Structured Retail Products Design Proposal

The objective of this SRP is to safeguard investors from unlimited losses. In exchange for this security, investors offer us the possibility of additional profits. As a result, we have designed a derivative product that ensures a guaranteed minimum return, denoted as g, with a maximum cap at G.

Regarding to the $Payoff(S_T) = L(1+g)^T + N * [\max(0, S_T - (1+g)^T S_0) - \max(0, S_T - (1+G)^T S_0)]$, where $N = \frac{L}{S_0}$, we can consider this product as a portfolio of a bond and two options with different strikes. To prevent arbitrage opportunities, the present value of the final payoff must be equivalent to the initial investment.

$$L = price\ of\ bond + N * (price\ of\ option\ 1 - price\ of\ option\ 2)$$
$$L = L(1+g)^T e^{-rt} + BScall(S_0, K1, T, r, \sigma) - BScall(S_0, K2, T, r, \sigma)$$

We gathered price data for the S&P 500 index over the past 10 years and used the current 3-year Treasury rate as our benchmark for interest rates. The estimated annualized volatility for the S&P 500 is 0.172, and the current interest rate is 4.09%. By applying the Black-Scholes model with constant coefficients to calculate the price of options, and with a given set of guaranteed returns g{-1%, -0.5%, 0%, 0.5%, 1%, 1.5%, 2%}, we derived the corresponding values for the maximum return G. The results are as follows:

g	-1%	-0.5%	0%	0.5%	1%	1.5%	2%
G	10.98%	10.1%	9.5%	8.8%	8.2%	7.6%	6.9%
mean	4.9%	4.8%	4.75%	4.65%	4.6%	4.55%	4.45%

The average of g and G is approximately equivalent to the risk-free rate. This suggests that it may be challenging to generate profits without reducing the upper limit. To address this, we can decrease the initially calculated G by 0.5%.

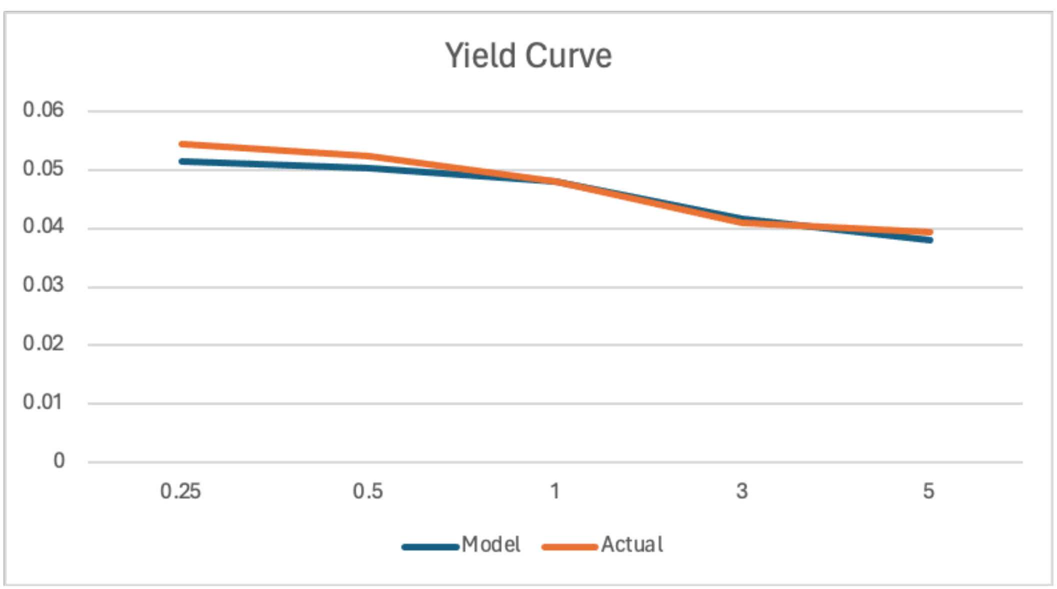
Given that the stock market can experience significant fluctuations—either sharp declines or rapid growth—we face risks due to changes in stock prices, which can affect the value of our options. To mitigate this risk, we recommend holding a combination of stocks and depositing cash in the bank as a hedging strategy. We developed our hedging strategy based on the Black-Scholes model and conducted a backtest to validate its effectiveness. The test used data from 1-1-2021, through 12-31-2023, with g=0, G=0.47%.

