Analysis and Generation of Pseudo-Industrial MaxSAT Instances

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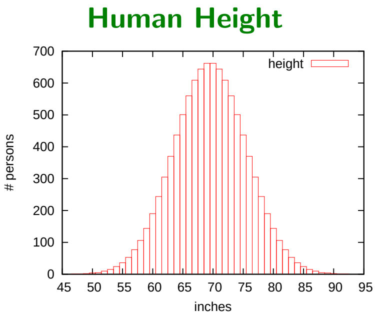
Modeling “Real” Problems

- SAT and MaxSAT are central problems in AI
- They are NP-complete... but this is in the **worst** case
- State-of-the-art SAT and MaxSAT solvers (heuristics, backjumping, learning, restarts, SAT-base MaxSAT solvers ...) are of practical use with **real-world** SAT and MaxSAT instances... **Why???**
- Generating **pseudo-real** instances can help us to solve **real** instances

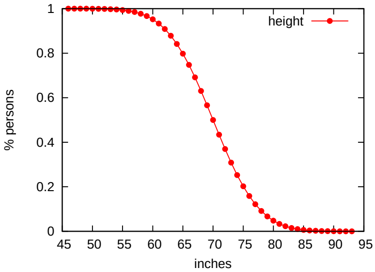
Exponential vs Powerlaw Distributions



- Human heights follows a normal distribution with mean 70 inches

Exponential vs Powerlaw Distributions

Human Height

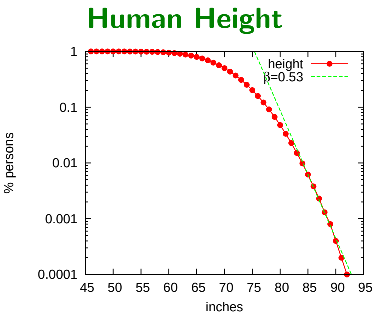


- **Cumulative Distribution:** $F(x) =$% persons with height $\geq x$

$$F(x) = \int_x^\infty f(x)dx$$

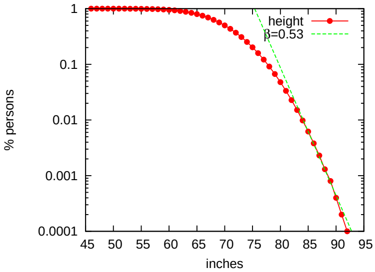
- How decays the **tail**:
  Exponentially: $f^{exp}(x) \sim e^{-\beta x}$ $F^{exp}(x) \sim e^{-\beta x}$
  Powerlaw: $f^{pow}(x) \sim x^{-\alpha}$ $F^{pow}(x) \sim x^{-\alpha+1}$

Exponential vs Powerlaw Distributions



- How decays the **tail**:
  Exponentially: $f^{exp}(x) \sim e^{-\beta\, x}$ $F^{exp}(x) \sim e^{-\beta\, x}$
  Powerlaw: $f^{pow}(x) \sim x^{-\alpha}$ $F^{pow}(x) \sim x^{-\alpha+1}$
- Modify the scale:
  - **Exponential:** we get a line in the **logarithmic** scale
  - **Powerlaw:** we get a line in the **double-logarithmic** scale

Human Height

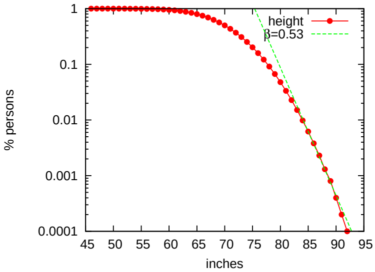


- logarithmic scale
- Decay as

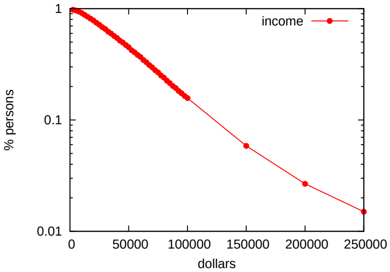
$$F^{exp}(x) \sim e^{-0.53\,x}$$

Giants are very rare

Human Height



Personal Income



- logarithmic scale
- Decay as

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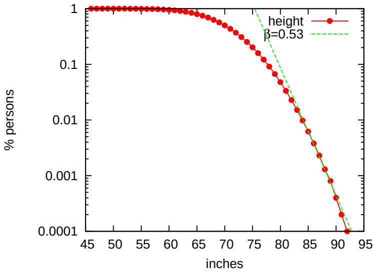
Giants are very rare

