Stepwise Randomized Combinatorial Auctions Achieve Revenue Monotonicity

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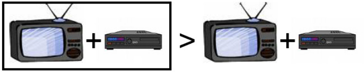
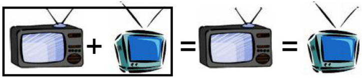
Outline

Combinatorial Auctions

- There are multiple goods for sale.
- Bidders may have non-additive valuations over goods.



Superadditive valuation



Subadditive valuation

Definition (CA mechanism)

In a combinatorial auction (CA) mechanism, multiple goods are sold simultaneously and bidders are allowed to place bids on bundles, rather than just on individual goods.
The mechanism decides on the allocation of goods and the payments given the bids.

Revenue Monotonicity

It is natural to imagine that

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Definition (Revenue Monotonicity)

A CA mechanism is revenue monotonic if adding a bidder never reduces the auction's revenue.

- Revenue monotonicity holds in single-good settings.
- Does revenue monotonicity hold in combinatorial auctions?

Which Combinatorial Auctions Should We Consider?

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Definition (Participation)

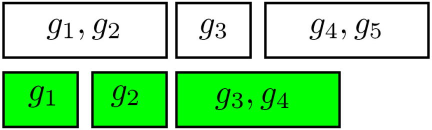
A bidder makes zero payment if she does not win.

Definition (Consumer sovereignty)

Any bidder can win any bundle she desires, if she bids high enough.

Definition (Maximality)

The chosen allocation is maximal: it cannot be augmented to make some bidders better off while making none worse off.



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Definition (Strategyproofness)

It is a dominant strategy for any bidder to declare her true valuation.

Our Past Result

Theorem (RCL, AAAI'07)

Let M be a deterministic CA mechanism that satisfies

- *strategyproofness;*
- *participation;*
- *consumer sovereignty; and*
- *maximality.*

Then M is not revenue monotonic.

Related Work

- Revenue monotonicity
  - *Ausubel and Milgrom* (2002, 2006)
  - *Day and Milgrom* (2007)

- Design of strategyproof CA mechanisms
  - *Archer and Tardos* (2001)
  - *Mu'alem and Nisan* (2002)
  - *Lehmann et al.* (2002)
  - *Bartel et al.* (2003)
  - *Babaioff et al.*(2006)
  - *Andelman and Mansour* (2006)
  - ...

Plan of this talk

We are interested in whether revenue monotonicity is achievable if we relax the assumption that mechanisms are deterministic.

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In the rest of the talk I'll:

- Extend our desirable properties to randomized CA mechanisms,
- Show that there exist randomized CA mechanisms defined for known single-minded bidders, that satisfy our properties and are revenue monotonic.

Outline

1. Introduction
2. Randomized Mechanisms
3. Revenue Monotonic Mechanisms
4. Conclusion

Setting

- G: a set of m goods for sale
- $N = \{1, \ldots, n\}$: the universal set of n bidders
    - each may or may not participate in a given auction

Definition (Single-minded bidder)

A bidder i is single minded if she has the valuation function:

$$\forall s \in 2^G, \qquad v_i(s) = \begin{cases} v_i > 0 & \text{if } s \supseteq b_i; \\ 0 & \text{otherwise.} \end{cases}$$

Definition (Known single-minded setting)

In a known single-minded setting, all bidders are single-minded and the bundles b_i are known to the auctioneer.

Randomized Mechanisms

- $\hat{v}$: bidders' declared valuation profile
- A randomized CA mechanism maps from declared valuation profiles both to a distribution over allocations and to payments.
  - $\pi_{\hat{v}}(\mathrm{a})$: the probability that allocation a will be chosen
  - $p_i(\hat{v})$: expected payment from bidder i
- $w_i(\hat{v})$: the probability that single-minded bidder i wins (is allocated at least b_i)

Which Randomized CAs Should We Consider?

Definition (Revenue Monotonicity)

Adding a bidder never reduces the auction's expected revenue.

Definition (Participation)

A bidder makes zero expected payment if she does not win.

Definition (Maximality)

The chosen allocation is maximal: it cannot be augmented to make some bidders better off while making none worse off.

Definition (Strategyproofness)

It is a dominant strategy for any bidder to declare her true valuation in the game induced by expectation.

Consumer Sovereignty (I)

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Any bidder can win any bundle she desires with probability one if she bids high enough.

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Any bidder can win any bundle she desires with probability one if she bids high enough.

We recover the same impossibility result as with deterministic mechanisms.

Theorem

Let M be a randomized CA mechanism defined for known single-minded bidders that satisfies strategyproofness, participation, consumer sovereignty (I), and maximality. Then M is not revenue monotonic.

Consumer Sovereignty (II)

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Consumer Sovereignty (II)

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Any bidder can win any bundle she desires with some probability above zero if she bids high enough.

There exists a (degenerate) mechanism that satisfies all our desired properties and consumer sovereignty (II).

