Constraint Satisfaction

CS161
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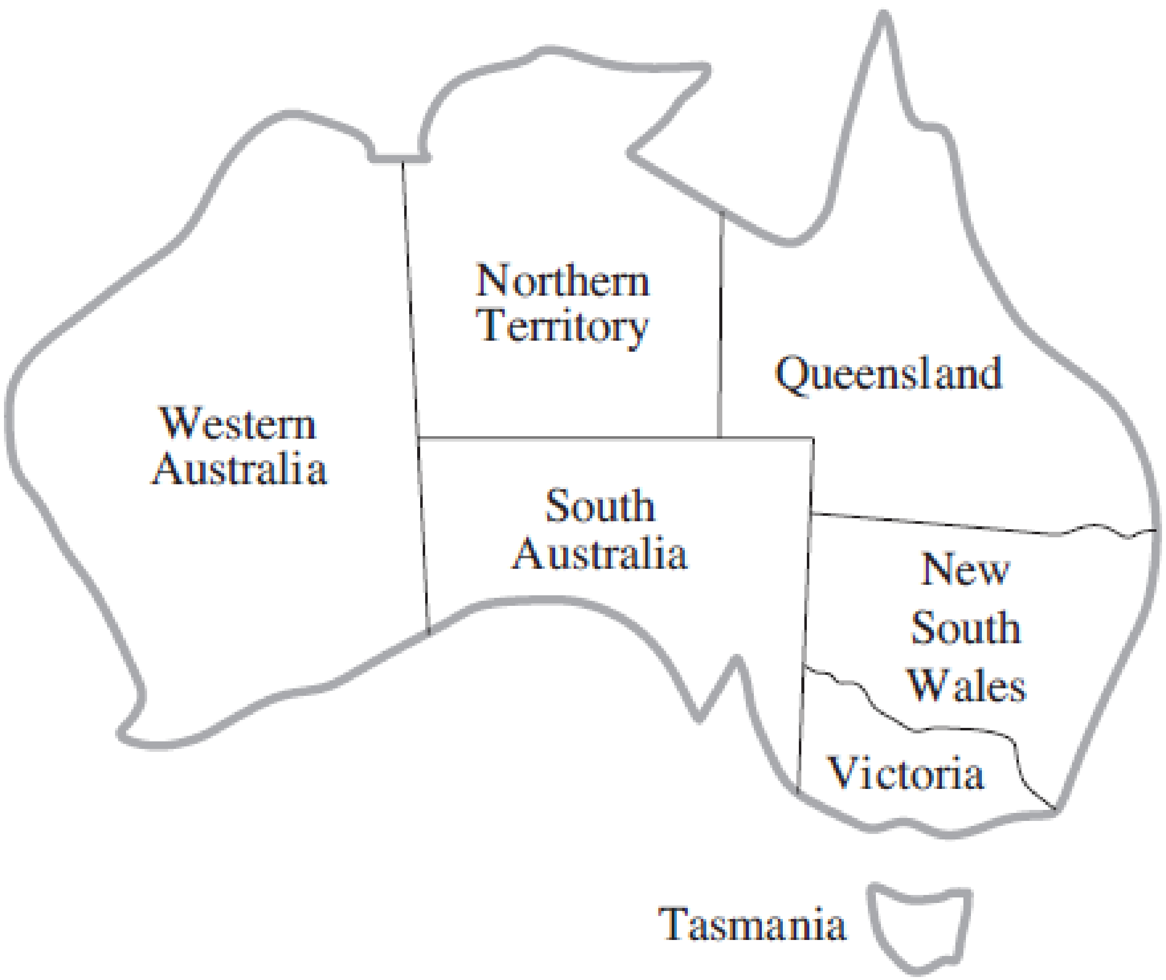
Constraint Satisfaction Problems

- What? n-queens, cryptoarithmetic, map coloring, SAT, etc.
- Factored problem formulation

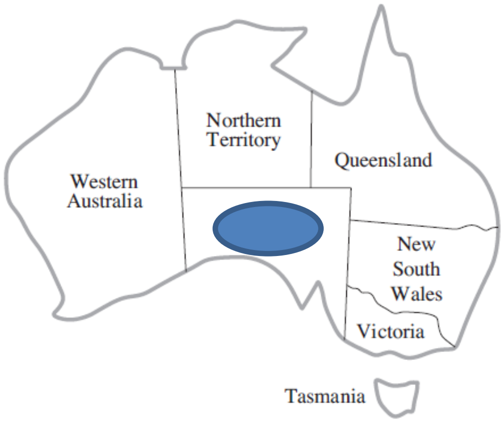
  Look inside state structure (no longer black box)
- Representation (formal language)
  - Variables X
  - Domains D
  - Constraints (=goal test)
- More powerful general-purpose solvers

Example: Graph Coloring



What is the benefit of a factorized representation?