- logarithmic scale

Exponential vs Powerlaw Distributions

Human Height



- logarithmic scale
- Decay as

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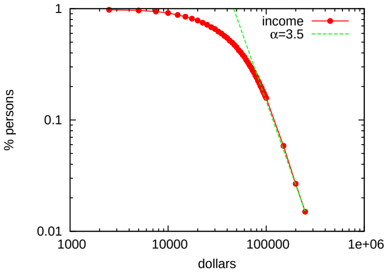
Personal Income



- double-logarithmic scale
- Decay as

$$F^{pow}(x) \sim x^{-2.5}$$

Riches are not so rare

Estimation of the Parameters α, β, Δ and k_{min}

Empirical data $k_1, \dots, k_n$,

- **Estimate α (or β)** maximizing $P(k_1, \dots, k_n \mid \alpha)$

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$$P^{pow}(k_1, \ldots, k_n \mid \alpha) = \prod_{i=1}^{n} \frac{\alpha-1}{k_{min}} \left(\frac{k_i}{k_{min}}\right)^{-\alpha}$$

$$\frac{\partial}{\partial \alpha} \log P^{pow}(x_1, \ldots, x_n \mid \alpha) =$$
$$= \frac{n}{\alpha-1} - \sum_{i=1}^{n} \log \frac{k_i}{k_{min}} = 0$$

We get

$$\hat{\alpha} = 1 + \frac{n}{\sum_{i=1}^{n} \log(k_i / k_{min})}$$

For the discrete case, a good approx. for larges k_{min} is

$$\hat{\alpha} = 1 + \frac{n}{\sum_{i=1}^{n} \log \frac{k_i}{k_{min}-1/2}}$$

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$$P^{exp}(k_1, \dots, k_n \mid \beta) = \prod_{i=1}^{n} (1 - e^{-\beta})\, e^{-\beta(k_i - k_{min})}$$

$$\frac{\partial}{\partial\alpha} \log P^{pow}(x_1, \dots, x_n \mid \alpha) = \frac{n}{\alpha-1} - \sum_{i=1}^{n} \log \frac{k_i}{k_{min}} = 0$$

We get

$$\hat{\alpha} = 1 + \frac{n}{\sum_{i=1}^{n} \log(k_i / k_{min})}$$

$$\frac{\partial}{\partial\beta} \log P^{exp}(k_1, \dots, k_n \mid \beta) = \frac{n\, e^{-\beta}}{1-e^{-\beta}} - \sum_{i=1}^{n} (k_i - k_{min}) = 0$$

We get

$$\hat{\beta} = \log\left(\frac{n}{\sum_{i=1}^{n} (k_i - k_{min})} + 1\right)$$

For the discrete case, a good approx. for larges k_{min} is

$$\hat{\alpha} = 1 + \frac{n}{\sum_{i=1}^{n} \log \frac{k_i}{k_{min}-1/2}}$$

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- Compute the error **for every possible k_{min}** as

$$\Delta^{pow}(k_{min}) = \max_{k \geq k_{min}} \{|F^{emp}(k) - F^{pow}(k; \hat{\alpha}, k_{min})|\}$$

Estimation of the Parameters α, β, Δ and k_{min}

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- Compute the error **for every possible k_{min}** as

$$\Delta^{pow}(k_{min}) = \max_{k \geq k_{min}} \{|F^{emp}(k) - F^{pow}(k; \hat{\alpha}, k_{min})|\}$$

