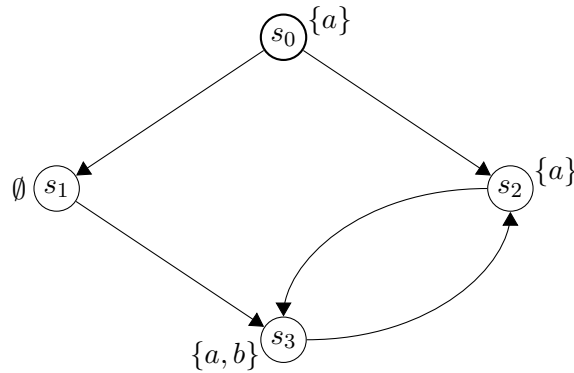# Chapter 4

LTL logic

Exercise 4.1. Let the following transition system TS be given ($I = \{s_0\}$).

Check (do not use **nuXmv**) whether TS satisfies the following LTL formulas:

a) $\mathsf{GF}a$

b) $(\neg a \wedge \neg b) \Rightarrow \mathsf{XG}a$

c) $\mathsf{FXG}a$

d) $b \Rightarrow (\mathsf{G}a \wedge \mathsf{GF}b)$

e) $\mathsf{F}(\mathsf{G}b \vee \mathsf{G}a)$

Use the following SMV model and the **nuXmv** toolbox to check your answers.

Listing 4.1: SMV model

```
MODULE main
VAR
  s : {s0, s1, s2, s3};
  a : boolean;
  b : boolean;
ASSIGN
  init(s) := s0;

  next(s) := case
    s = s0 : {s1, s2};
    s = s1 : s3;
    s = s2 : s3;
    s = s3 : s2;
  esac;
```

```
a := case
   s = s1 : FALSE;
   TRUE   : TRUE;
esac;

b := case
   s = s3 : TRUE;
   TRUE   : FALSE;
esac;
```

Exercise 4.2. Assume $AP = \{a, b, c\}$. Define LTL formulas describing the following properties:

a) a never occurs.

b) a should occur exactly once.

c) a should eventually be followed by b.

d) a, b, and c never occur together in the same state.

e) a, b, and c always occur together.

f) If a occurs in the given state then b occurs in the next state.

g) a never occurs in two consecutive states.

Exercise 4.3. Let the following transition system TS be given ($I = \{s_1, s_2\}$).
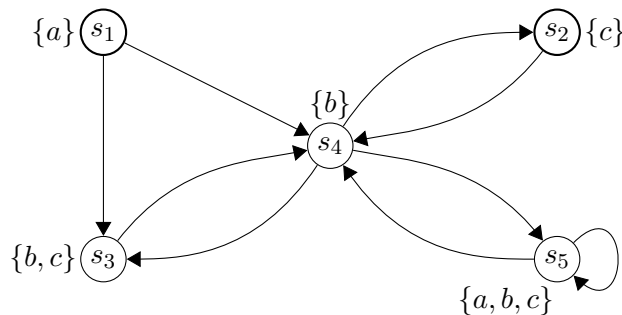
Check (do not use **nuXmv**) whether TS satisfies the following LTL formulas:

a) $\mathsf{FG}c$

b) $\mathsf{GF}c$

c) $\mathsf{X}\neg c \Rightarrow \mathsf{XX}c$

d) $\mathsf{G}a$

e) $a \,\mathsf{U}\, \mathsf{G}(b \vee c)$

f) $(\mathsf{XX}b) \,\mathsf{U}\, (b \vee c)$

g) $\mathsf{FG}(a \vee c)$

h) $(a \Rightarrow \mathsf{X}(b \wedge ((b \vee c) \,\mathsf{U}\, a)))$

i) $(a \vee c) \wedge \mathsf{XG}(b \vee c)$

j) $\mathsf{XG}(\neg b \vee ((b \vee c) \,\mathsf{U}\, a))$

Use the following SMV model and the **nuXmv** toolbox to check your answers.