- Example: Coloring Australia
  - Suppose we know that SA=blue
  - Still 3^6 possible states left
  - No neighboring state can be blue
  - Really only $2^5 \cdot 3$ states left!



- Check inconsistency early
- Be smarter about which actions to try

Constraint Satisfaction Problems

- Constraint types
  - Unary
  - Binary
  - Higher-order
  - Global constraints
  - Soft constraints

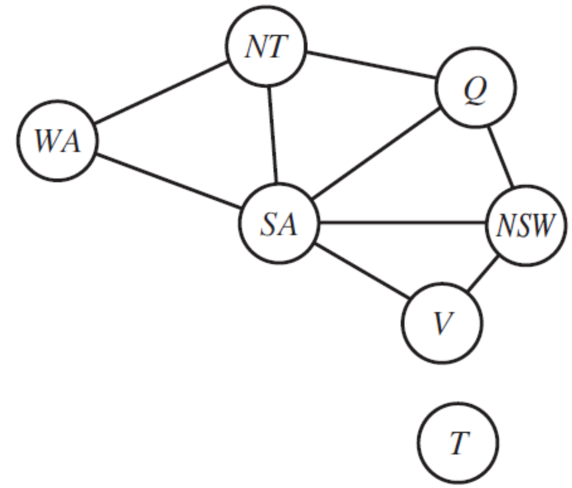
- Linear/Mathematical programming

Examples

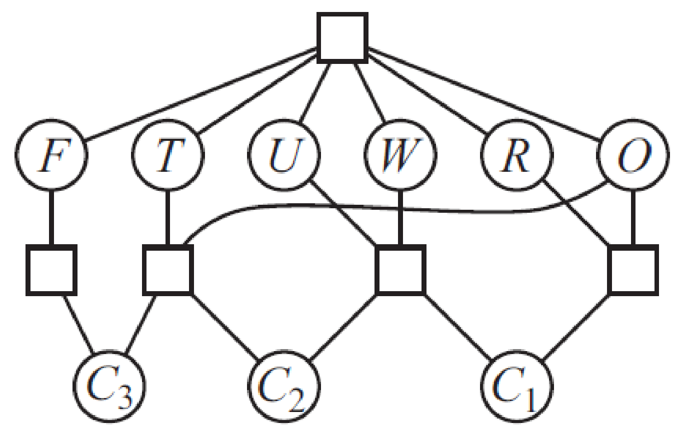
- Cryptarithmetic
- SAT solving

Constraint graphs

- Binary constraints:
  Binary constraint graphs



- Higher-order constraints:
  Constraint hypergraphs/
  bipartite graphs



Real-World Examples

- Assignment problems (who teaches this class)
- Timetabling problems (when and where)
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Floorplanning
- Etc.

CSP Search, the Naïve Way

- Initial state: empty assignment {}
- Actions: assign to a variable X a value v
  - Any choice of unassigned X
  - Any choice of v
  - Only allowed if no constraint becomes violated
- Successor function: add {X=v} to state
- Goal test:
  - all variables get assignment
  - all constraints are satisfied

CSP Search, the Naïve Way

- Search tree
- Which search algorithm to use?
  - Why not IDS? DLS? BFS?
  - DFS is perfect for CSP: optimal and complete
- Complexity?
  - Number of variables N=|X|
  - Number of values V=|D|
  - Size of tree:
    $(NV) \cdot \big((N-1)V\big) \cdot \big((N-2)V\big) \cdot \ldots = N!\,V^N$
  - Note: only V^N distinct states! ☹