- **Estimate k_{min}** as the value resulting into the smallest error

$$\Delta^{pow} = \min_{k_{min} \geq 1} \Delta(k_{min})$$

- If $\Delta^{pow} > \Delta^{exp}$ the distribution is powerlaw

Two Models of Graphs

Erdös-Rényi Graphs

- For every pair $< v_i, v_j >$, put an edge with probability p
- Arity of nodes follows a binomial distribution

$$P(k) = \binom{n-1}{k} p^k \, (1-p)^{n-1-k}$$

Scalefree Graphs

- There is not a formal definition
- Arity of nodes follows a powerlaw distribution

$$P(k) = C \, k^{-\alpha}$$

- They are also self-similar

SAT Formulas as Bi-partite Graphs

Bi-partite graph: Nodes: are variables **v** and clauses **c**

Edges: **v** — **c** if clause **c** contains variable **v**

SAT Formulas as Bi-partite Graphs

Bi-partite graph: Nodes: are variables **v** and clauses **c**

Edges: **v**—**c** if clause **c** contains variable **v**

Arity of nodes: **v** number of occurrences

c size

SAT Formulas as Bi-partite Graphs

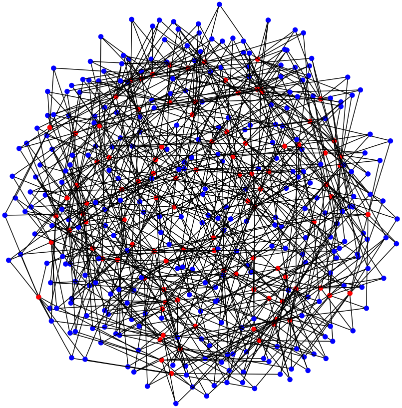
Bi-partite graph: Nodes: are variables **v** and clauses **c**

Edges: **v**—**c** if clause **c** contains variable **v**

Arity of nodes: **v** number of occurrences $\approx 4.25 * 3 = 12.75$

c size $= 3$

Random 3-CNF



Number of var occurrences very close to the mean

SAT Formulas as Bi-partite Graphs

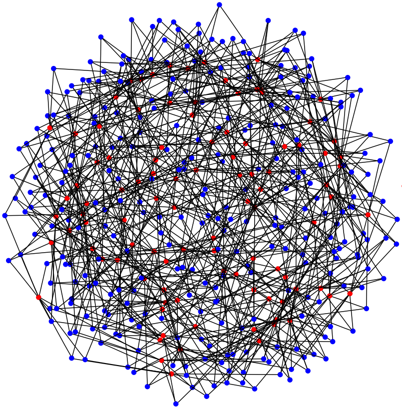
Bi-partite graph: Nodes: are variables v and clauses c

Edges: v—c if clause **c** contains variable **v**

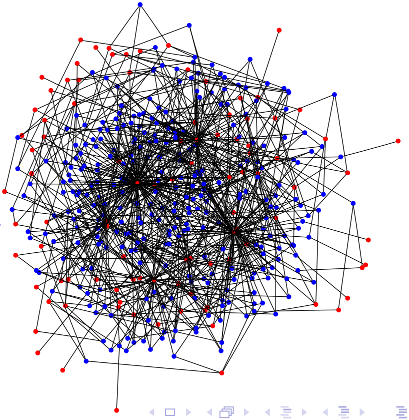
Hubs: v very frequent variables

c very large clauses

Random 3-CNF



Industrial Instance



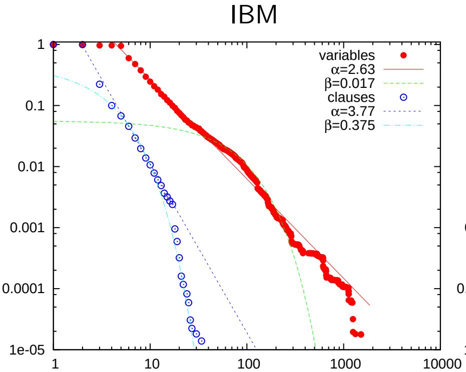
Study of some Families (Variables)

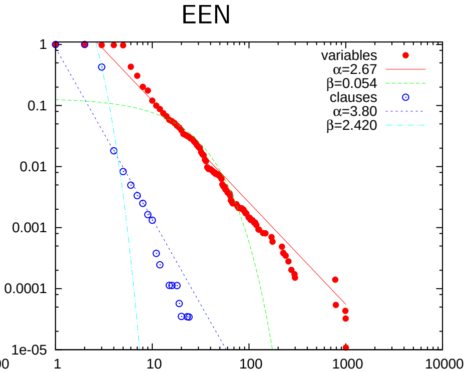
Family	inst	n	$E[V]$	$Var[V]$	Powerlaw α	k_{min}^{pow}	Δ^{pow}	Exponential β	k_{min}^{exp}	Δ^{exp}
cmu	3	16678	7.95	12.11	3.49	5	**0.072**	0.224	4	0.176
een	12	739744	7.60	13.26	2.67	10	**0.043**	0.054	15	0.136
fuhs	2	73486	9.05	14.56	2.79	62	**0.075**	0.181	4	0.158
goldb	11	114038	21.02	88.71	2.05	21	**0.042**	0.003	100	0.204
grieu	9	6914	364.21	42.23	1.77	100	0.577	0.004	100	0.538
ibm	38	4985723	10.75	23.97	2.63	7	**0.027**	0.017	45	0.083
manol	59	7827736	6.93	16.24	2.95	57	0.059	0.017	76	**0.033**
mizh	13	725644	12.49	148.12	4.09	15	0.172	0.034	22	0.247
narai	6	9642548	9.72	16.38	3.85	5	0.152	0.109	1	0.347
palac	2	298266	10.82	60.55	1.84	20	0.087	0.003	100	0.074
post	10	12906872	7.90	44.15	2.57	12	0.132	0.135	1	0.334
schup	7	2196731	8.09	12.40	2.59	41	0.120	0.063	9	0.182
simon	12	798804	7.78	11.96	2.53	14	**0.028**	0.022	50	0.065
uts	10	1420464	13.01	74.70	1.76	69	0.111	0.003	75	**0.088**
velev	60	8442829	88.31	379.04	1.82	13	**0.030**	0.003	87	0.287
random	40	400000	12.75	3.57	18.65	24	0.019	0.777	25	**0.008**
SAT'08	100	27964721	13.30	113.48	2.29	12	**0.051**	0.003	73	0.254

Study of some Families (Clauses)

Family	inst	m	$E[C]$	$Var[C]$	Powerlaw α	k_{min}^{pow}	Δ^{pow}	Exponential β	k_{min}^{exp}	Δ^{exp}
cmu	3	53769	2.46	1.21	5.35	3	0.126	1.778	3	**0.048**
een	12	2278059	2.47	0.69	3.80	4	**0.044**	2.420	3	0.046
fuhs	2	256742	2.59	0.82	4.89	5	0.041	2.182	3	**0.020**
goldb	11	710559	3.37	1.46	10.48	5	0.158	4.803	5	**0.008**
grieu	9	961030	2.62	0.76	8.54	26	0.108	3.878	3	**0.020**
ibm	38	21084555	2.54	1.57	3.77	6	**0.023**	0.375	4	0.032
manol	59	23244626	2.33	0.47						
mizh	13	3036234	2.98	0.91	1.58	1	0.328	0.408	1	0.334
narai	6	37639556	2.49	2.05	3.33	2	**0.088**	1.113	2	0.090
palac	2	1274356	2.53	9.33	1.71	4	0.116	1.055	2	0.116
post	10	42441234	2.40	1.39	3.33	2	0.143	2.884	33	**0.053**
schup	7	6947242	2.56	1.36	4.30	4	0.093	2.585	3	**0.046**
simon	12	2675233	2.32	0.90	3.76	4	0.033	0.498	5	**0.026**
uts	10	7101806	2.60	11.56	3.63	2	0.114	0.004	35	0.116
velev	60	253221473	2.94	9.01	3.35	72	0.042	0.021	28	**0.040**
random	40	1700000	3.00	0.00						
SAT'08	100	140942860	2.64	5.68	3.03	17	**0.054**	0.074	10	0.068

Cumulative Distributions for some Families (1)

