The Community Structure of SAT Formulas

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SAT Conference 2012
June 20, 2012

Introduction

- SAT is a central problem in computer science and AI with theoretical and **practical** applications.
- SAT competitions: *good* solvers for random SAT instances are *bad* for industrial instances, and vice versa.
- SAT solvers efficiency solving industrial instances has undergone a great advance, mainly motivated by the introduction of lazy data-structures, learning mechanisms and activity-based heuristics.
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- We propose the use of **modularity** (of graphs) for detecting the **community structure** of SAT instances.
  - The notion of *community* is more general than the notion of *connected components*.
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How does learning modify it?

SAT Formulas as Graphs

Clause-Variable Incidence Graph (CVIG):

Nodes: are variables **v** and clauses **c**

Edges: **v**—**c** if clause **c** contains variable **v** with weight $w = \frac{1}{|c|}$

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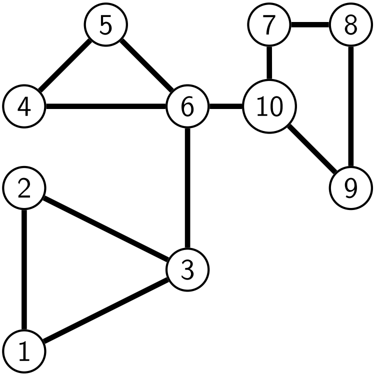
the sum of the weights of the edges generated by a clause is one

Partitions in Graphs



$$Q = \frac{inner\ \ edges}{total\ \ edges} - \frac{expected\ \ inner\ \ edges}{total\ \ edges}$$

$$Q = \frac{10}{12} - \frac{7 \cdot \frac{7}{24} + 8 \cdot \frac{8}{24} + 9 \cdot \frac{9}{24}}{12} \approx 0.8333 - 0.3368 = 0.4965$$

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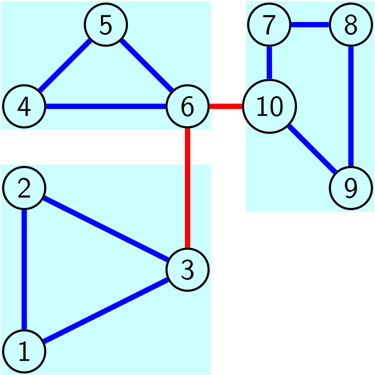

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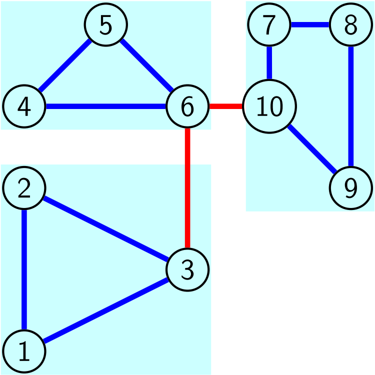

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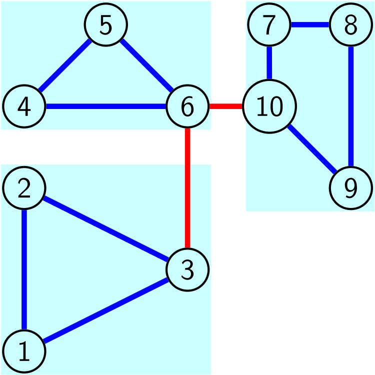

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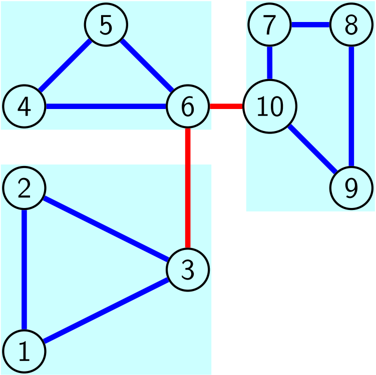

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Modularity $Q \in [-1,\ 1]$ measures **how good** is partition $P = \{P_i\}$

For **weighted graphs** (Newman et al. (2004)):

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where $\deg(x) = \sum_{y \in V} w(x,y)$

