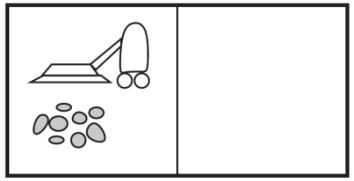
Worlds or Truth Assignments

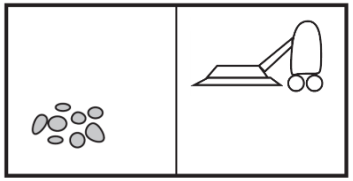
1 

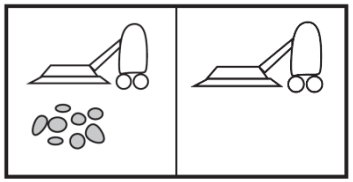
2 

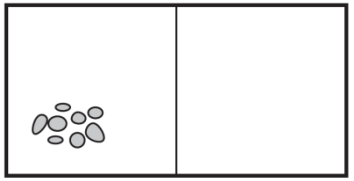
9 

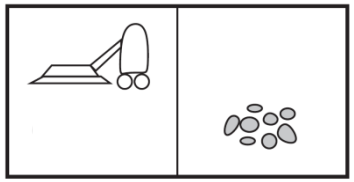
10 

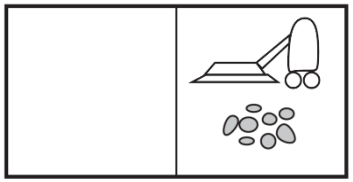
3 

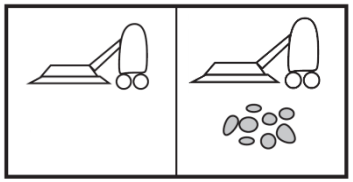
4 

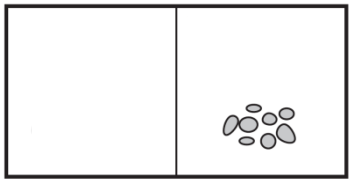
11 

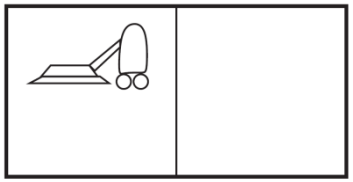
12 

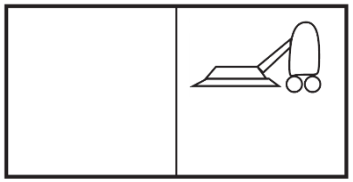
5 

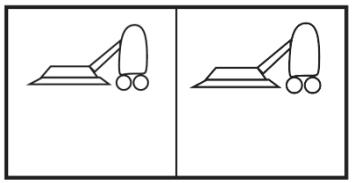
6 

13 

14 

7 

8 

15 

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Sentences and Worlds

- Syntax:
  LR, RR, LD, RD
- Exactly one robot: sentences α

- Relationship between sentence α and world ω
  - α is true at ω

$$\omega \vDash \alpha$$

α holds in ω

ω satisfies α

  - α is false at ω

$$\omega \nvDash \alpha$$

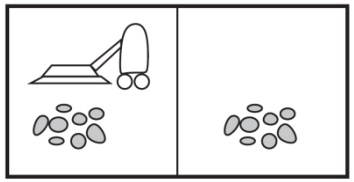
Semantics

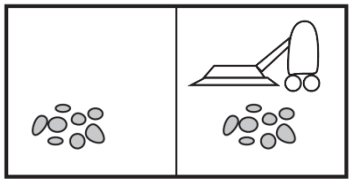
- Knowledge/meaning is a set of worlds ω
- Meaning of α is that the "set of worlds ω where α is true" are the only ones possible:

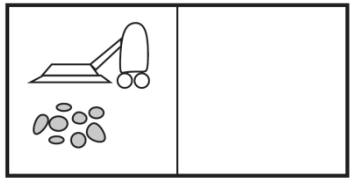
$$M(\alpha) = \{\omega : \omega \vDash \alpha\}$$