The results indicate that our strategy performs quite well, effectively hedging against risk and preventing significant losses during downturns in the stock market. At the end of the test period, our strategy yielded a return of $1,424.60.

However, in the real world, we can't overlook daily fluctuations in interest rates. Thus, assuming a constant interest rate is unrealistic, making it critical to consider interest rate dynamics when designing a strategy. By applying the Vasicek Model, which describes the evolution of interest rates, we derived a forecast of future rates and used it to discount our payoff. Using the current term structure from January 1, 2024, we calculated the Vasicek parameters (α=0.5, β=0.03, γ=0.02). With these parameters, we can then determine the correlation (ρ=0.075) between changes in stock prices and interest rates.

$$L = L(1+g)^T p(t, T, r_t) + V\_call1(S_0, K1, T, r, \sigma, r_t) - BScall(S_0, K2, T, r, \sigma, r_t)$$

In the model, we employed the mean squared error (MSE) as our loss function to minimize errors and optimize our estimates for the parameters α,β,γ.



We then recalculated the original set of assumed values for g, and derived the corresponding values for G.

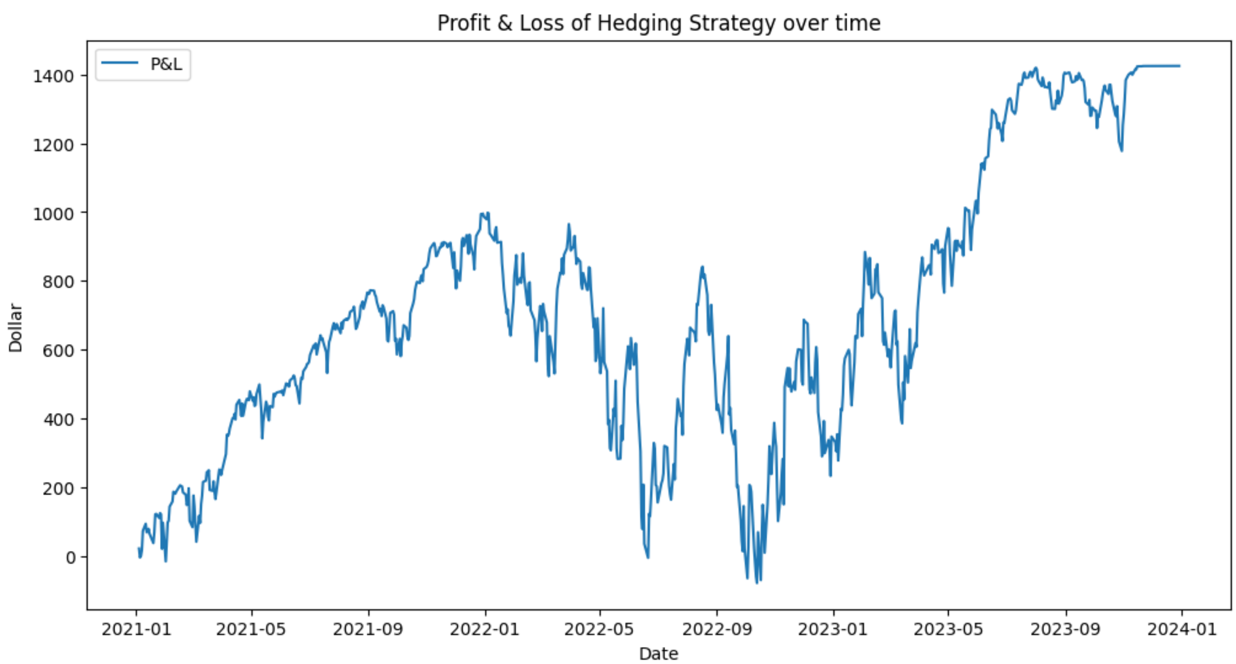
g	-1%	-0.5%	0%	0.5%	1%	1.5%	2%
G	4.9%	4.3%	3.6%	3%	2.3%	1.7%	1%
mean	1.95%	1.9%	4.75%	1.7%	1.65%	1.6%	1.5%

The recalculated values of G are significantly lower than the previous ones, indicating a potential impact from a downward-sloping term structure. This suggests that the economic outlook may be bleak, leading governments to lower interest rates to stimulate investment. The results demonstrate that ignoring interest rate fluctuations could result in significant losses for our company. Therefore, it is crucial to factor in interest rates when modeling and designing our financial products.

