Solving (Weighted) Partial MaxSat Through Satisfiability Testing

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SAT 2009



Partial MaxSat

```
C_1
...
...
Soft Clauses
...
C_n
C_{n+1}
...
Hard Clauses
...
C_{n+m}
```

Partial MaxSat is the problem of finding an *assignment* to the variables of \mathcal{C} such that no hard clause is falsified and the minimum number of soft clauses are falsified.



Partial MaxSat

```
1: C_1 ... Soft Clauses ... 1: C_{n+1} \infty: C_{n+1} ... Hard Clauses ... \infty: C_{n+m}
```

Partial MaxSat is the problem of finding an *assignment* to the variables of C that minimizes the cost of the falsified clauses.



Weighted Partial MaxSat

```
w_1: C_1 ... Soft Clauses ... w_n: C_n ... C_{n+1} ... Hard Clauses ... \infty: C_{n+m}
```

Weighted Partial MaxSat is the problem of finding an assignment to the variables of $\mathcal C$ that minimizes the cost of the falsified clauses.



Solving Partial MaxSat Through Satisfiability Testing

General approach:

```
1: C_1 \vee b_1
                             Soft Clauses
 1: C_n \vee b_n
\infty: C_{n+1}
                            Hard Clauses
\infty: C_{n+m}
\infty: CNF(\sum b_i \leq k)
```

if SAT(k-1) is unsatisfiable and SAT(k) satisfiable, then k is the optimum.

Solving Weighted Partial MaxSat Through Satisfiability Testing

General approach:

```
W_1: C_1 \vee b_1
                                  Soft Clauses
W_n: C_n \vee b_n
\infty: C_{n+1}
                                 Hard Clauses
\infty: C_{n+m}
\infty: CNF(\sum b_i * w_i < k)
```

if SAT(k - 1) is unsatisfiable and SAT(k) satisfiable, then k is the optimum.

MaxSat Solvers Based On Satisfiability Testing

Solvers at MaxSat Evaluation 2008:

- Weighted Partial MaxSat:
 - SAT4Java, D. L. Berre
- Partial MaxSat:
 - Msu1.2. (implementation of FU&MALIK algorithm)
 J. Marques-Silva, V. Manquinho and J. Planes.
 - Msu4.0. J. Marques-Silva and J. Planes.

Our contribution:

- A Weighted version of the Fu&Malik algorithm (WPM1) together with its proof of correctness
- Another Partial MaxSat algorithm variant of the Fu&MALIK algorithm (PM2), and the proof of its correctness

The FU&MALIK algorithm

```
input: \varphi = \{C_1, \ldots, C_m\}
cost := 0
                                                             Optimal
while true do
       (st, \varphi_c) := SAT(\varphi)
                                                             Call to the SAT solver
       if st = SAT then return cost
       BV := \emptyset
                                                             Set of blocking variables
       for each C \in \varphi_c do
              if C is soft then
                      b := new blocking variable
                      \varphi := \varphi \setminus \{C\} \cup \{C \lor b\}
                                                             Add blocking variable
                      BV := BV \cup \{b\}
       if BV = \emptyset then return UNSAT
                                                             No soft clauses in the core
       \varphi := \varphi \cup \mathit{CNF}(\sum_{b \in \mathit{BV}} b = 1)
                                                             Add cardinality as hard clauses
       cost := cost + 1
```

FU&MALIK algorithm as Complete Inference

Let φ be a Partial MaxSat Formula, given that φ is unsatisfiable

$$\frac{\varphi}{1:\Box}$$
 φ'

 φ and $\varphi' \wedge (1 : \Box)$ are MaxSat equivalent, i.e., the cost of the optimal assignment of φ is equal to the optimal cost of $\varphi' \wedge (1 : \Box)$

FU&MALIK algorithm as Complete Inference

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1:	X
1:	$\neg x$
1:	
1:	$x \vee b_1$
1:	$\neg x \lor b_2$
∞ :	$b_1 + b_2 = 1$