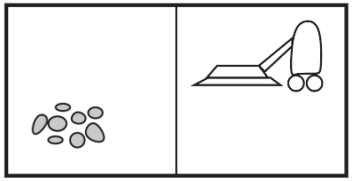
- Example:
  - Initially all 16 worlds

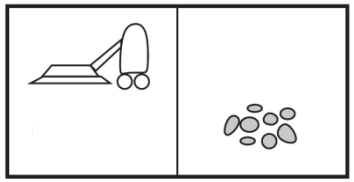
Meaning/Semantics

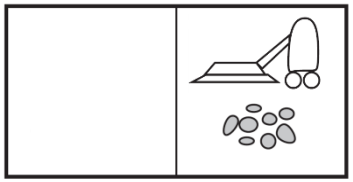
1 

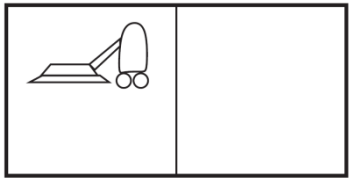
2 

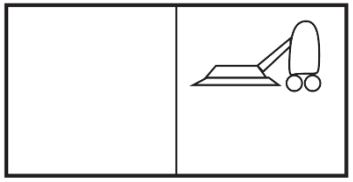
3 

4 

5 

6 

7 

8 

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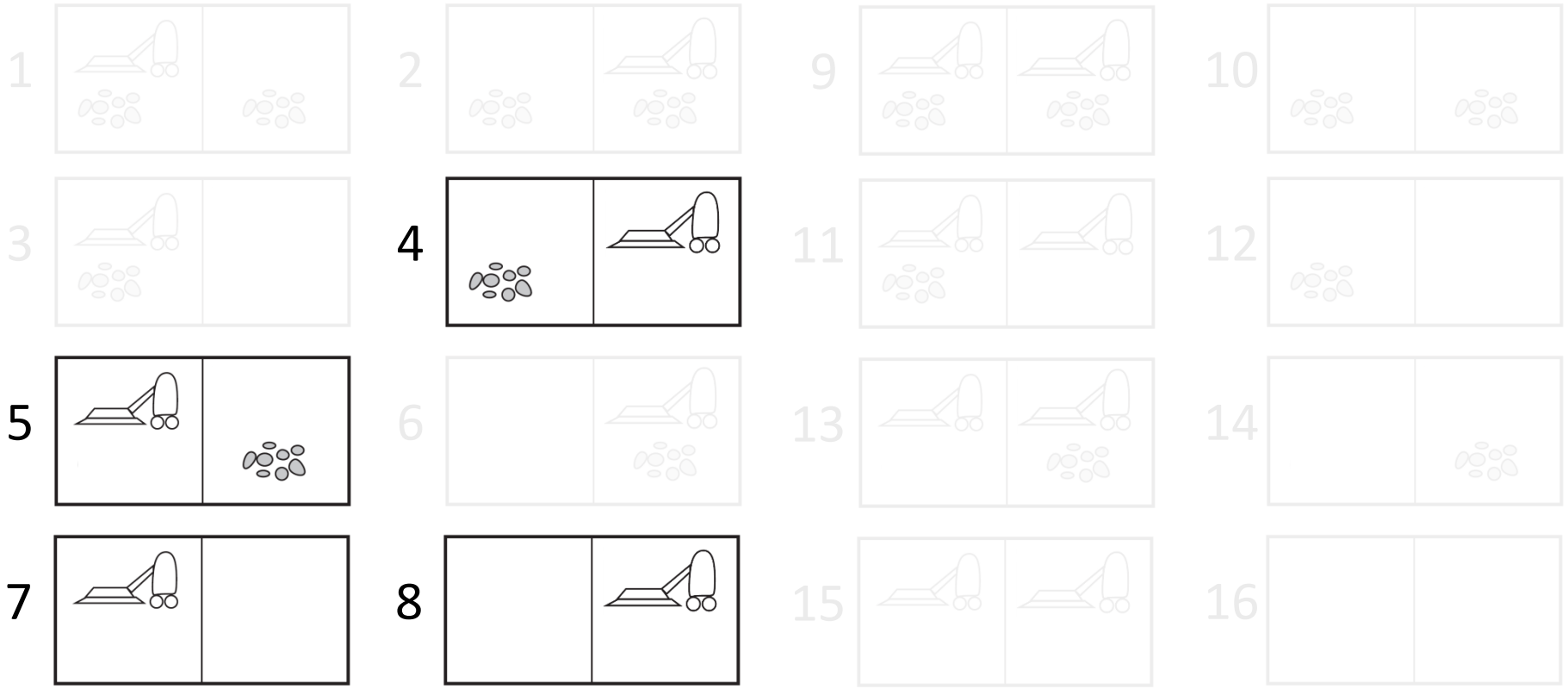
- Example:
  - Initially all 16 worlds
  - Sentence α removes 10,12,14,16 and 9,11,13,15
  - Sentence: Dirt and robot cannot co-exist