Similar to our previous hedging strategy, where we used a mix of stocks and bank deposits, incorporating a variable interest rate into our model requires a different approach. To effectively hedge our position against interest rate fluctuations, we should utilize bonds alongside stocks and cash in bank accounts.

The number of shares to buy or sell: $N(d1)$
The amount of bond to buy or sell: $-K * N(d2)$
The amount of cash to deposit or withdraw: $V_0 - S_0 * N(d1) + K * N(d2)$

This diversification helps us manage risk in a more dynamic interest rate environment. Our results indicate that this modified strategy continues to perform quite well, yielding a final profit of $6,786.25.



However, we overlooked a critical factor: volatility. In our earlier model, we assumed constant volatility, which is both unrealistic and impractical. The actual stock market experiences continual fluctuations with the potential for large daily swings. Therefore, it is essential to account for changing volatility. To address this, we propose using the Heston model, which allows for stochastic volatility. Using the parameters (α, β, γ, ρ) from earlier, we finalize our model with (r_t, σ_t), where r_t, represents the interest rate and σ_t represents the volatility at a given time. We used today's VIX index as a representation of the "volatility of volatility," serving as a gauge for market uncertainty and expected future fluctuations in the S&P 500. The VIX index is widely recognized as a barometer of market sentiment and helps us capture the level of implied volatility over time.

g	-1%	-0.5%	0%	0.5%	1%	1.5%	2%
G	9.7%	9.0%	8.4%	7.7%	7.1%	6.4%	5.8%
mean	4.35%	4.25%	4.20%	4.10%	4.05%	3.95%	3.9%

Today, with the rapid advancements in Machine Learning, these technologies can be applied to our pricing and hedging processes. By incorporating Machine Learning algorithms, we can consider a wider range of potential factors, leading to more accurate estimations and enhanced risk management. This approach allows for greater adaptability to market dynamics and can improve the robustness of our financial models.

We can employ the Random Forest model to improve the accuracy of option pricing. Random Forest relies on decision trees, which inherently perform "automatic" feature (predictor/variable) selection during the model-building process. This characteristic helps explain why the model is effective; it tends to ignore noisy predictors and focuses on selecting the most relevant ones. Meanwhile, it helps us focus our attention to what

By considering a variety of factors, such as the S&P 500 index, the VIX index, VIX futures with different maturities, and treasury yields of various durations, the Random Forest model allows us to identify the most significant predictors for option pricing while filtering out irrelevant or redundant factors. This model not only considers factors with measurable values, like market news counts and stock trading volumes, but also integrates other elements not traditionally used in option pricing. This broader perspective allows the model to encompass a wider range of market dynamics, resulting in an option price that more accurately reflects the true market value. By accounting for both conventional and unconventional factors, the model provides a more nuanced view of market conditions and thus produces more precise pricing. Therefore, this approach can lead to more accurate and robust option pricing, enhancing our ability to manage risk and optimize strategies.

This model allows me to concentrate on the market factors that are most likely to impact option pricing. This has a dual benefit: first, it results in more accurate prices, reducing the potential for arbitrage; second, it enables us to design new hedging strategies to mitigate these risk factors, making our risk management approach more comprehensive.

The most challenging part of building this model is identifying the most relevant factors that influence option pricing within a random forest framework. Despite pinpointing some strongly correlated factors, there's always a risk of overlooking important influences. To address this, the company should consider hiring industry researchers, PhDs in economics, and other experts who can help identify the appropriate features. These professionals can conduct thorough research to uncover hidden patterns, ensuring that the model captures a comprehensive set of factors impacting option pricing.

Moreover, even when some key influences are identified, quantifying them can be challenging, and data for some influences might be unavailable or incomplete. This creates significant obstacles for machine learning models, which rely on measurable inputs. Without sufficient data or consistent metrics for these influences, building accurate and reliable models becomes more complex. Companies must develop alternative approaches to address these gaps, such as using proxies, relying on expert opinions, or combining quantitative and qualitative data sources, to ensure the model remains robust despite data limitations.

In addition, computational power poses a significant challenge. Unlike software companies, financial firms often lack access to powerful cloud servers and dedicated computer experts who can continuously train models to enhance their accuracy. To meet this challenge, companies need to invest in renting servers or cloud computing resources. They also need to hire professionals with expertise in machine learning to maintain, optimize, and train the models, ensuring they perform at a high level of accuracy. This investment in infrastructure and expertise is critical for successful model development and implementation.