Time-Bounded Sequential Parameter Optimization

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Automatically find good instantiation of parameters

- ▶ Eliminate most tedious part of algorithm design and end use
- Save development time & improve performance

Parameter Optimization Methods

- Lots of work on numerical parameters, e.g.
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 - Iterated Local Search, ParamILS [Hutter et al., AAAI '07 & JAIR'09]
- Success of parameter optimization
 - Many parameters (e.g., CPLEX with 63 parameters)
 - Large speedups (sometimes orders of magnitude!)
 - For many problems: SAT, MIP, time-tabling, protein folding, ...

Limitations of Model-Free Parameter Optimization

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Response surface models can help

 Predictive models of algorithm performance with given parameter settings

- Original SPO [Bartz-Beielstein et al., '05-present]
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 - Multiple benchmark instances
 - Very promising results for both

Outline

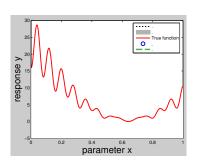
1. Sequential Model-Based Optimization

2. Reducing the Computational Overhead Due To Models

3. Conclusions

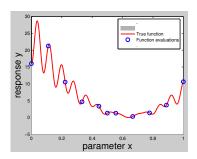
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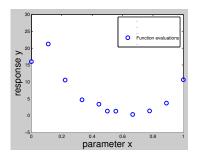
Blackbox function optimization; function = algo. performance

0. Run algorithm with initial parameter settings

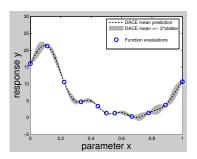


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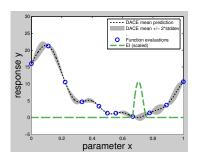
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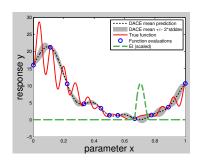
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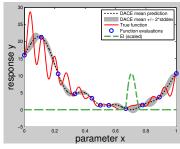
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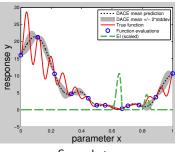
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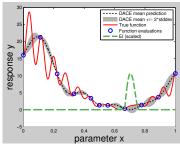


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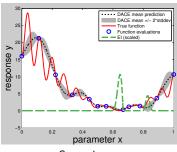


Second step

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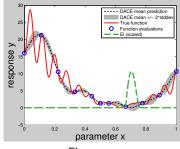


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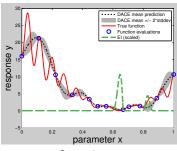


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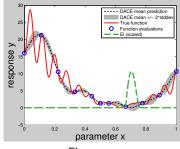


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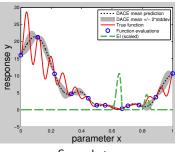


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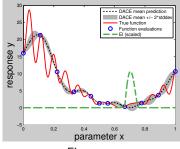


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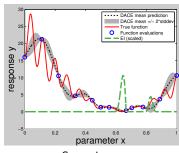


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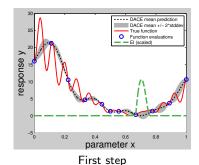


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Example times

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purpose of the parameter x

---- DACE mean prediction

Function evaluations

DACE mean +/- 2*stdde

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- ▶ How to choose number of param. settings in initial design?
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- ▶ How to choose number of param. settings in initial design?
 - ▶ Too large: take too long to evaluate all of the settings
 - ► Too small: poor first model, might not recover
- Our solution: simply drop the initial design
 - ▶ Instead: interleave random settings during the search
 - Much better anytime performance

Overhead due to Models

Central SMBO algorithm loop

- ► Repeat: Example times
 - 1. Fit model using performance data gathered so far 50s
 - 2. Use model to select promising parameter setting 20s
 - 3. Perform algorithm run(s) with that parameter setting 10s
- → Only small fraction of time spent actually running algorithms

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Solution 1

- Do more algorithm runs to bound model overhead
 - Select not one but many promising points (little overhead)
 - Perform runs for at least as long as phases 1 and 2 took