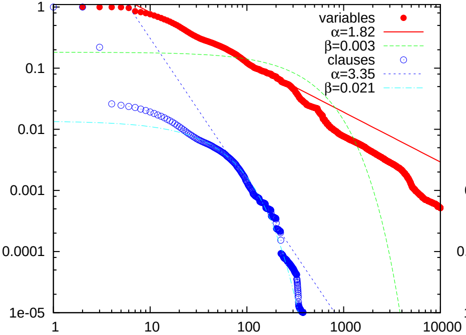




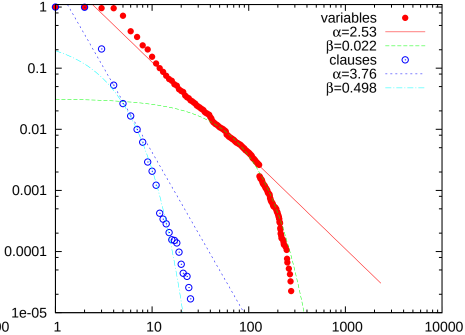
- **Vars** follows a **powerlaw** with $\alpha \approx 2.6$
- **Clauses** follows a **powerlaw** with $\alpha \approx 3.8$

Cumulative Distributions for some Families (2)

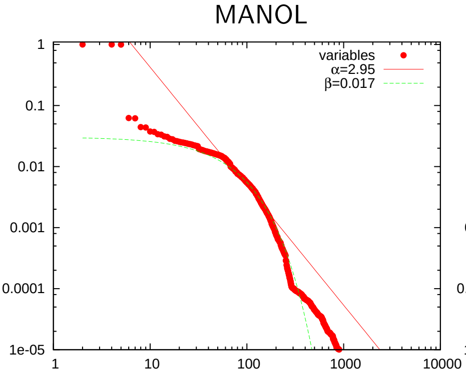
VELEV

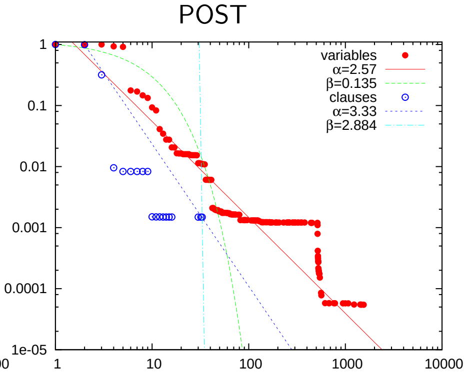


SIMON

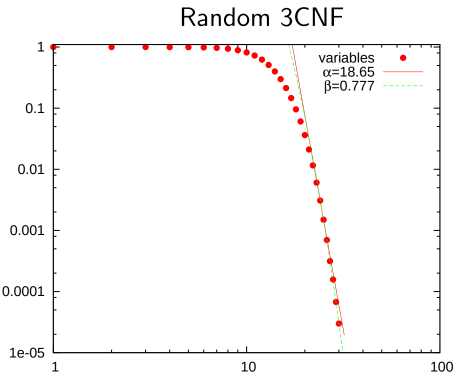


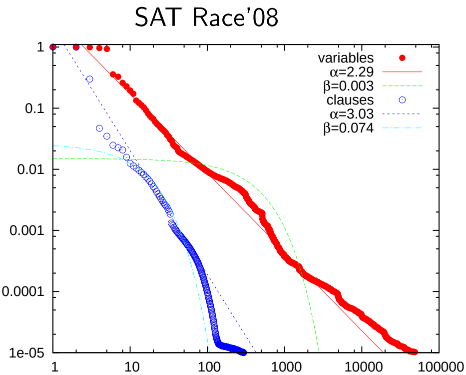
- **Vars** follows a **powerlaw** with $1.8 \leq \alpha \leq 3.5$
- **Clauses** follows an **exponential**





- **Vars** does not seem to follow any "simple" distribution
- neither **Clauses**, or there are very few values

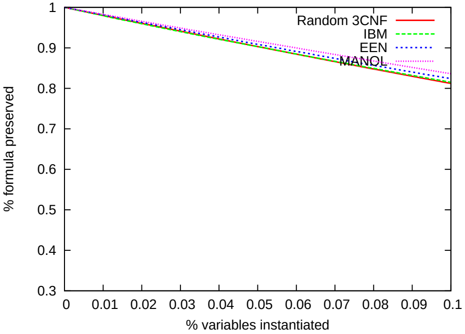




In general:

- **Vars** follows a **powerlaw** distribution in more cases than **clauses**
- α is bigger for **clauses** than for **vars**

Instantiating Variables: How Formula Decreases



- 3 heuristics: **Random Heuristics (RH)** , **Jeroslow-Wang (JW)** and **Most-Frequent (MF)**
- **RH** has the same effect in all families
- **JW** is better than **RH**
- **JW** is not so good in **random 3CNF** as in **industrial instances**, but still better than **RH**
- **JW** and **MF** have the same effect

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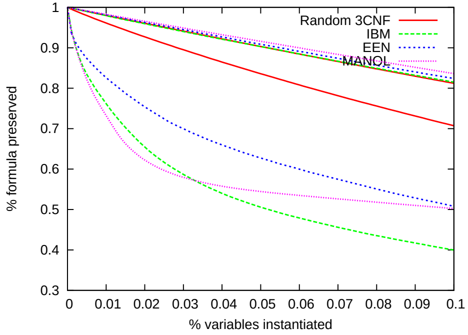

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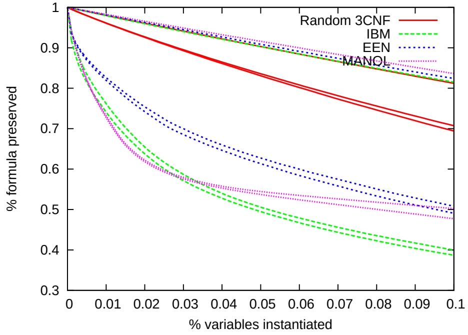

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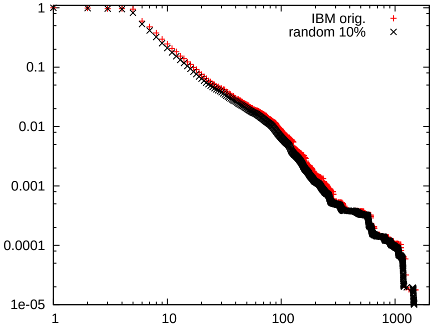
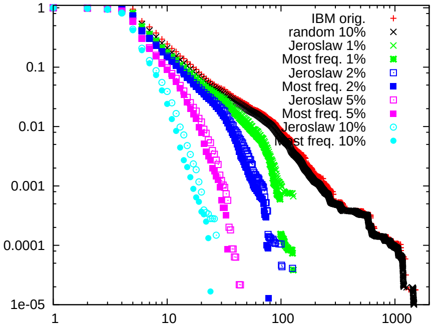
Instantiating Variables: How Formula is Modified



- Experiments with the IBM family
- RH preserve the scalefree structure

Instantiating Variables: How Formula is Modified



- Experiments with the IBM family
- RH preserve the scalefree structure
- JW and MF instantiate frequent variables and could destroy the scalefree structure

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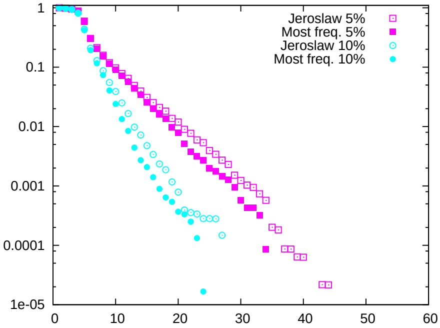

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Instantiating Variables: How Formula is Modified

	random				Jeroslow-Wang				Most freq.			
	Powerlaw		Exponential		Powerlaw		Exponential		Powerlaw		Exponential	
	α	Δ^{pow}	β	Δ^{exp}	α	Δ^{pow}	β	Δ^{exp}	α	Δ^{pow}	β	Δ^{exp}
0%	**2.63**	**0.027**	0.017	0.083	**2.63**	**0.027**	0.017	0.083	**2.63**	**0.027**	0.017	0.083
1%	**2.56**	**0.027**	0.017	0.078	**2.72**	**0.017**	0.046	0.036	**2.79**	**0.020**	0.052	0.034
2%	**2.57**	**0.025**	0.017	0.076	**2.82**	**0.015**	0.083	0.026	**2.89**	**0.012**	0.093	0.030
5%	**2.59**	**0.020**	0.018	0.075	3.27	0.029	0.218	**0.019**	3.39	0.029	0.250	**0.014**
10%	**2.62**	**0.021**	0.019	0.077	5.79	0.023	0.407	**0.014**	5.90	0.023	0.510	**0.021**

