FACULTY OF COMPUTING

SEMESTER 1 2023/2024

SECI1013 – DISCRETE STRUCTURE

SECTION 3

ASSIGNMENT 1 – CHAPTER 1

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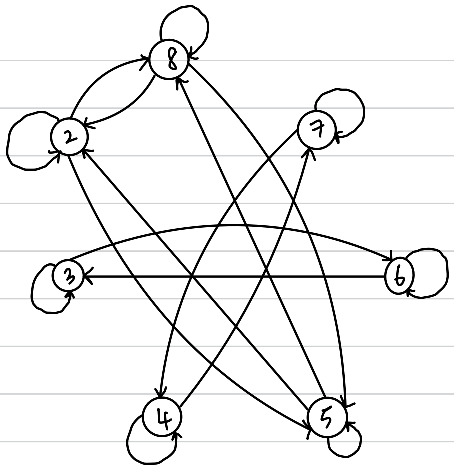
Q1. Relation

1. Given A = {2, 3, 4, 5, 6, 7, 8} and R a relation over A. Draw the directed graph of R after realising that xRy iff x−y = 3n for some n ∈ Z. Find all possible equivalence relations for R.

(5 marks)



if n=0
$3(0)=0$
$2-2=0$
$3-3=0$
$4-4=0$
$5-5=0$
$6-6=0$
$7-7=0$
$8-8=0$

if n=1
$3(1)=3$
$8-5=3$
$7-4=3$
$6-3=3$
$5-2=3$

if n=−1
$3(-1)=-3$
$5-8=-3$
$4-7=-3$
$3-6=-3$
$2-5=-3$

if n=2
$3(2)=6$
$8-2=6$

if n=−2
$3(-2)=-6$
$2-8=-6$

R = {(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,2), (5,5), (5,8), (6,3), (6,6), (7,4), (7,7), (8,2), (8,5), (8,8)}

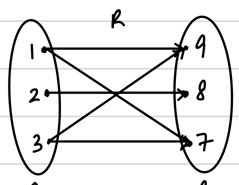
2. Let A = {1, 2, 3} and B = {9, 8, 7}.
Let R: A to B. For all (a, b) ∈ A × B, and given a R b ⇔ a+b is an even number,

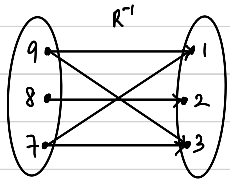
a. Determine R and R^{-1}.
b. Draw arrow diagrams for both.
c. Describe R−1 in words.

(10 marks)

a) R = {(1,9), (1,7), (2,8), (3,9), (3,7)}
R^{-1} = {(9,1), (7,1), (8,2), (9,3), (7,3)}

b)





c) The inverse relation of R is 9 to 1, 7 to 1, 8 to 2, 9 to 3 and 7 to 3

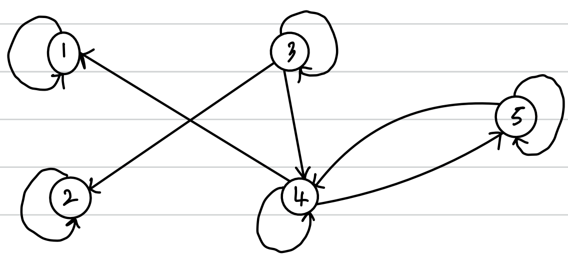
3. Let *A* = {1, 2, 3, 4, 5}, and let *R* be the relation on *A* that has the matrix (given below)

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Construct the digraph of *R*, and list in-degrees and out-degrees of all vertices.

(6 marks)

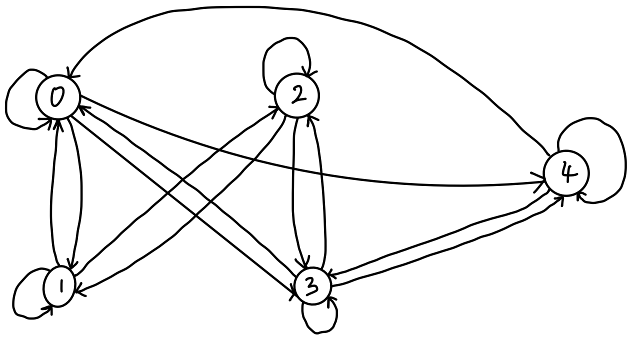
R = {(1,1), (2,2), (3,2), (3,3), (3,4), (4,1), (4,4), (4,5), (5,4), (5,5)}



	1	2	3	4	5
in-degree	2	2	1	3	2
out-degree	1	1	3	3	2

4. Given A = {0, 1, 2, 3, 4}, and R = {(0, 0),(0, 1),(0, 3),(0, 4),(1, 0),(1,1), (1,2), (2,1), (2, 2),(2,3), (3, 0),(3, 2),(3,3),(3,4),(4,0),(4,3),(4,4)}. Draw the relation graph and find is R reflexive, symmetric, or transitive?

(12 marks)



∴ R is reflexive and symmetric but not transitive because (2,3), (3,0) ∈ R but (2,0) ∉ R

5. Relation R in the set A = {1, 2, 3…13, 14} defined as R = {(x, y): 3x − y = 0}, Determine whether the relation is
   a. Reflexive
   b. Symmetric
   c. Transitive

   Support your answer with the reason.

(9 marks)

R = {(1,3),(2,6),(3,9),(4,12)}

a) Not reflexive because for each element x ∈ A, (x,x) ∉ R

b) Not symmetric because (1,3) ∈ R but (3,1) ∉ R

c) Not transitive because (1,3) and (3,9) ∈ R but (1,9) ∉ R

6. Suppose that the given is a relation matrix for R and S,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

   Using Boolean Arithmetic, Find
   a. RS
   b. SR

(8 marks)

6. a)
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Q2. Function

7. What is the different between Relation and Function?

(2 Marks)

Relation is a list of connection between things while function is specific connection where each element is linked to only each other.

8. If A = {2, 3, 4, 5}, then write whether each of the following relations on set A is a function or not. Give reasons also.
   (i) {(2, 3), (3, 4), (4, 5), (5, 2)}
   (ii) {(2, 4), (3, 4), (5, 4), (4, 4)}
   (iii) {(2, 3), (2, 4), (5, 4)}
   (iv) {(2, 3), (3, 5), (4, 5)} (v) {(2, 2), (2, 3), (4, 4), (4, 5)}

(8 marks)

i) Function because domain of f is equal to A

ii) Function because $f(x_1) = f(x_2)$ but $x_1 \neq x_2$

iii) Not function because domain element has 2 corresponding elements

iv) Not function because the first part is not function as the domain of f is not equal to A, the second part is not function as domain element has 2 corresponding elements. Hence, not function $\vee$ not function, so it is not function.

9. Given the relation of $R = \{(x, y) | y = x + 5, x \text{ is } \mathbb{Z}^+ \text{ less than } 6\}$. Depict this relationship using roster form. Write down the domain and the range.

(3 marks)

$R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain = $\{1, 2, 3, 4, 5\}$

Range = $\{6, 7, 8, 9, 10\}$

10. In the following cases, state whether the function is one-one, onto or bijective. Justify your answer.
   (v) $f = R \rightarrow R, f(x) = 1 - 2x$
   (vi) $f = R \rightarrow R, f(x) = 5x^2 - 1$
   (vii) $f = R \rightarrow R, f(x) = x^4$
   (viii) $f = R \rightarrow R, f(x) = \left(\frac{x-2}{x-3}\right)$

(8 marks)

v) $f(x_1) = f(x_2)$

$1 - 2x_1 = 1 - 2x_2$

$x_1 = x_2$

(one-to-one)

Every element in codomain is covered

$x = \frac{1}{2}(1-y)$

$f(x) = 1 - 2\left[\frac{1}{2}(1-y)\right] = y$

(onto)

$\therefore$ It is bijective

(vi) $f(x) = 5x^2 - 1$

$f(-2) = f(2)$

but $-2 \neq 2$

$\therefore$ Onto function

(vii) $f(x_1) = f(x_2)$

$x_1^4 = x_2^4$

$x_1 = x_2$

(one-to-one)

if $x = \sqrt[4]{y}$, $f(x) = (\sqrt[4]{y})^4 = y$

(onto function)