Let φ be a Weighted Partial MaxSat Formula, given that SAT(φ) is unsatisfiable

$$\frac{\varphi}{\mathbf{w}:\square}$$
$$\varphi'$$

$$\varphi$$
 and $\varphi' \wedge (w : \Box)$ are MaxSat equivalent

Let φ be a Weighted Partial MaxSat Formula, given that SAT(φ) is unsatisfiable

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$$\varphi'$$

$$\varphi$$
 and $\varphi' \wedge (w : \Box)$ are MaxSat equivalent

100 : *x* 100 : ¬*x*

Let φ be a Weighted Partial MaxSat Formula, given that SAT(φ) is unsatisfiable

$$\frac{\varphi}{\mathbf{w}:\square}$$
 φ'

 φ and $\varphi' \wedge (w : \Box)$ are MaxSat equivalent

100:
$$x$$

100: $\neg x$
100: \Box
100: $x \lor b_1$
100: $\neg x \lor b_2$
 ∞ : $b_1 + b_2 = 1$

Let φ be a Weighted Partial MaxSat Formula, given that SAT(φ) is unsatisfiable

$$\frac{\varphi}{\mathbf{w}:\square}$$
 φ'

 φ and $\varphi' \wedge (w : \Box)$ are MaxSat equivalent

100 :	X	100 :	X
100 :	$\neg x$	200 :	$\neg x$
100 :		100 :	
100 :	$x \vee b_1$	100 :	$x \vee b_1$
100 :	$\neg x \lor b_2$	100 :	$\neg x \lor b_2$
∞ :	$b_1 + b_2 = 1$	100 :	$\neg x$
		∞ :	$b_1 + b_2 = 1$

The WPM1 algorithm

```
input: \varphi = \{(C_1, w_1), \dots, (C_m, w_m), w_i > 0\}
cost := 0
                                                                     Optimal
while true do
      (st, \varphi_c) := SAT(\{C_i \mid (C_i, w_i) \in \varphi\})
                                                                     Call SAT solver without weights
      if st = SAT then return cost
      BV := \emptyset
                                                                     Blocking variables of the core
      w_{min} := \min\{w_i \mid C_i \in \varphi_c \text{ and } C_i \text{ is soft}\}\
     for each C_i \in \varphi_c do
            if C<sub>i</sub> is soft then
                  b_i := \text{new blocking variable}
                 \varphi := \varphi \setminus \{(C_i, W_i)\} \cup \{(C_i, W_i - W_{min})\} \cup \{(C_i \vee b_i, W_{min})\}
                                                                     Duplicate soft clauses
                  BV := BV \cup \{b_i\}
      if BV = \emptyset then return UNSAT
                                                                     No soft clauses in the core
     else \varphi := \varphi \cup CNF(\sum_{b \in BV} b = 1)
                                                                     Add cardinality as hard clauses
      cost := cost + W_{min}
```

```
\begin{array}{cccc} 1: & x \\ 2: & y \\ 3: & z \\ \infty: & \neg x \lor \neg y \\ \infty: & x \lor \neg z \\ \infty: & y \lor \neg z \end{array}
```

```
3:
                              2:
 2 :
 3:
                              3:
                                        z \vee b_1
\infty :
        \neg x \lor \neg y
                            \infty :
                                     \neg x \lor \neg y
\infty:
       x \vee \neg z
                            \infty: X \vee \neg Z
                            \infty: y \vee \neg z
\infty :
       y \vee \neg z
                             \infty: b_1 = 1
```

```
4:
                          3:
                                                    3:
                                                              z \vee b_1
               Х
 2:
                          2:
                                                   1:
                                                        x \vee b_2
                                                   1:
 3:
                          3:
                                 z \vee b_1
                                                          y \vee b_3
                                \neg x \lor \neg y
                                                  \infty: \neg x \lor \neg y
\infty :
       \neg x \lor \neg y
                         \infty :
       x \lor \neg z
                        \infty: X \vee \neg Z
                                                        x \lor \neg z
\infty:
                                                  \infty :
\infty: y \lor \neg z
                         \infty: y \vee \neg z
                                                  \infty :
                                                        y \vee \neg z
                                                        b_1 = 1
                         \infty: b_1 = 1
                                                  \infty :
                                                  \infty: b_2 + b_3 = 1
```

```
\begin{array}{ccc}
1: & x \\
2: & y \\
3: & z \\
\infty: & \neg x \lor \neg y \\
\infty: & x \lor \neg z \\
\infty: & y \lor \neg z
\end{array}
```

```
2 :
2 :
 2:
                                   x \vee b_1
 3 :
                                    z \vee b_2
        \neg x \lor \neg y
\infty:
                                  \neg x \lor \neg y
                            \infty :
\infty :
      x \vee \neg z
                                   x \vee \neg z
                            \infty :
\infty: y \lor \neg z
                            \infty: y \lor \neg z
                                   b_1 + b_2 = 1