Heuristic Mechanism

lacktriangle Compare one configuration heta at a time to the incumbent $heta_{\it inc}$

Stop once time bound is reached

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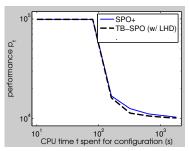
- ▶ TB-SPO
 - Get ordered list of promising parameter settings using model
 - Interleave random settings: 2nd, 4th, etc
 - Compare one param. setting at a time to incumbent
 - Nice side effect: additional runs on good random settings
- "Strawman" algorithm: TB-Random
 - Only use random settings
 - Compare one param. setting at a time to incumbent

Experimental validation: setup

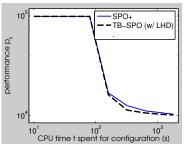
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 - Prominent SAT solver with 4 continuous parameters
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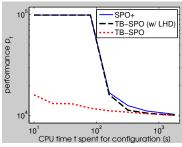
- Optimizing SLS algorithm SAPS
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- Seven different SAT instances
 - 1 Quasigroups with holes (QWH) instance used previously
 - 3 instances from Quasigroup completion (QCP)
 - 3 instances from Graph colouring based on smallworld graphs (SWGCP)



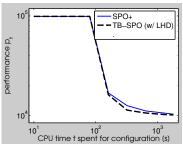
Both methods with same LHD



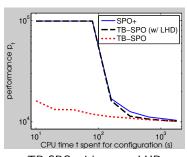
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TB-SPO with empty LHD

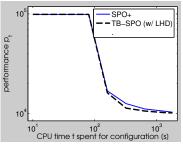


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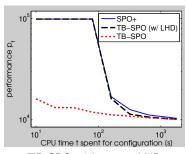


TB-SPO with empty LHD

Scenario	SPO^+	TB-SPO	pval1
SAPS-QCP-MED $[\cdot 10^{-2}]$	4.50 ± 0.31	$\textbf{4.32} \pm \textbf{0.21}$	$4 \cdot 10^{-3}$
Saps-QCP-q075	3.77 ± 9.72	0.19 ± 0.02	$2 \cdot 10^{-6}$
Saps-QCP-q095	49.91 ± 0.00	$\boldsymbol{2.20\pm1.17}$	$1 \cdot 10^{-10}$
Saps-QWH [·10 ³]	10.7 ± 0.76	10.1 ± 0.58	$6 \cdot 10^{-3}$
Saps-SWGCP-MED	49.95 ± 0.00	0.18 ± 0.03	$1\cdot 10^{-10}$
Saps-SWGCP-Q075	50 ± 0	$\textbf{0.24} \pm \textbf{0.04}$	$1 \cdot 10^{-10}$
Saps-SWGCP-Q095	50 ± 0	$\boldsymbol{0.25 \pm 0.05}$	$1\cdot 10^{-10}$



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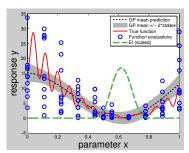
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Scenario	SPO^+	TB-SPO	TB-Random	pval1	pval2
Saps-QCP-MED $[\cdot 10^{-2}]$	4.50 ± 0.31	$\textbf{4.32} \pm \textbf{0.21}$	4.23 ± 0.15	$4 \cdot 10^{-3}$	0.17
Saps-QCP-q075	3.77 ± 9.72	0.19 ± 0.02	0.19 ± 0.01	$2 \cdot 10^{-6}$	0.78
Saps-QCP-Q095	49.91 ± 0.00	$\boldsymbol{2.20 \pm 1.17}$	2.64 ± 1.24	$1\cdot 10^{-10}$	0.12
Saps-QWH [·10 ³]	10.7 ± 0.76	10.1 ± 0.58	9.88 ± 0.41	$6 \cdot 10^{-3}$	0.14
Saps-SWGCP-MED	49.95 ± 0.00	0.18 ± 0.03	0.17 ± 0.02	$1\cdot 10^{-10}$	0.37
Saps-SWGCP-Q075	50 ± 0	$\textbf{0.24} \pm \textbf{0.04}$	$\boldsymbol{0.22 \pm 0.03}$	$1 \cdot 10^{-10}$	0.08
Saps-SWGCP-Q095	50 ± 0	0.25 ± 0.05	0.28 ± 0.10	$1 \cdot 10^{-10}$	0.89