For **bipartite graphs** (Barber (2007)):

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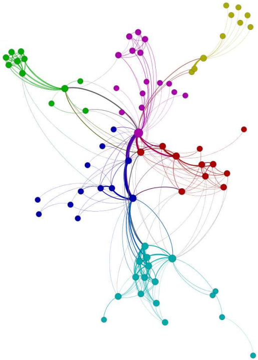
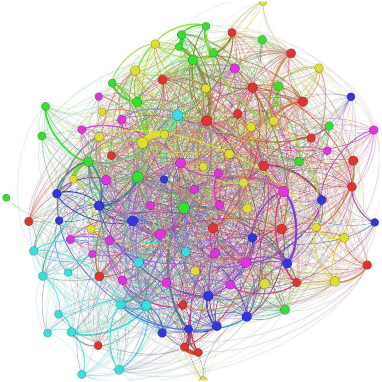
Example



$Q = 0.560$



$Q = 0.097$

"In practice, Q values for networks showing a strong community structure range from 0.3 to 0.7, higher values are rare."

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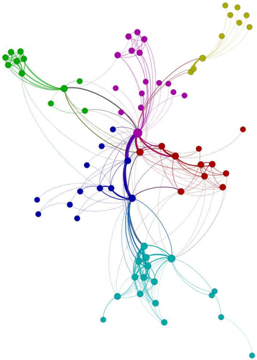

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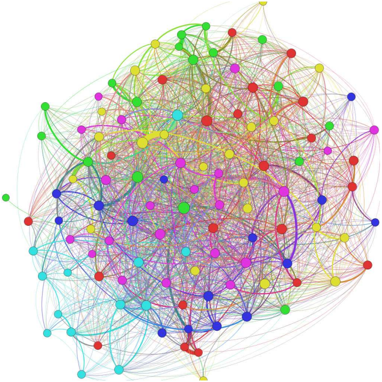

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Label Propagation Algorithm (LPA)

Input: Graph $G = (X, w)$
Output: a labelling L for X

```
for x ∈ X do L[x] := x; endfor
do
    changes := FALSE;
    for i ∈ |X| in random order do
        label := most_freq_label(i,neighbors(i));
        if label ≠ L[i] then
            changes := TRUE;
            L[i] := label;
    endfor
while changes
return L
```

Raghavan et al. (2007).

Graph Folding Algorithm (GFA)

Input: Graph $G = (X, w)$
Output: a labelling L_1 for X
for $x \in X$ **do** $L_1[x] := x$; **endfor**
$L_2 :=$ **OneLevel**(G);
while $Mod(G, L_1) < Mod(G, L_2)$ **do**
  $L_1 := L_1 \circ L_2$;
  $G :=$ **Fold**(G, L_2);
  $L_2 :=$ **OneLevel**(G);
return L_1

function OneLevel(Graph $G = (X, w)$): Label L
  do
    $changes := \texttt{FALSE}$;
    foreach $i \in X$ **do**
      $bestInc := 0$;
      foreach $c \in \{c \mid \exists j . w(i,j) \neq 0 \wedge L[j] = 0\}$ **do**
        $inc := \sum_{L[j]=c} w(i,j) - arity(i) \cdot \sum_{L[j=c]} arity(j) / \sum_{j \in X} arity(j)$;
        if $inc > bestInc$ **then**
          $L[i] := c$;
          $bestInc := inc$;
          $changes := \texttt{TRUE}$;
  until $\neg changes$
  return L;

function Fold(Graph G_1, Label L): Graph G_2
  $X_2 = \{c \mid \forall i, j \in c . L[i] = L[j]\}$;
  $w_2(c_1, c_2) = \sum_{i \in c_1, j \in c_2} w_1(i,j)$;
  return $G_2 = (X_2, w_2)$;

Blondel et al. (2008).

Modularity of industrial SAT instances

- Modularity of the SAT instances used in the **2010 SAT Race Finals**.
    - 100 instances.
    - Grouped into 16 families.
    - Families classified as *cryptography*, *hardware verification*, *software verification*, and *mixed*, according to their application area.
    - All instances are *industrial* in the sense that their solubility has an industrial or practical application.
    - However, they are expected to show a **distinct nature** from random instances.