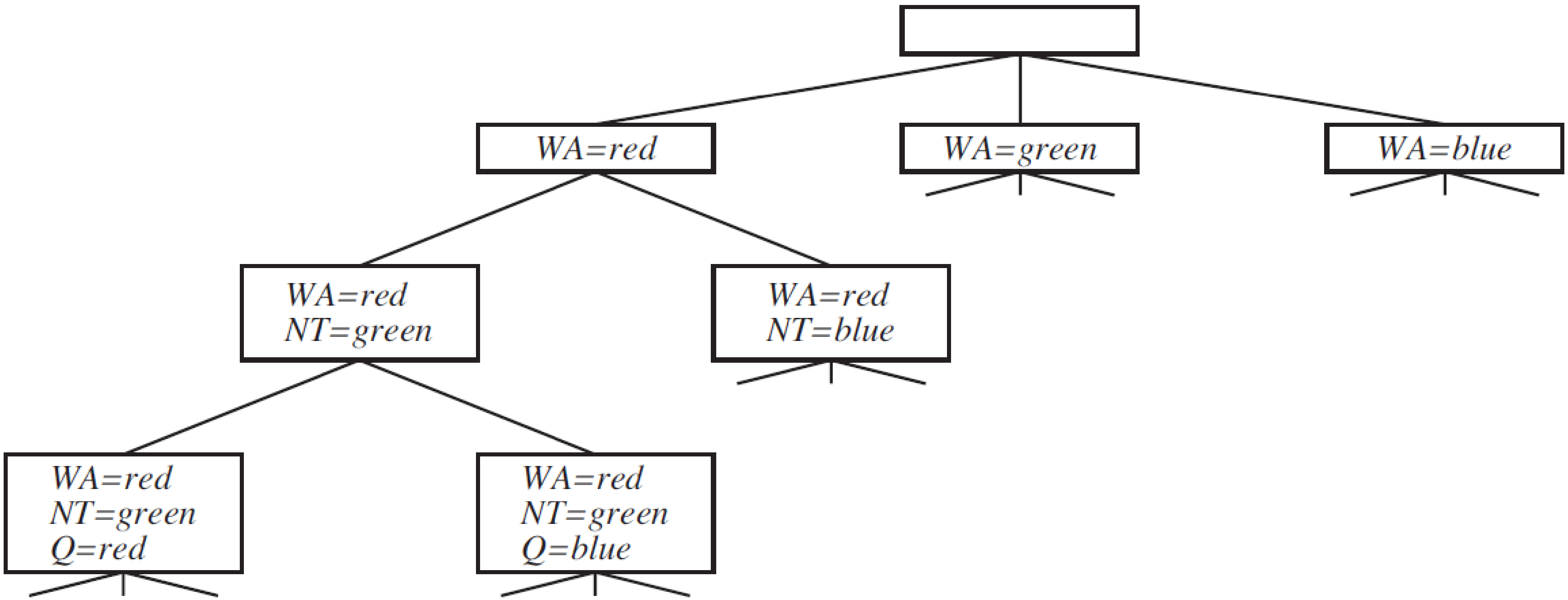
Can we do better?

- Variable assignments are commutative

- Per node, only need consider assignment to **one** variable
- Branching factor V
- Complexity: V^N

- This is called “backtracking search”.

Backtracking Search



Improvements?