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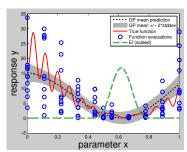
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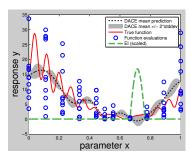


Model I: noisy fit of original response

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- ▶ Model II (used in SPO, SPO⁺, and TB-SPO)
 - Compute empirical mean of responses at each param. setting
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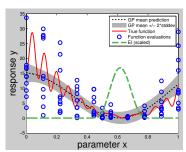


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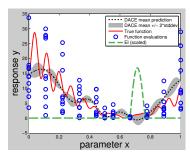


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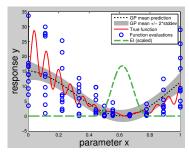


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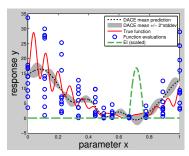


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 - But assumes empirical means are perfect (even when based on just 1 run!)
 - Cheaper (here 11 means vs 110 raw data points)



Model I: noisy fit of original response



Model II: noise-free fit of empir. means

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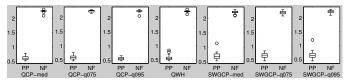
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Complexity of projected process (PP) approximation

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Empirical Evaluation of the Model

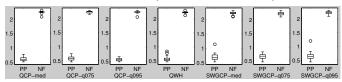
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 Log_{10} of CPU time (in seconds)

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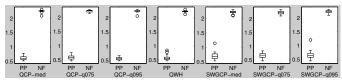
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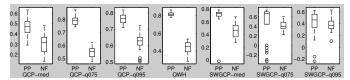
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Correlation (high is good, 1 is optimal)

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 - ▶ P: TB-SPO(PP)

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 - ► F: FocusedILS (variant of ParamILS; limited by discretization)

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► F: FocusedILS (variant of ParamILS; limited by discretization)

Scenario	TB-Random	TB-SPO	TB-SPO(PP)	FOCUSEDILS
Saps-QCP-MED $[\cdot 10^{-2}]$	4.23 ± 0.15	4.32 ± 0.21	$\textbf{4.13} \pm \textbf{0.14}$	5.12 ± 0.41
Saps-QCP-q075	0.19 ± 0.01	0.19 ± 0.02	0.18 ± 0.01	0.24 ± 0.02
Saps-QCP-q095	2.64 ± 1.24	2.20 ± 1.17	1.44 ± 0.53	2.99 ± 3.20
Saps-QWH $[\cdot 10^3]$	9.88 ± 0.41	10.1 ± 0.58	$\textbf{9.42} \pm \textbf{0.32}$	10.6 ± 0.49
Saps-SWGCP-MED	0.17 ± 0.02	0.18 ± 0.03	0.16 ± 0.02	0.27 ± 0.12
Saps-SWGCP-q075	0.22 ± 0.03	0.24 ± 0.04	$\textbf{0.21} \pm \textbf{0.02}$	$\textbf{0.35} \pm \textbf{0.08}$
Saps-SWGCP-Q095	0.28 ± 0.10	0.25 ± 0.05	0.23 ± 0.05	0.37 ± 0.16

- ▶ TB-SPO(PP) best on all 7 instances
- Good models do help

Outline

- 1. Sequential Model-Based Optimization
- 2. Reducing the Computational Overhead Due To Models
- 3. Conclusions

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 - Uses efficient approximate Gaussian process model
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- Per-instance approaches
 - Build joint model of instance features and parameters
 - Given a new unseen instance:
 - + Compute instance features (fast)
 - + Use parameter setting predicted to be best for those features