                            \infty:
```

```
3 :
                                                                        x \vee b_1
                             2:
                                                                     z \vee b_2
                             2:
                                                             2:
 2:
                                                                     y \vee b_3
                             1:
                                       x \vee b_1
 3:
                                                             2:
                                                                        z \vee b_4
                                          z \vee b_2
        \neg x \lor \neg y
                                                            \infty: \neg x \lor \neg y
\infty :
                                      \neg x \lor \neg y
                            \infty :
       x \vee \neg z
                                                                    x \vee \neg z
\infty :
                                                            \infty :
                            \infty :
                                     x \vee \neg z
         y \vee \neg z
                                                                   y \vee \neg z
\infty :
                                                            \infty:
                            \infty :
                                     y \vee \neg z
                                                            \infty: b_1 + b_2 = 1
                                   b_1 + b_2 = 1
                            \infty :
                                                            \infty: b_3 + b_4 = 1
```

```
4:
                                                                           1:
                                                                                                   z \vee b_2
                                      3:
 1:
                                                                                                  y \vee b_3
                                                      x \vee b_1
 2:
                                                                            1:
                                                                                                   z \vee b_4
                                      1:
                                                      z \vee b_2
 2:
                                                                           1:
                                                                                           x \vee b_1 \vee b_5
                                      2:
                                                     y \vee b_3
 1:
                                                                            1:
                                                                                           y \vee b_3 \vee b_6
                 x \vee b_1
                                      2:
                                                      z \vee b_4
 1:
                                                                           1:
                                                                                           z \vee b_4 \vee b_7
                 z \vee b_2
                                                 \neg x \lor \neg y
                                     \infty :
              \neg x \lor \neg y
                                                                                              \neg x \lor \neg y
\infty:
                                                                          \infty:
                                             X \vee \neg Z
                                     \infty :
\infty:
            x \vee \neg z
                                                                                                 X \vee \neg Z
                                                                          \infty :
                                                    V \vee \neg Z
                                     \infty :
                y \vee \neg z
                                                                                                 y \vee \neg z
\infty:
                                                                          \infty:
                                     \infty :
                                             b_1 + b_2 = 1
         b_1 + b_2 = 1
                                                                                         b_1 + b_2 = 1
\infty :
                                                                          \infty :
                                     \infty:
                                             b_3 + b_4 = 1
                                                                                         b_3 + b_4 = 1
                                                                          \infty:
                                                                                  b_5 + b_6 + b_7 = 1
```