Proposition (Uniform-random allocation, no payments)

The following mechanism satisfies strategyproofness, participation, consumer sovereignty (II), maximality and revenue monotonicity:

- *choose a maximal allocation uniformly at random;*
- *charge bidders nothing.*

Consumer Sovereignty (III)

Idea: require that any bidder can increase her probability of winning by δ at least γ times unless it reaches one.

Definition ((γ-step, δ) Consumer Sovereignty)

For every bidder i, there exist constants $0 = c_{i,0} < \ldots < c_{i,\gamma+1} = \infty$ such that w_i's are monotonic and furthermore that either:

- $w_i(c_{i,j+1}, \hat{v}_{-i}) \geq w_i(c_{i,j}, \hat{v}_{-i}) + \delta$, or
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- Note: the constants $c_{i,j}$'s are independent of $\hat{v}$.
- If the mechanism designer has information about the valuation distribution(s), it can be used for setting these constants.

Stepwise Randomized Mechanism

A stepwise randomized mechanism partitions the valuation space into a finite number of equivalence classes.

Definition (γ-step Randomized Mechanism)

For every bidder i, there exist constants $0 = c_{i,0} < \ldots < c_{i,\gamma+1} = \infty$ such that for all $\hat{v}$ and all bidders k,

$$w_k(\hat{v}) = w_k(c_{1,j_1}, c_{2,j_2}, \ldots, c_{n,j_n}),$$

where $c_{i,j_i} \leq \hat{v}_i < c_{i,j_i+1}, \ \forall i$.

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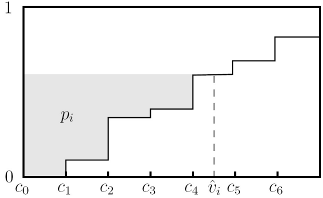
- In a strategyproof, stepwise randomized mechanism the payment functions are stepwise-linear.



Outline

Randomized Revenue Monotonic Mechanisms

Theorem

For any given $\gamma \geq 0$, there exists a γ-step randomized mechanism defined for known single-minded bidders that satisfies

- *strategyproofness;*
- *participation;*
- *(γ-step,δ) consumer sovereignty, for some $\delta > 0$;*
- *maximality;*

and that is revenue monotonic.

Proof Sketch

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1. Give a nonlinear feasibility program F whose solutions correspond to the mechanisms that satisfy our properties.

Proof Sketch

$$\xrightarrow[\gamma,\ \{c_{i,j}\}]{\forall\ N,\ G,\ \{b_i\},} \qquad \xrightarrow[\delta]{\pi_{\hat{v}}(\mathrm{a}),\ p_i(\hat{v}),}$$

1. Give a nonlinear feasibility program F whose solutions correspond to the mechanisms that satisfy our properties.
2. Construct a quadratically constrained linear program (QCLP) P that can be used to check for a solution to F.

Proof Sketch

1. Give a nonlinear feasibility program F whose solutions correspond to the mechanisms that satisfy our properties.
2. Construct a quadratically constrained linear program (QCLP) P that can be used to check for a solution to F.
3. Analytically construct a solution to the QCLP that is also a solution to F.
   - In fact, show that there exist infinitely many such solutions.

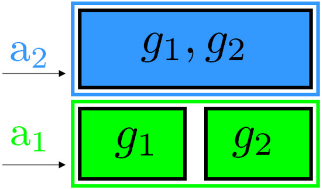
1. Feasibility Program

- We create variables $\pi_{\hat{v}}(\mathrm{a})$ and $p_i(\hat{v})$ for each $\hat{v}$, a and i.
- We write constraints expressing our desired properties.
- This feasibility program has
  - an infinite number of both variables and constraints;
  - nonlinear constraints; and
  - (some) strict inequality constraints.

2. Quadratically Constrained Linear Program

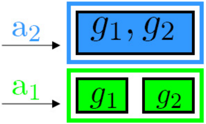
- We are interested in finding a stepwise mechanism.
- Thus, it is enough to consider one $\pi_{\hat{v}}(\mathrm{a})$ and $p_i(\hat{v})$ for each equivalence class.
- The QCLP has
  - a finite number of both variables and constraints;
  - linear and quadratic constraints; and
  - (only) weak inequality constraints.

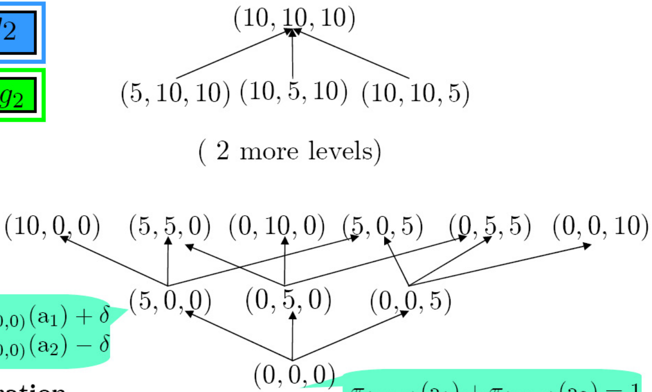
QCLP - Example



- 3 bidders: $N = \{1, 2, 3\}$
- 2 goods: $G = \{g_1, g_2\}$
- Bundles: $b_1 = \{g_1\}$, $b_2 = \{g_1, g_2\}$, and $b_3 = \{g_2\}$
- $\gamma = 2$
- 2 steps: $c_{i,1} = 5, c_{i,2} = 10$, for all $i \in N$.