- Experiments with the IBM family
- RH preserve the scalefree structure
  α **is preserved**
- JW and MF destroy the structure after instantiating 5% of the variables
  α **is increased**

Instantiating Variables: How Formula is Modified

	random				Jeroslow-Wang				Most freq.			
	Powerlaw		Exponential		Powerlaw		Exponential		Powerlaw		Exponential	
	α	Δ^{pow}	β	Δ^{exp}	α	Δ^{pow}	β	Δ^{exp}	α	Δ^{pow}	β	Δ^{exp}
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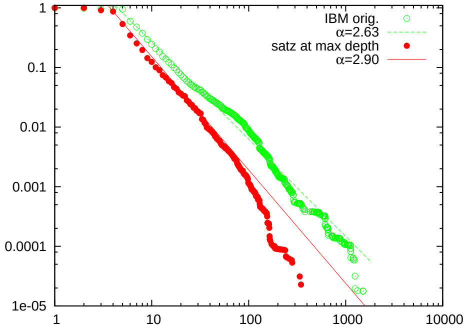
- RH preserve the scalefree structure
- JW and MF destroy the structure after instantiating 5% of the variables

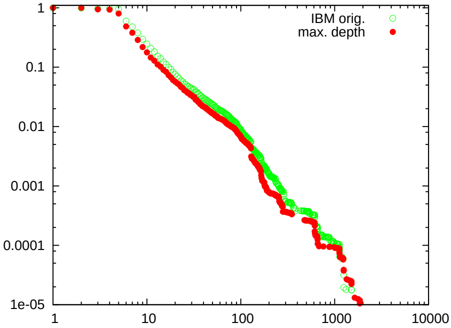
After instantiating 5% vars the assignment is inconsistent

Experiments with Real SAT Solvers



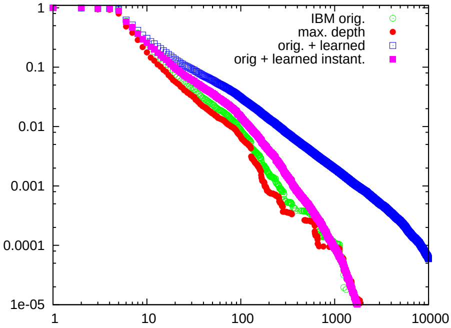
- Solver: Satz Family: IBM
- Formula under the longest assignment found after 1h of search: α is increased but the formula is still scalefree

Experiments with Real SAT Solvers



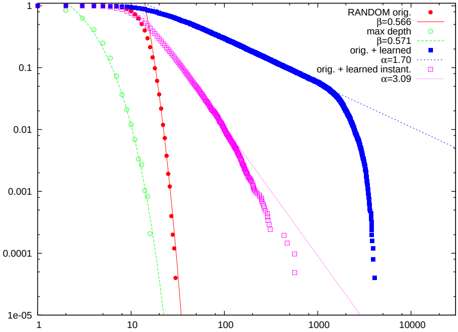
- Solver: Minisat Family: IBM
- Formula under the longest assignment found after 1000s of search: α is increased but the formula is still scalefree

Experiments with Real SAT Solvers



- Solver: Minisat Family: IBM
- Formula under the longest assignment found after 1000s of search: α is increased but the formula is still scalefree
- Original formula plus clauses learned after 200000 decisions: α is decreased substantially
- Instantiated or working formula (plus learned clauses) after 200000 decisions: α is approximately as in the original formula

Experiments with Random 3CNF Formulas



- Solver: Minisat Family: Random 3CNF
- **Learned clauses makes the formula scalefree**
- For the working formula $\alpha \approx 3$

Conclusions

- **Most industrial** SAT formulas seems to be **scalefree**
- **Contrarily to what one could expect** formulas does **not loose scalefree** structure after instantiating **most frequent** variables
- **Learning seems** to help to **preserve scalefree** structure
- Even more, **learning seems** to make **random 3CNF formulas scalefree**
- Solvers specialized in industrial instances **seem** to take profit of their scalefree structure

Open Question:

- **Why** many industrial SAT instances are scalefree? Preferential attachment?
- **Why** the scalefree structure helps? Is it true?
- **How** we can take more advantage of the structure?