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Modularity of industrial SAT instances

	Family (#instanc.)	Variable IG LPA Q	LPA $\|P\|$	LPA larg.	LPA iter.	GFA Q	GFA $\|P\|$	GFA larg.	GFA iter.	Clause-Variable IG LPA Q	LPA $\|P\|$	LPA larg.	LPA iter.	Connect. Comp. $\|P\|$	larg.
cripto.	desgen(4)	**0.88**	532	0.8	36	**0.95**	97	2	37	**0.75**	3639	1	25	1	100
	md5gen(3)	**0.61**	7176	0.1	16	**0.88**	38	7	40	**0.78**	7904	0.1	42	1	100
	mizh(8)	0.00	33	99	6	**0.74**	30	9	33	**0.67**	5189	31	43	1	100
hard. ver.	ibm(4)	**0.81**	3017	0.6	9	**0.95**	723	4	32	**0.77**	19743	0.2	140	70	99
	manolios(16)	0.30	66	81	9	**0.89**	37	9	81	**0.76**	6080	1	26	1	100
	velev (10)	**0.47**	8	68	9	**0.69**	12	30	24	0.30	1476	77	31	1	100
mixed	anbulagan(8)	**0.55**	30902	0.1	11	**0.91**	90	2	43	**0.72**	46689	0.6	26	1	100
	bioinf(6)	**0.61**	87	44	3	**0.67**	60	17	22	**0.64**	94027	15	10	1	100
	diagnosis(4)	**0.61**	20279	0.7	15	**0.95**	68	3	43	**0.65**	85928	0.1	42	1	100
	grieu(3)	0	1	100	2	0.23	9	14	11	0	1	100	14	1	100
	jarvisalo(1)	**0.57**	260	5	8	**0.76**	19	9	26	**0.71**	336	1	11	1	100
	palacios(3)	0.14	1864	96	58	**0.93**	1802	6	13	**0.76**	2899	0.4	35	1	100
soft. ver.	babic(2)	**0.68**	34033	8	54	**0.90**	6944	10	141	**0.73**	59743	4	53	41	99
	bitverif(5)	**0.48**	3	57	4	**0.87**	24	6	29	**0.76**	33276	0.4	8	1	100
	fuhs(4)	0.02	18	99	43	**0.81**	43	7	30	**0.67**	12617	0.8	28	1	100
	nec(17)	0.07	107	96	31	**0.93**	65	14	124	**0.79**	23826	0.8	114	1	100
	post(2)	**0.36**	$3{\cdot}10^6$	53	54	**0.81**	$3{\cdot}10^6$	9	262	**0.72**	$3{\cdot}10^6$	6	49	224	99

Modularity of 2010 SAT Race instances.

Modularity after preprocessing formulas

Family		Orig. Form.	Preprocessed Formula				
			Modularity			Connect. Comp.	
		Q	Q	$\|P\|$	larg.	$\|P\|$	larg.
cripto.	desgen (4)	**0.951**	**0.929**	81	3.1	1	100
	md5gen (3)	**0.884**	**0.884**	18	8.5	1	100
	mizh (8)	**0.741**	**0.741**	18	9.5	1	100
hard. ver.	ibm (4)	**0.950**	**0.905**	26	6.1	1	100
	manolios (16)	**0.890**	**0.800**	16	14.9	1	100
	velev (10)	**0.689**	**0.687**	6	30.3	1	100
mixed	anbulagan (8)	**0.909**	**0.913**	47	5.1	1	100
	bioinf (6)	**0.673**	**0.657**	25	11.1	2	99.9
	diagnosis (4)	**0.952**	**0.950**	65	3.6	1	100
	grieu (3)	0.235	0.235	9	14.3	1	100
	jarvisalo (1)	**0.758**	**0.722**	11	13.1	1	100
	palacios (3)	**0.928**	**0.848**	17	10.76	1	100
soft. ver.	babic (2)	**0.901**	**0.875**	23	9.7	1	100
	bitverif (5)	**0.875**	**0.833**	19	7.3	1	100
	fuhs (4)	**0.805**	**0.743**	32	9.6	1	100
	nec (17)	**0.929**	**0.879**	37	10.3	1	100

Modularity after preprocessing the formula with *Satelite* preprocessor.