QCLP - Example





$\pi_{(5,0,0)}(a_1) \geq \pi_{(0,0,0)}(a_1) + \delta$
$\pi_{(5,0,0)}(a_2) \leq \pi_{(0,0,0)}(a_2) - \delta$

$\pi_{(0,0,0)}(a_1) + \pi_{(0,0,0)}(a_2) = 1$

Lattice illustration

- Nodes: valuation profiles
- Edges: between v and v' whenever v and v' differ only in one bidder's valuation and this difference is exactly 5

3. Analytic Construction of Solutions to the QCLP

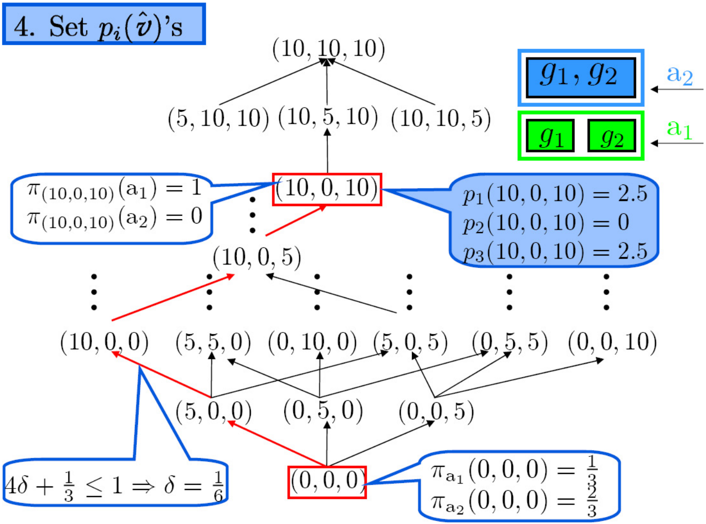
1. Set $\pi_{(0,\ldots,0)}(\mathrm{a})$'s nearly arbitrarily requiring that $\pi_{(0,\ldots,0)}(\mathrm{a}) = 0$ if a is not maximal.
   - E.g., it's always OK to set $\pi_{(0,\ldots,0)}(\mathrm{a}) = \langle \epsilon_1, \ldots, \epsilon_n \rangle$, $0 < \epsilon_i < 1$, for all maximal a.
   - many other settings also work; restrictions apply

2. Pick a δ that satisfies the hardest path from $(0, \ldots, 0)$.

3. Inductively set $\pi_{\hat{v}}(\mathrm{a})$'s using δ and realizing weak inequality constraints as equalities.

4. Set payments $p_i(\ldots, \hat{v}_i, \ldots) = \sum_{\substack{1 \le \ell \le j_i | \\ c_{i,j_i} \le \hat{v}_i < c_{i,j_i+1}}} \delta . c_{i,\ell}$

Analytic Construction - Example



A Polynomial Time Algorithm

- Constructing the mechanism may require exponential time in $|N|$ and $|G|$.
  - $\pi_{\hat{v}}(\mathrm{a})$'s may induce an exponential number of maximal allocations in the support of the mechanism.

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  - $\pi_{\hat{v}}(\mathrm{a})$'s may induce an exponential number of maximal allocations in the support of the mechanism.
- We give a polynomial-time construction algorithm that
  - picks a polynomial-size set of maximal allocations which can preserve our properties of interest, and
  - induces $\pi_{\hat{v}}(\mathrm{a})$'s given this set.

Theorem

We can construct a γ-step randomized mechanism M_γ in time polynomial in $|N|$ and $|G|$ such that M_γ is strategyproof and revenue monotonic and satisfies participation, maximality and (γ-step, $\frac{1}{n^2\gamma}$) consumer sovereignty.

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Summary

- There is no deterministic CA mechanism that satisfies strategyproofness, participation, consumer sovereignty, maximality, and revenue monotonicity.
    - In deterministic CA mechanisms, more bidders does not necessarily mean more competition.

- There exist stepwise randomized CA mechanisms defined for known single-minded bidders that satisfy strategyproofness, participation, consumer sovereignty, maximality and revenue monotonicity.
    - We characterized the class of all such mechanisms.
    - We gave a polynomial-time algorithm for constructing such a mechanism.

Future Work

- Identify stepwise randomized mechanisms that maximize objective functions of interest.
  - E.g. identify those that maximize revenue.
- Investigate optimally setting the parameters over which we have design freedom:
  - $\pi_{(0,\ldots,0)}(\mathrm{a})$'s;
  - set of maximal allocations in the support of the mechanism;
  - γ;
  - δ (as long as it is small enough); and
  - $c_{i,j}$'s.
- Prove or disprove the conjecture that we can allow $c_{i,j}$'s to depend on $\hat{v}$.
- Extend our result to unknown single-minded bidders or prove that such an extension is impossible.