CS161: Homework #6
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Problem 1

(a) $P(A, B, B), P(x, y, z)$
Unifier = $\{x/A, y/B, z/B\}$

(b) $Q(y, G(A, B)), Q(G(x, x), y)$
No unifier exists. The y variable of the second sentence is forced to be $G(A, B)$. However, there's no possible way $G(x, x)$ of the second sentence can be a function of two separate variables. So this is a contradiction.

(c) $Older(Father(y), y), Older(Father(x), John)$
Unifier = $\{x/John, y/John\}$

(d) $Knows(Father(y), y), Knows(x, x)$
No unifier exists. $Knows(x, x)$ forces the two variables of $Knows()$ to be the same. However, in the first atomic sentence, $Knows(Father(y), y)$ has two different variables, one of which is a function of the other. Therefore, this causes a contradiction.

Problem 2

(a) (i)

$$\forall x Food(x) \Rightarrow Likes(John, x)$$

(ii)

$$Food(Apples)$$

(iii)

$$Food(Chicken)$$

(iv)

$$\forall z \forall y (Eats(y, z) \land \neg Sickens(z, y)) \Rightarrow Food(z)$$

(v)

$$\forall r \forall s Sickens(r, s) \Rightarrow \neg Well(s)$$

(vi)

$$Eats(Bill, Peanuts) \land Well(Bill)$$

(vii)

$$\forall t Eats(Bill, t) \Rightarrow Eats(Sue, t)$$

(b) First, we turn the above sentences into disjunctions.

(i)
$$\neg Food(x) \vee Likes(John, x)$$

(ii)
$$Food(Apples)$$

(iii)
$$Food(Chicken)$$

(iv)
$$\neg Eats(y, z) \vee Sickens(z, y) \vee Food(x)$$

(v)
$$\neg Sickens(r, s) \vee \neg Well(s)$$

(vi)
$$Eats(Bill, Peanuts)$$

(vii)
$$Well(Bill)$$

(viii)
$$\neg Eats(Bill, t) \vee Eats(Sue, t)$$

We now assume John doesn't like Apples.
(ix)
$$\neg Likes(John, Apples)$$

Resolve (ix) with (i) we get:
(x)
$$\neg Food(Apples)$$

We resolve (x) and (ii) and get an empty clause. So, John likes apples.

We then assume that John doesn't like Chicken.
(xi)
$$\neg Likes(John, Chicken)$$

Resolve (xi) with (i) we get:
(xii)
$$\neg Food(Chicken)$$

We resolve (xii) and (ii) and get an empty clause. So, John likes chicken.

(c) We assume that Sue doesn't eat peanuts.
(xiii)
$$\neg Eats(Sue, Peanuts)$$

We use unifier $\{t/Peanuts\}$ to resolve (xiii) and (viii) and gets:
(xiv)
$$\neg Eats(Bill, Peanuts)$$

We resolve (xiv) with (vi) and get an empty clause. So, Sue eats peanuts.

Problem 3

We first convert the sentences into disjunctives:

(i)
$$Mother(Mary, Tom)$$

(ii)
$$Alive(Mary)$$

(iii)
$$\neg Mother(x, y) \vee Parent(x, y)$$

(iv)
$$\neg Parent(x, y) \vee \neg Alive(x) \vee Older(x, y)$$

We resolve (iv) with (iii) and get:
(v)
$$\neg Mother(x, y) \vee \neg Alive(x) \vee Older(x, y)$$

We use unifier $\{x/Mary\}$ to resolve (ii) and (v) and get:
(vi)
$$\neg Mother(Mary, y) \vee Older(Mary, y)$$

We use unifier $\{y/Tom\}$ to resolve (i) and (iv), we get:
(vii)
$$Older(Mary, Tom).$$

Thus, Mary is older than Tom. If we prove by refutation, and assume $\neg Older(Mary, Tom)$. we resolve it with (vii) to get an empty clause, which is another way to prove.

Problem 4

(a) $H(Origin) = B(\frac{p}{p+n}) = B(\frac{2}{5})$=0.97
$Remainder(A_1) = H(A_1 = 1) + H(A_1 = 0) = \frac{4}{5}B(\frac{1}{2}) + \frac{1}{5}B(0) = \frac{4}{5} = 0.8$
$Gain(A_1) = H(Origin) - Remainder(A_1) = 0.97 - 0.8 = 0.17$
$Remainder(A_2) = H(A_2 = 1) + H(A_2 = 0) = \frac{3}{5}B(\frac{2}{3}) + \frac{2}{5}B(0) = 0.552$
$Gain(A_2) = H(Origin) - Remainder(A_2) = 0.97 - 0.55 = 0.42$
$Reamainder(A_3) = H(A_3 = 1) + H(A_3 = 0) = \frac{2}{5}B(\frac{1}{2}) + \frac{3}{5}B(\frac{1}{3}) = 0.4 + 0.552 = 0.952$
$Gain(A_3) = H(Origin) - Remainder(A_3) = 0.97 - 0.95 = 0.02$
$Gain(A_2) > Gain(A_1) > Gain(A_3)$
So we choose A_2 to split on.
The tree is presented as below: