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1 Bourbaki Theory of Sets

1.1 Introduction

Continuing from the section break on page 12.

Consistency or non-contradiction as a major question for logicians. Mathematical languages should be non-contradictory, that is, they should be consistent.

To be non-contradictory either a statement can be true, or it's negation, but not both at the same time. (https://en.wikipedia.org/wiki/Law_of_noncontradiction)

A related concept is The Law of the Excluded Middle

$https://en.wikipedia.org/wiki/Law\_of\_excluded\_middle$.

The difference between the two ideas is that non-contradiction slices possible states of the the world into two, one where p is true and one where $\neg p$ is. The Law of Excluded Middle says at least one of "p is true" or "$\neg p$ is true".

Non-contradiction gives us mutual exclusion. Excluded Middle gives us exhaustive covering.

> The law of non-contradiction and the law of excluded middle create a dichotomy in "logical space", wherein the two parts are "mutually exclusive" and "jointly exhaustive". The law of non-contradiction is merely an expression of the mutually exclusive aspect of that dichotomy, and the law of excluded middle is an expression of its jointly exhaustive aspect. - Wikipedia Law of Noncontradiction, retrieved 2023-08-25

Bourbaki uses metamathematics to examine consistency with metamathematical methods.

> To say that a theory is contradictory means in effect that it contains a correct formalized proof which leads to the conclusion $0 \neq 0$. Now, metamathematics can attempt, using methods of reasoning borrowed from mathematics, to investigate the structure of such a formalized text, in the hope of "proving" that such a text cannot exist. p 12