Modularity of Learnt Clauses

- Formulas are transformed during the proof or the satisfying assignment search.
  - Even if the original formula shows a community structure, could it be the case that this structure is quickly destroyed during the search process?
  - How new learnt clauses affect the community structure of the formula?
  - Even if the value of the modularity is not altered, could it be the case that communities are changed?

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Modularity of resulting Formulas adding Learnt Clauses

Family		Orig. Form. Q	Orig. + Learnt Formula Modularity Q	$\|P\|$	larg.
cripto.	desgen (2)	**0.951**	**0.561**	53	13.0
	md5gen (3)	**0.884**	**0.838**	19	8.0
	mizh (1)	**0.741**	**0.705**	28	11.4
hard. ver.	ibm (3)	**0.950**	**0.912**	752	6.7
	manolios (14)	**0.890**	**0.776**	31	11.2
	velev (1)	**0.689**	**0.558**	6	30.1
mixed	anbulagan (4)	**0.909**	**0.876**	84	2.6
	bioinf (5)	**0.673**	0.287	32	58.7
	grieu (3)	0.235	0.085	6	35.0
	palacios (2)	**0.928**	**0.851**	2289	7.4
soft. ver.	babic (2)	**0.901**	**0.904**	6942	10.7
	fuhs (1)	**0.805**	**0.670**	24	7.6
	nec (15)	**0.929**	**0.936**	62	7.3

Modularity of the formula with learnt clauses included (using *PicoSAT*).

Modularity of Learnt Clauses using Original Partitions

Family	VIG orig.	VIG first 100	VIG all	CVIG orig	CVIG first 100	CVIG all
desgen (1)	**0.89**	**0.74**	0.08	**0.77**	0.28	0.09
md5gen (1)	**0.61**	**0.74**	0.02	**0.78**	**0.96**	0.02
ibm (2)	**0.84**	**0.60**	**0.47**	**0.81**	**0.58**	0.29
manolios (10)	0.21	0.04	0.10	**0.76**	0.11	0.09
anbulagan (2)	**0.56**	0.16	0.01	**0.87**	0.10	0.04
bioinf (4)	**0.62**	**0.46**	0.06	**0.68**	**0.69**	0.15
grieu (1)	0.00	0.00	0.00	0.00	0.00	0.00
babic (2)	**0.68**	**0.36**	**0.36**	**0.71**	**0.33**	**0.33**
fuhs (1)	**0.66**	**0.59**	0.14	**0.71**	**0.78**	0.07
nec (10)	0.12	0.01	0.12	**0.78**	**0.49**	0.24

Family	VIG orig.	VIG first 100	VIG all	CVIG orig.	CVIG first 100	CVIG all
md5gen (1)	**0.61**	**0.74**	0.02	**0.78**	**0.96**	0.02
ibm (2)	**0.84**	**0.50**	**0.43**	**0.81**	**0.58**	**0.42**
anbulagan (1)	**0.55**	0.03	0.00	**0.87**	0.13	0.05
manolios (9)	0.20	0.07	0.10	**0.76**	0.27	0.15
nec (10)	0.12	0.05	0.05	**0.78**	**0.30**	0.22

Modularity of (only) learnt clauses, computed using original partition, for all the instances that were solved (top), and only for unsatisfiable ones (bottom).