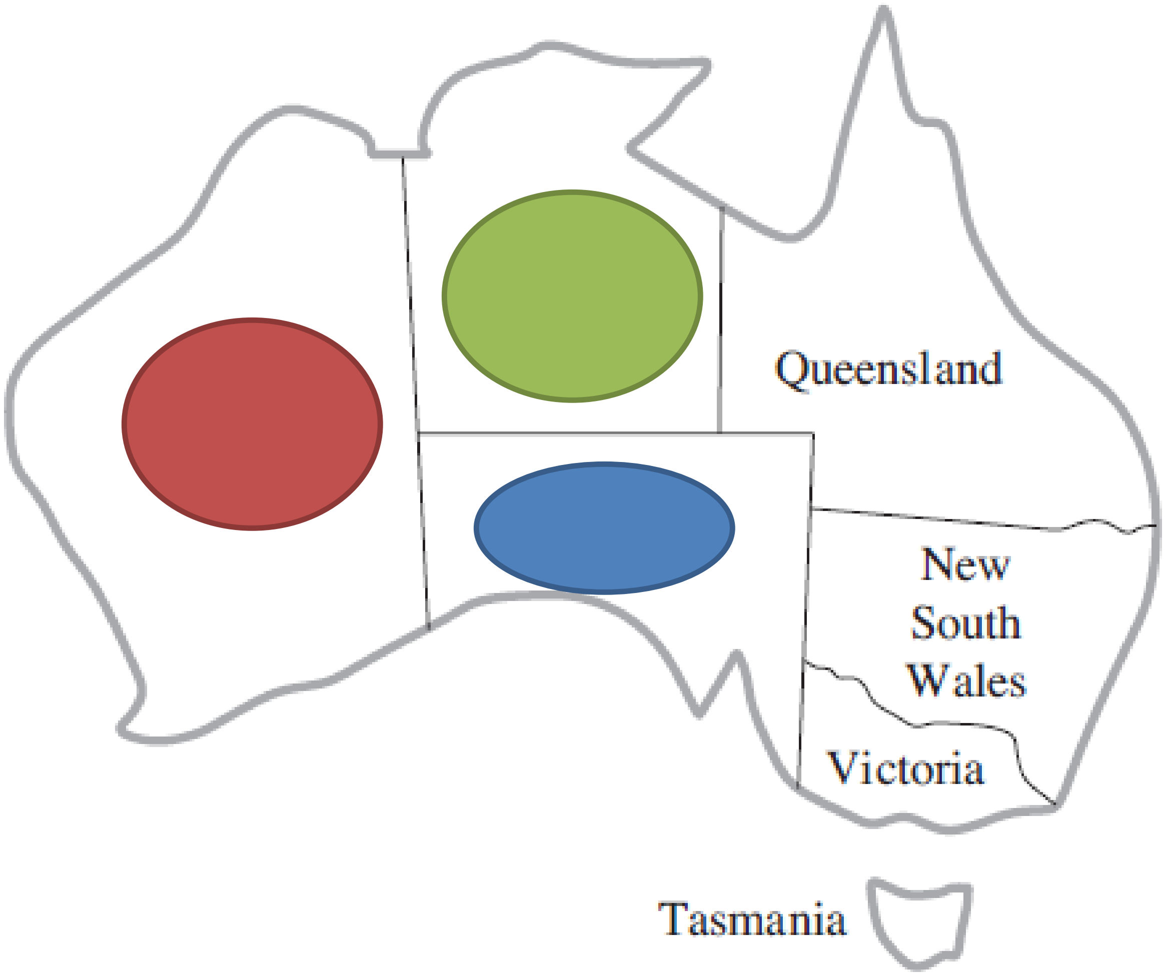
1. Which variable next?
2. Which value next?
3. Detect inevitable failure early
4. Exploit problem structure

Effect of Variable Order?

- Constraints: X=Y, Y=Z
- Constraint graph: X – Y – Z
- Which order to pick?
  - Order A: XZY
  - Order B: XYZ

- Variable order permutes the tree and changes its size!