 ∞ :

The PM2 algorithm

```
input: \varphi = \{C_1, \ldots, C_m\}
BV := \{b_1, \dots, b_m\}
\varphi_w := \{C_1 \vee b_1, \ldots, C_m \vee b_m\}
cost := 0
I := \emptyset
while true do
         (st, \varphi_c) := SAT(\varphi_w \cup CNF(\sum_{b \in BV} b \le cost))
         if st = SAT then return cost
         remove the hard clauses from \varphi_c
         if \varphi_c = \emptyset then return UNSAT
         B := \emptyset
         for each C = C_i \vee b_i \in \varphi_c do
                 B := B \cup \{b_i\}
         L := L \cup \{\varphi_c\}
         \mathbf{k} := |\{\psi \in \mathbf{L} \mid \psi \subseteq \varphi_{\mathbf{c}}\}|
         \varphi_w := \varphi_w \cup \mathit{CNF}(\sum_{b \in B} b \geq k)
         cost := cost + 1
```

Set of all blocking variables Protect all clauses Optimal Set of Cores

Call to SAT solver with at most cardinality

Blocking variables of the core

Num. of cores contained in φ_c Add at least cardinality constraint

```
\begin{array}{llll} 1: & C_1 \vee b_1 \\ 1: & C_2 \vee b_2 \\ 1: & & \dots \\ 1: & C_k \vee b_k \\ 1: & & \dots \\ \infty: & C_n \vee b_n \\ \infty: & C_{n+1} \\ \infty: & & \dots \\ \infty: & C_{n+m} \\ \infty: & \sum_{1}^{n} b_i \leq 0 \end{array}
```

```
C_1 \vee b_1
                                                 C_1 \vee b_1
            C_1 \vee b_1
                                                                                     C_2 \vee b_2
            C_2 \vee b_2
                                                 C_2 \vee b_2
                                                                                     C_i \vee b_k
                                         C_k \vee b_k
       C_k \vee b_k
                                                                                     C_n \vee b_n
                                                C_n \vee b_n
           C_n \vee b_n
                                                                                       C_{n+1}
                                                                      \infty :
                                                 C_{n+1}
            C_{n+1}
\infty :
                                 \infty :
                                                                      \infty :
\infty :
                                 \infty :
                                                                                       C_{n+m}
                                                                      \infty :
            C_{n+m} \infty:
                                                C_{n+m}
\infty :
                                                                             b_1 + b_2 > 1
                                                                      \infty :
        \sum_{1}^{n} b_{i} \leq 0
                            \infty :
                                         b_1 + b_2 > 1
\infty :
                                                                      \infty: \sum_{i=1}^{k} b_i \geq 2
                                          \sum_{i=1}^{n} b_i \leq 1
                                 \infty :
                                                                      \infty :
```

Solvers & Benchmarks

- Our MaxSat solvers: WPM1 and PM2
 - SAT solver: picosat
 - Cardinality constraints: regular encoding for WPM1, sequential counters for PM2.
- Other solvers based on satisfiability Testing: SAT4Java, msu1.2 and msu4.0.
- Benchmarks: crafted and industrial instances available from the MaxSat Evaluation 2008.

Exp: Unweighted MaxSat Category

set	best08	WPM1	PM2	msu1.2	msu4.0	SAT4J		
	Crafted							
Maxcut (62)	81.8(52)	0.03(4)	175(7)	0.28(4)	1.71(3)	0.93(2)		
Maxcut (58)	4.5(40)	-(0)	-(0)	- (0)	- (0)	- (0)		
Maxcut Spinglass (5)	1.62(3)	0.85(2)	102.5(2)	0.68 (2)	-(0)	-(0)		
Industrial								
SeanSafarpour(112)	57.5(72)	66.6(81)	90.2(75)	57.5(72)	64.4(50)	14.5(10)		

Exp: Unweighted MaxSat Category

set	best08	WPM1	PM2.1	msu1.2	msu4.0	SAT4J		
Crafted								
Maxcut (62)	81.8(52)	0.03(4)	72(10)	0.28(4)	1.71(3)	0.93(2)		
Maxcut (58)	4.5(40)	-(0)	-(0)	- (0)	- (0)	- (0)		
Maxcut Spinglass (5)	1.62(3)	0.85(2)	4(2)	0.68 (2)	-(0)	-(0)		
Industrial								
SeanSafarpour(112)	57.5(72)	66.6(81)	60(78)	57.5(72)	64.4(50)	14.5(10)		

Exp: Partial MaxSat Category

set	best08	WPM1	PM2	msu1.2	msu4.0	SAT4J		
Crafted								
Maxclique (96)	2.4(96)	50.4(1)	-(0)	-(0)	106(61)	114(52)		
Maxclique (62)	73(36)	41.2(11)	32.6(6)	4.9(7)	105.2(13)	50.5(13)		
Maxone (80)	0.46(80)	16(46)	105.7(79)	52.7(40)	118.2(35)	96.6(31)		
Maxone Structured (60)	10.1(60)	0.69(2)	547.5(13)	122.7(2)	3.34(1)	10.1(60)		
		ı	ndustrial					
Bcp (59)	49(46)	32 (57)	67.4(56)	49.2(46)	-(0)	13.3(10)		
$\frac{Bcp}{hipp-yRa1}$ (1183)	19(1111)	3(1122)	6(1163)	7.2(1105)	0.29(348)	12.2(1109)		
$\frac{Bcp}{msp}$ (148)	49(104)	15.5(26)	106(94)	4.9(25)	22.9(79)	8.8(93)		
$\frac{Bcp}{mtg}$ (215)	26(206)	5.8(170)	1.3(215)	17.5(164)	0.43(22)	57(196)		
$\frac{Bcp}{syn}(74)$	63(34)	14.1(32)	14(38)	51.1(31)	105.2(11)	67.4(21)		
$\frac{Pbo}{mqc-nencdr}$ (128)	167(115)	80.4(50)	125(84)	50.3(54)	167.5(115)	180.6(102)		
$\frac{Pbo}{mqc-nlogencdr}$ (128)	111(128)	67.1(75)	130(106)	53(65)	111(128)	117.5(126)		
$\frac{Pbo}{routing}$ (15)	2.9(15)	1(15)	24.7(15)	1.35(15)	54.9(15)	26.4(9)		