Modularity of Random Formulas

- **Random formulas** have a clear community structure?
  Obviously NO!!
- **Modularity** can be used as a reference value to compare how *modular* other kind of instances are:
  - Is it as high in random formulas as in industrial ones?
  - Is it dependent on the clause-variable ratio α?
  - Is it dependent on the number of variables?
  - How modularity is affected by the effects of:
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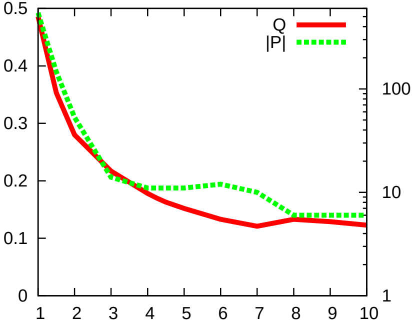
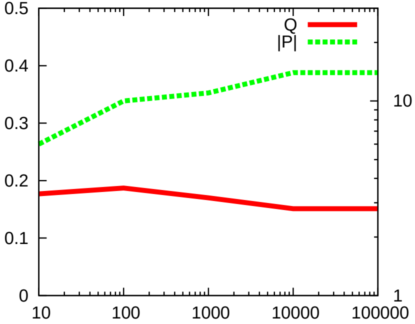
n	α	Q	$\|P\|$	larg.	iter
10^4	1	0.486	545	3.8	54
10^4	1.5	0.353	146	5.1	52
10^4	2	0.280	53	6.8	51
10^4	3	0.217	14	15.5	64
10^4	4	0.178	11	14.8	54
10^4	4.25	0.170	11	14.6	53
10^4	4.5	0.163	11	14.7	53
10^4	5	0.152	11	14.3	51
10^4	6	0.133	12	13.9	53
10^4	7	0.120	10	15.0	56
10^4	8	0.138	6	25.0	50
10^4	9	0.130	6	24.3	49
10^4	10	0.123	6	24.4	47



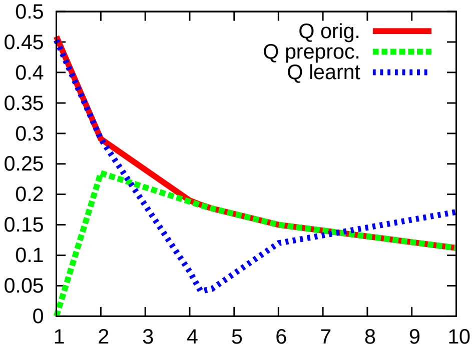
Modularity of random formulas varying the clause variable ratio α.

n	α	Q	$\|P\|$	larg.	iter
10^2	4.25	0.177	6	14.5	11
10^3	4.25	0.187	10.5	11.4	35
10^4	4.25	0.170	11	12.2	53
10^5	4.25	0.151	14	6.8	102
10^6	4.25	0.151	14	5.7	167



Modularity of random formulas at the peak transition region (clause-var. ratio $\alpha = 4.25$) varying the number of variables (n).

n	α	Orig.	Prep.	Learnt	C.C.
300	1	0.459	0	0.453	1
300	2	0.291	0.235	0.291	1
300	4	0.190	0.188	0.073	1
300	4.25	0.183	0.182	0.041	1
300	4.5	0.177	0.177	0.045	1
300	6	0.150	0.150	0.120	1
300	10	0.112	0.112	0.171	1



Modularity of random formulas (varying the clause variable ratio) after preprocessing, and including all learnt clauses.

Conclusions

- Industrial SAT instances have a **clear community structure**.
- Preprocessing these instances has almost no impact in this structure. However, it eliminates almost all the small unconnected components.
- Modularity weakly decreases with the learnt clauses. Therefore, learning does **not completely destroy the organization of the formula**. Nevertheless, it does **change the partitions structure**.
- Modularity of random instances is **only significant** for very low clause-variable ratios (α). Even for them, the modularity is **not as high as for industrial instances**. No abrupt change in the phase transition point is observed. This modularity does not depend on the number of variables (it seems to tend to a particular value).
- Preprocessing has almost no impact on the modularity of the formula. However, learning process **totally changes** this modularity. So, it totally changes the structure of the formula.

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Future Work

- To understand better how to **exploit** the community structure in SAT solvers (heuristics, learning, restarts, ...).
- **Generate** industrial-like random SAT instances with similar modularity to industrial instances.
- Study **other measures** of complex networks (such as the fractal dimension).

Thank you for your attention!!