Example

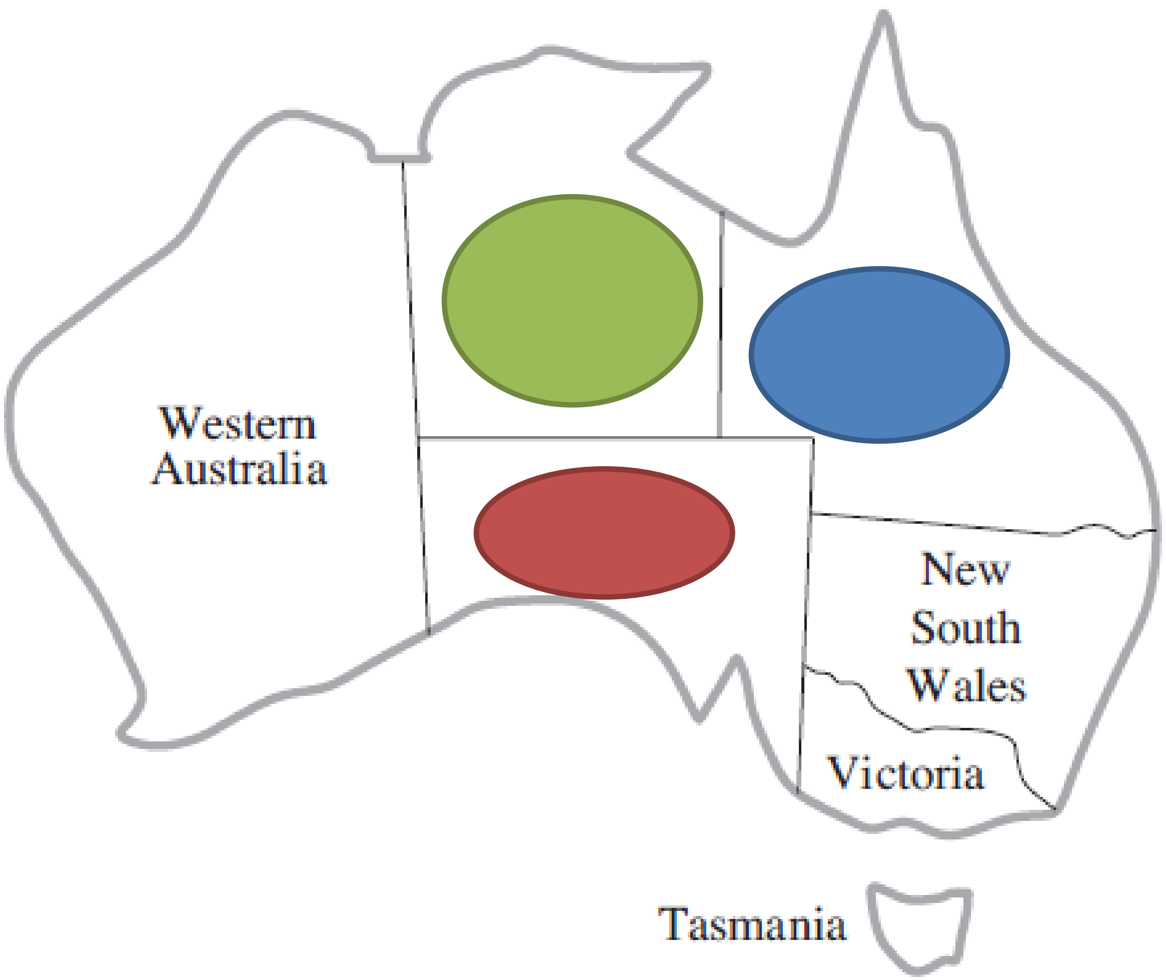


Variable Selection Heuristics

- Note: not a "heuristic" as in heuristic search
- General idea: first-fail principle

1. Most constrained variable heuristic:
   Pick variable with fewest legal values
   - aka Minimum remaining values heuristic
   - Reduces branching factor immediately