$\therefore$ It is bijective

(iv) $f(x) = \left(\frac{x-2}{x-3}\right)$

$f(x_1) = f(x_2)$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$x_1 = x_2$

(one-to-one)

Every element in codomain is covered

(onto)

$\therefore$ It is bijective

11. Given the following functions, find the function $f(g(x))$ and find the value of the function if $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

(x) $f(x) = x^2$; $g(x) = 5x - 6$

(xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

(9 marks)

(ix) $f(g(x)) = 3(x^2-1) - 1$
$= 3x^2 - 3 - 1$
$= 3x^2 - 4$

$f(g(0)) = 3(0)^2 - 4 = 4$
$f(g(1)) = 3(1)^2 - 4 = -1$
$f(g(2)) = 3(2)^2 - 4 = 8$
$f(g(3)) = 3(3)^2 - 4 = 23$

(x) $f(g(x)) = (5x-6)^2$
$= 25x^2 - 60x + 36$

$f(g(0)) = 25(0)^2 - 60(0) + 36 = 36$
$f(g(1)) = 25(1)^2 - 60(1) + 36 = 1$
$f(g(2)) = 25(2)^2 - 60(2) + 36 = 16$
$f(g(3)) = 25(3)^2 - 60(3) + 36 = 81$

(xi) $f(g(x)) = (x^3+1) - 1$
$= x^3$

$f(g(0)) = 0^3 = 0$
$f(g(1)) = 1^3 = 1$
$f(g(2)) = 2^3 = 8$
$f(g(3)) = 3^3 = 27$

Q3. Recurrence Relation

12. Solve the recurrence relation given;

(xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1$ and $a_1 = 6$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$;
initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1$, $a_1 = -2$ and $a_2 = -1$

(12 marks)

(xii) $a_0 = 1$, $a_1 = 6$

$a_2 = 6(a_1) - 9(a_0) = 6(6) - 9(1) = 27$
$a_3 = 6(a_2) - 9(a_1) = 6(27) - 9(6) = 108$
$a_4 = 6(a_3) - 9(a_2) = 6(108) - 9(27) = 405$
$a_5 = 6(a_4) - 9(a_3) = 6(405) - 9(108) = 1458$

1, 6, 27, 108, 405, 1458, ...

(xiii) $a_0 = 2$, $a_1 = 5$, $a_2 = 15$

$a_3 = 6(a_2) - 11(a_1) + 6(a_0) = 6(15) - 11(5) + 6(2) = 47$
$a_4 = 6(a_3) - 11(a_2) + 6(a_1) = 6(47) - 11(15) + 6(5) = 147$
$a_5 = 6(a_4) - 11(a_3) + 6(a_2) = 6(147) - 11(47) + 6(15) = 455$

2, 5, 15, 47, 147, 455, ...

(xiv) $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3(a_2) - 3(a_1) + a_0 = -3(-1) - 3(-2) + 1 = 10$
$a_4 = -3(a_3) - 3(a_2) + a_1 = -3(10) - 3(-1) + (-2) = -29$
$a_5 = -3(a_4) - 3(a_3) + a_2 = -3(-29) - 3(10) + (-1) = 56$

1, -2, -1, 10, -29, 56, ...

13. A sequence a_1, a_2 , a_3 , a_4 , … is given by

$$a_{n+1} = 5a_n - 3 \ ; \ a_1 = k$$

where k is a non-zero constant.

(i) Find the value of a_4 in terms of k .
(ii) Given that a_4= 7 , determine the value of k .

(8 marks)

i) $a_1 = k$

$a_2 = 5k - 3$

$a_3 = 5(5k-3) - 3$

$= 25k - 15 - 3$

$= 25k - 18$

$a_4 = 25(25k - 18) - 3$

$= 125k - 93$

ii) $a_4 = 7$

$125k - 93 = 7$

$125k = 100$

$k = 0.8$