Exp: Partial MaxSat Category

set	best08	WPM1	PM2.1	msu1.2	msu4.0	SAT4J		
Crafted								
Maxclique (96)	2.4(96)	50.4(1)	126(54)	-(0)	106(61)	114(52)		
Maxclique (62)	73(36)	41.2(11)	62(12)	4.9(7)	105.2(13)	50.5(13)		
Maxone (80)	0.46(80)	16(46)	22(80)	52.7(40)	118.2(35)	96.6(31)		
Maxone Structured (60)	10.1(60)	0.69(2)	253(34)	122.7(2)	3.34(1)	10.1(60)		
			Industrial					
$\frac{Bcp}{fir}$ (59)	49(46)	32 (57)	18(58)	49.2(46)	-(0)	13.3(10)		
<u>Вср</u> hipp−yRa1 (1183)	19(1111)	3(1122)	13.5(1163)	7.2(1105)	0.29(348)	12.2(1109)		
$\frac{Bcp}{msp}$ (148)	49(104)	15.5(26)	384.2(36)	4.9(25)	22.9(79)	8.8(93)		
$\frac{Bcp}{mtg}$ (215)	26(206)	5.8(170)	10.5(214)	17.5(164)	0.43(22)	57(196)		
$\frac{Bcp}{syn}(74)$	63(34)	14.1(32)	71.2(34)	51.1(31)	105.2(11)	67.4(21)		
$\frac{Pbo}{mqc-nencdr}$ (128)	167(115)	80.4(50)	142(78)	50.3(54)	167.5(115)	180.6(102)		
$\frac{Pbo}{mqc-nlogencdr}$ (128)	111(128)	67.1(75)	140.3(97)	53(65)	111(128)	117.5(126)		
$\frac{Pbo}{routing}$ (15)	2.9(15)	1(15)	24.7(15)	2.9(15)	54.9(15)	26.4(9)		

Exp: Weighted (Partial) MaxSat Categories

set	set # best08							
Weighted MaxSat Category								
Crafted								
KeXu/ 15 IncWMaxsatz - 126.5(15) 478(1) 7.7(4								
Ramsey/	48	lb-psat - 1.63(37)	0.05(34)	16(35)				
WMaxcut/dimacs_mod/	62	ToolBar3 - 59(56)	0.12(3)	0.84(2)				
WMaxcut/Random/	40	MiniMaxSAT - 5.43(40)	-(0)	-(0)				
WMaxcut/Spinglass/	5	MiniMaxSAT - 27.6(4)	-(0)	-(0)				
V	leighte	d Partial MaxSat Category						
		Crafted						
Auctions/Auc_paths/	88	IncWMaxsatz - 8.4(88)	-(0)	497(15)				
Auctions/Auc_regions/	88	MiniMaxSAT - 1.7(84)	-(0)	166(76)				
Auctions/Auc_Sched/	84	MiniMaxSAT - 46(84)	-(0)	317(49)				
Random-net/	350	Clone - 72(236)	194(91)	331(13)				
Pseudo-factor/	186	IncWMaxsatz - 0.07(186)	16(124)	3.3(186)				
Pseudo- miplib/	16	SAT4J - 13(6)	0.29(3)	13(6)				
QCP/	25	SAT4J - 6.14(25)	0.27(25)	6.14(25)				
WCSP/Planning/	71	SAT4J - 6.55(71)	0.9(46)	6.55(71)				
WCSP/Spot5/Dir/	21	Clone - 87.6(6)	2.31(4)	76(3)				
WCSP/Spot5/Log/	21	Clone - 15(6)	0.52(5)	63.8(3)				
Industrial								
Protein_ins	12	MiniMaxSat - 482(8)	42(1)	6.05(1)				

Thanks!