Example

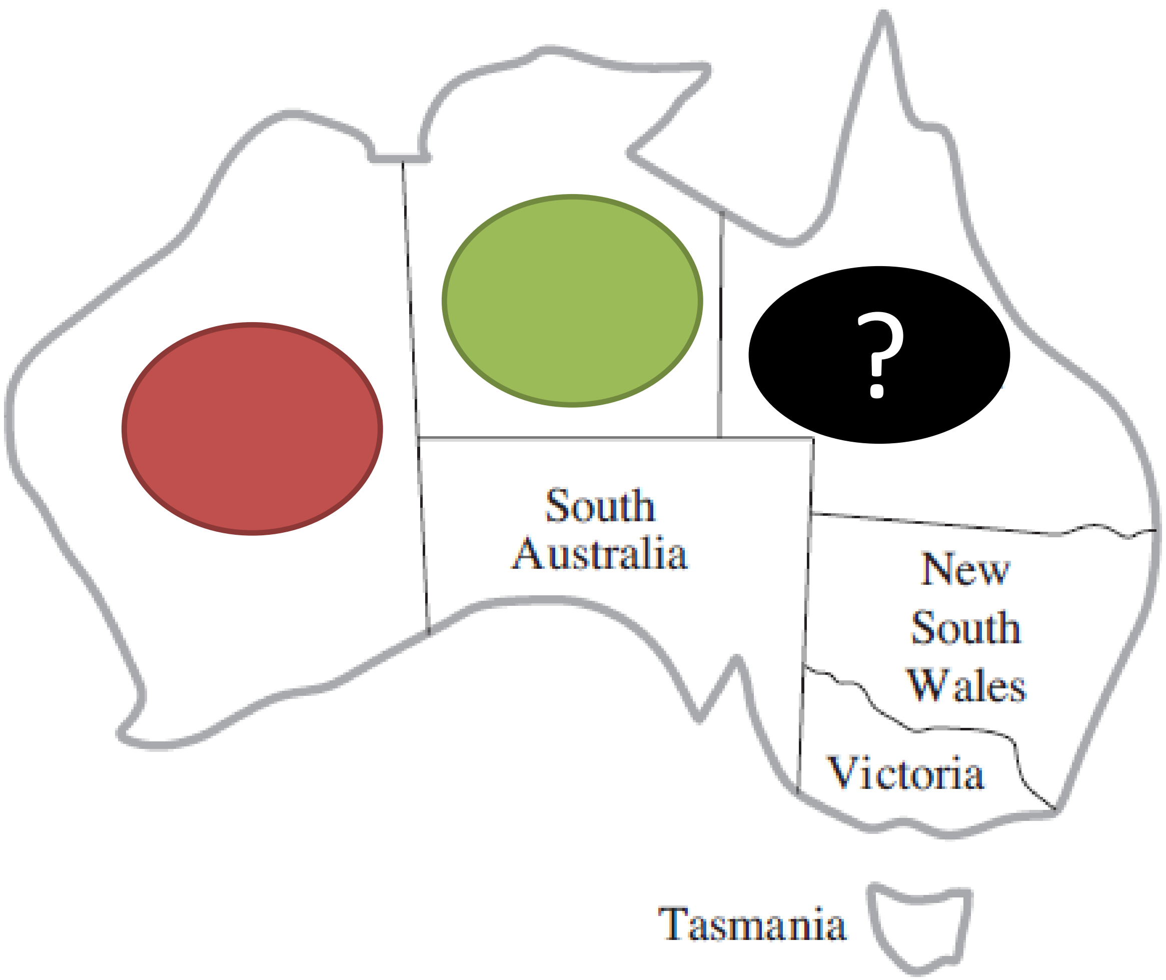


Variable Selection Heuristics

- Note: not a “heuristic” as in heuristic search
- General idea: first-fail principle

2. Degree heuristic: Pick variable with most constraints on remaining variables
   - Tie-breaker among most constrained variables
   - Easily computed on constraint graph (how?)
   - Reduces branching factor further down