Meaning/Semantics



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  - Sentence: Dirt and robot cannot co-exist
  - Sentence: Robot in left cell

Meaning/Semantics

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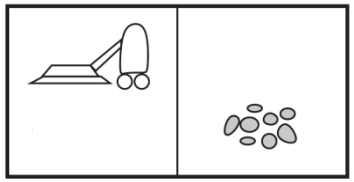

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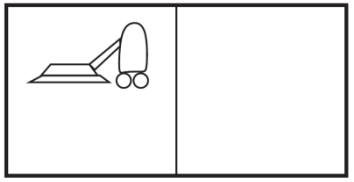

Formal Semantics + Algorithm

$$\omega \models \alpha?$$

Syntax Conversion Rules

Wumpus World Example

Relationships Between Sentences

Entailment

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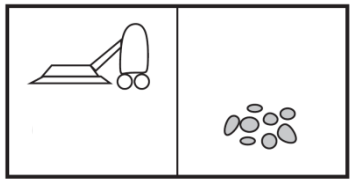

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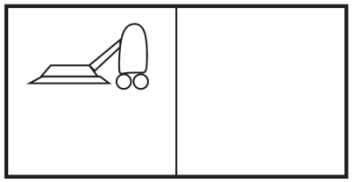
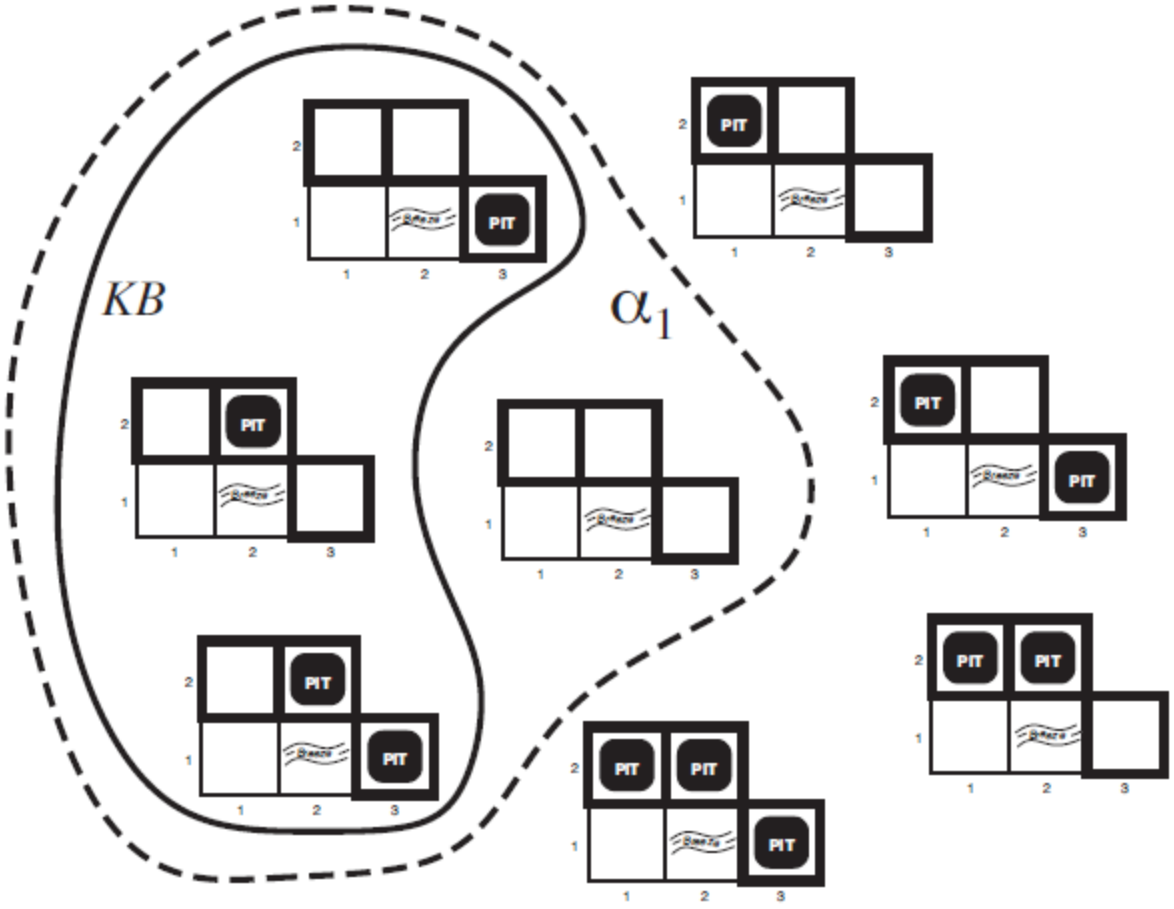
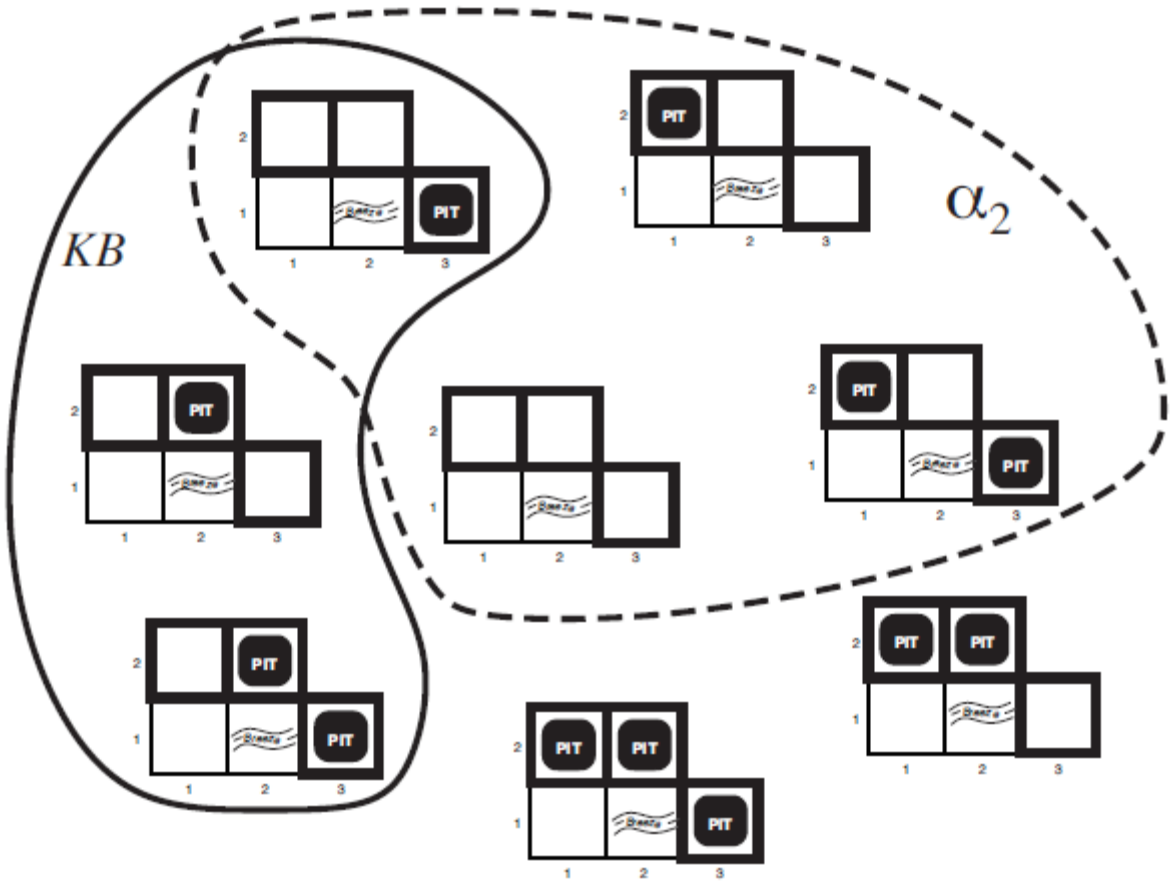



KB
α_1
PIT
1
2
3


KB
α_2
PIT
Breeze

Propositional Reasoning