Listing 4.2: SMV model

```
MODULE main

VAR
  s : {s1, s2, s3, s4, s5};
  a : boolean;
  b : boolean;
  c : boolean;
ASSIGN
  init(s) := {s1, s2};

  next(s) := case
    s = s1 : {s3, s4};
    s = s2 : s4;
    s = s3 : s4;
    s = s4 : {s2, s3, s5};
    s = s5 : {s4, s5};
  esac;

  a := case
    s = s1 : TRUE;
    s = s5 : TRUE;
    TRUE   : FALSE;
  esac;

  b := case
    s = s3 : TRUE;
    s = s4 : TRUE;
    s = s5 : TRUE;
    TRUE   : FALSE;
  esac;

  c := case
    s = s2 : TRUE;
    s = s3 : TRUE;
    s = s5 : TRUE;
    TRUE   : FALSE;
  esac;
```
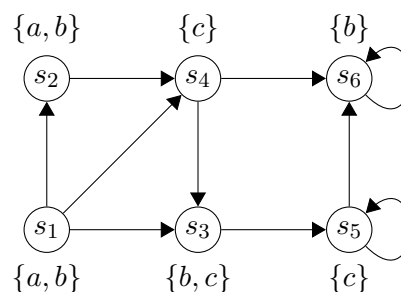
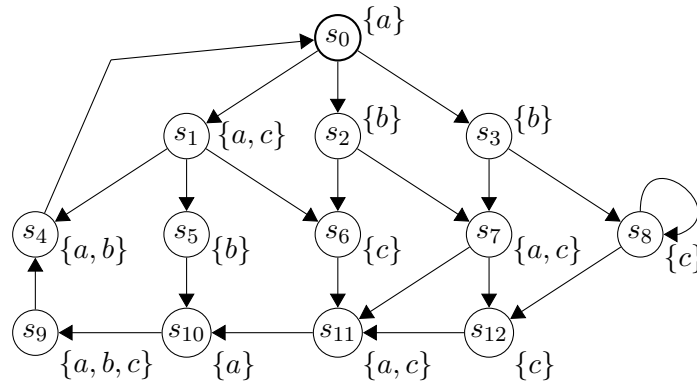**Exercise 4.4.** Let the following transition system TS be given.

For each of the following formulas find all states that satisfy the given formula.

a) $\mathsf{X}\mathsf{G}\neg a$

b) $b\,\mathsf{W}\,c$

c) $\mathsf{F}\mathsf{X}a \vee \mathsf{F}\mathsf{G}c$

d) $c \mathbin{\mathsf{R}} b$

Develop an **SMV** model for the transition system and check your answers.

Exercise 4.5. Let the following transition system TS be given ($I = \{s_0\}$).

Check whether TS satisfies the following LTL formulas:

a) $\mathsf{G}\mathsf{F}((b \vee c) \wedge \neg a)$

b) $\mathsf{X}(\neg a \mathbin{\mathsf{U}} (a \wedge c)) \vee \mathsf{G}(\neg a \vee \neg c)$

c) $\mathsf{F}\mathsf{X}c$

d) $\mathsf{X}(\neg a \mathbin{\mathsf{U}} (a \wedge c))$

Exercise 4.6. Which of the following equivalences are correct? Prove the equivalence or provide a counterexample that illustrates that the formula on the left and the formula on the right are not equivalent ([1]).

a) $\mathsf{G}\mathsf{G}(a \vee \neg b) \;\equiv\; \neg\mathsf{F}(\neg a \wedge b)$

b) $\mathsf{F}(a \wedge b) \;\equiv\; \mathsf{F}a \wedge \mathsf{F}b$

c) $\mathsf{F}a \wedge \mathsf{X}\mathsf{G}b \;\equiv\; \mathsf{F}b$

d) $\mathsf{G}\mathsf{F}a \Rightarrow \mathsf{G}\mathsf{F}b \;\equiv\; \mathsf{G}(a \Rightarrow \mathsf{F}b)$

e) $\neg(a \mathbin{\mathsf{U}} b) \;\equiv\; \neg b \mathbin{\mathsf{W}} (\neg a \wedge \neg b)$

f) $\mathsf{X}\mathsf{F}a \;\equiv\; \mathsf{F}\mathsf{X}a$

g) $(\mathsf{F}\mathsf{G}a) \wedge (\mathsf{F}\mathsf{G}b) \;\equiv\; \mathsf{F}(\mathsf{G}a \wedge \mathsf{G}b)$

h) $(a \mathbin{\mathsf{U}} b) \mathbin{\mathsf{U}} b \;\equiv\; a \mathbin{\mathsf{U}} b$