Value Selection Heuristics



Value Selection Heuristics

- <u>Note</u>: not a “heuristic” as in heuristic search

1. Least constraining value heuristic:
   Pick value ruling out fewest values in remaining variables

Combined with variable heuristics:
1000-queens feasible

Is there something wrong?

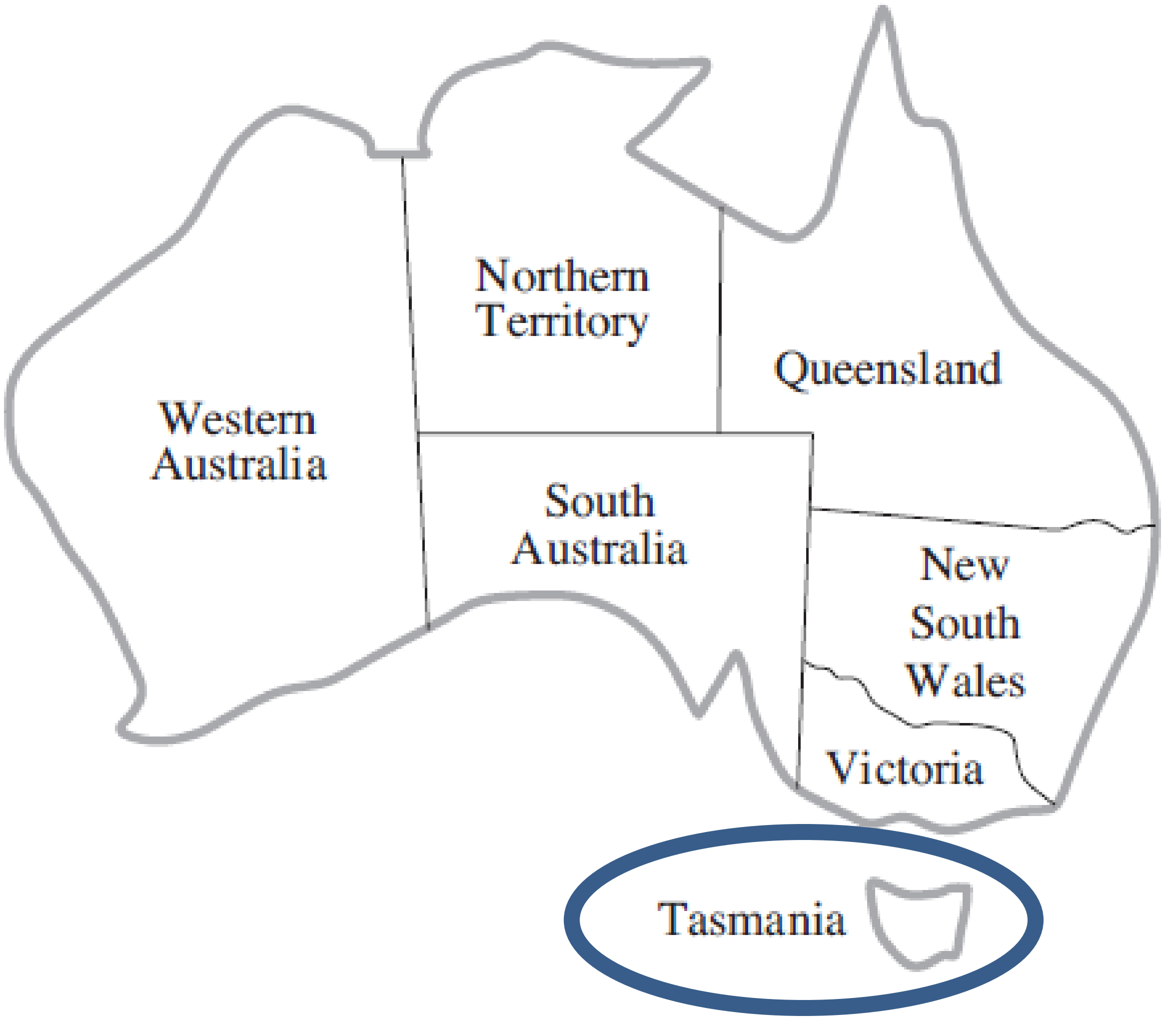
- Most constrained variable
- Least constraining value

Are these contradicting?

What if there is no solution?

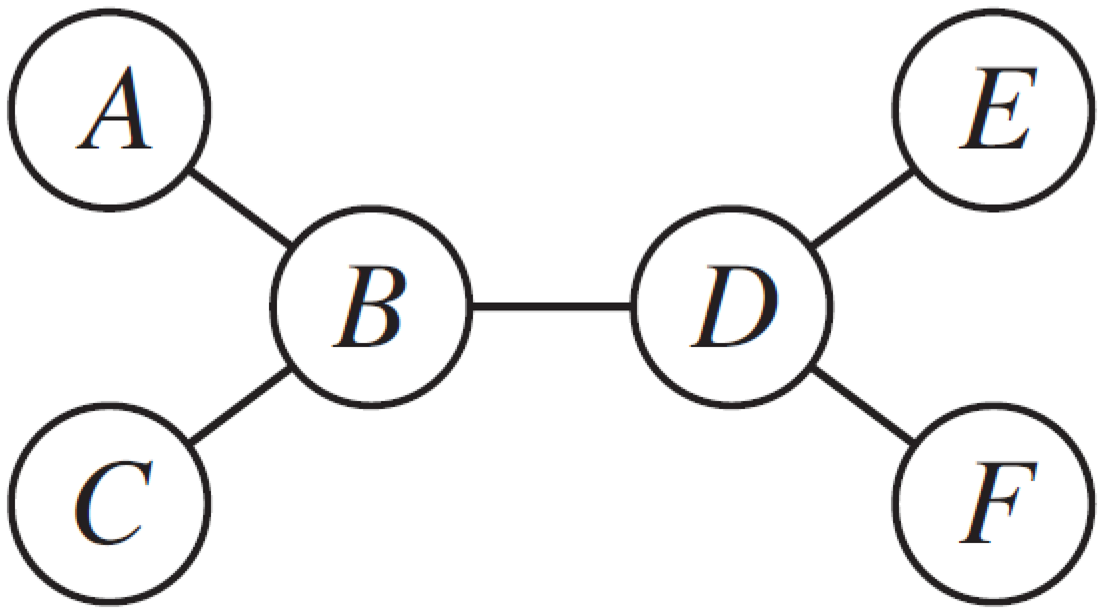
Detect Failure Early

Structure

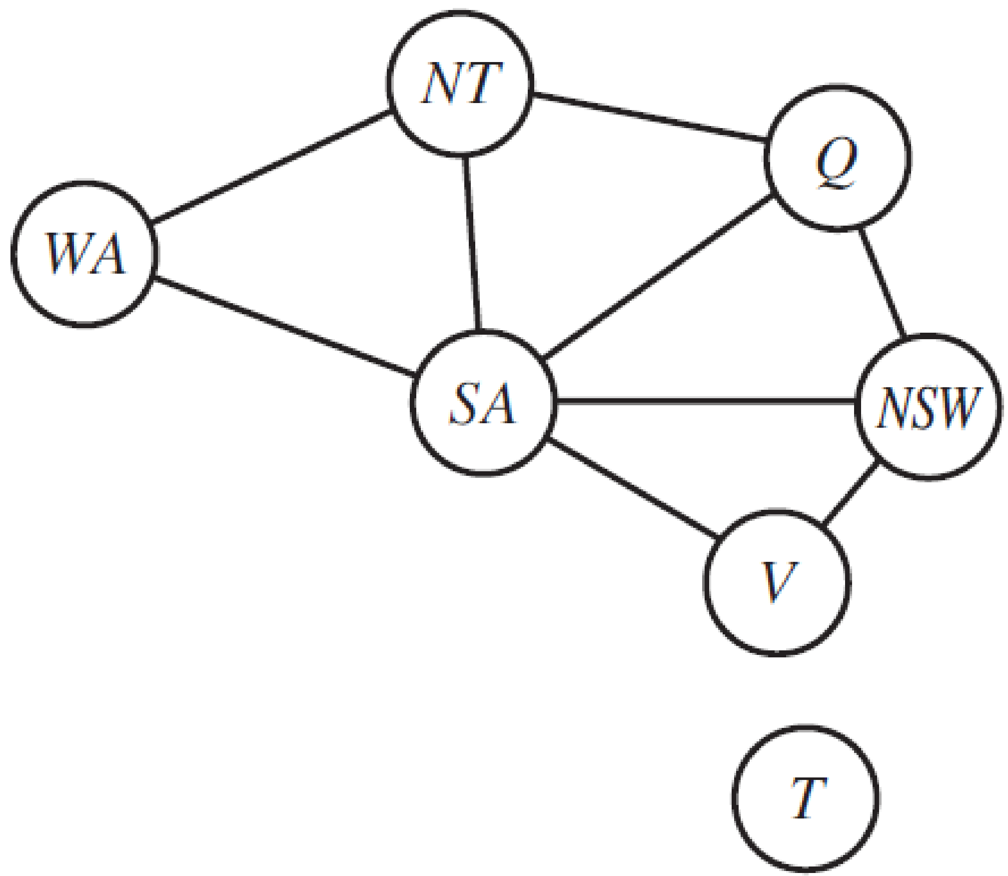


Northern Territory
Western Australia
Queensland
South Australia
New South Wales
Victoria
Tasmania

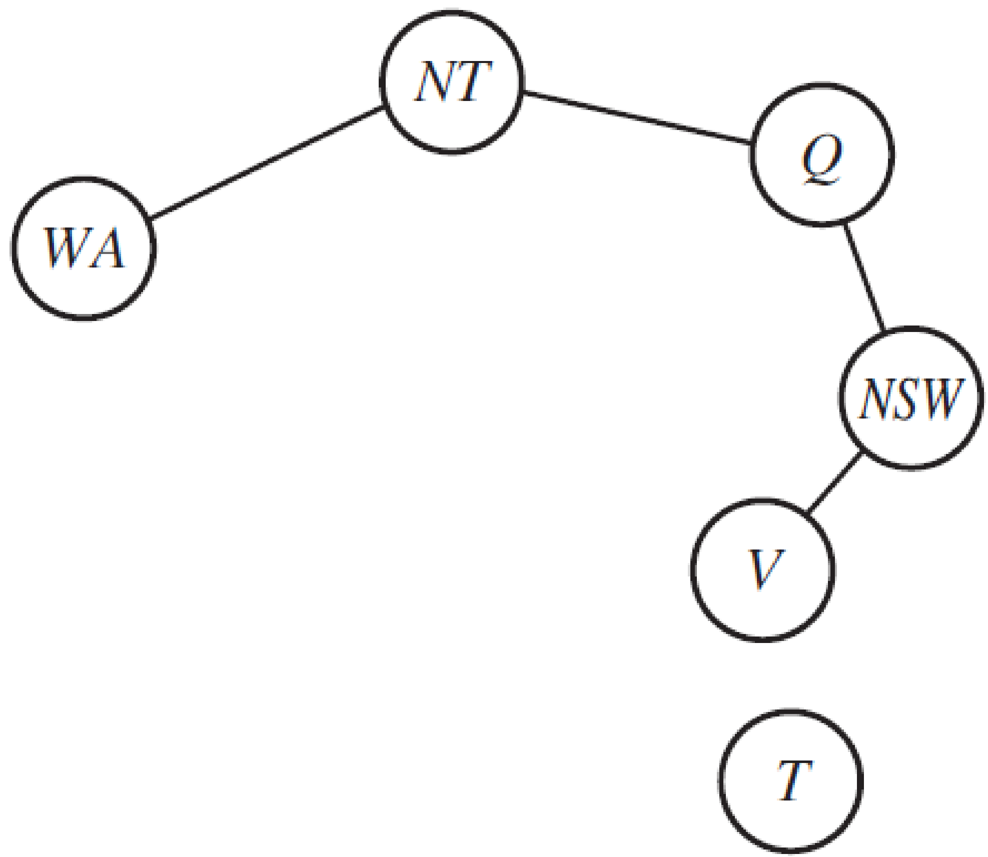
Structure: Tree



Structure?



Structure: Cutset Conditioning



Structure: Tree Decomposition