Laboratory exercise no. 4: Taylor model – explicit method

Aim: The aim of the laboratory is to create a simple numerical model simulating one dimensional transport of the pollutants in the river using QUICKEST explicit method.

Laboratory programme:

1. Defining of the physical model of simulated object.
2. Formulating of the QUICKEST numerical method.
3. Writing of the programme code.
4. Testing of the numerical stability of algorithm
5. Calculation of the spatial and temporal distribution of conservative tracer in the river.

Taylor model equation:

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} - D\frac{\partial^2 c}{\partial x^2} = 0$$

Parameters of the physical object:

Straight and rectangular part of the canal having the following parameters:

- length	100m
- width	5m
- depth	1m
- mean flow velocity	0.1m/s
- dispersion coefficient	0.01m^2/s
- location of the injection point	10m
- location of the measurement point	90m
- amount of injected tracer	1kg

Boundary conditions:

- left side – Dirichlet condition

$$c(0,t) = 0$$

- right side - von Neumann condition

$$\frac{\partial c}{\partial x}(L,t) = 0$$

Initial condition:

$$c(x,0) = f(x)$$

For the calculations the square shape f(x) will be assumed

$$\begin{cases} f(x) = 0 \; dla \; x \neq x_i \\ f(x) = m \; dla \; x = x_i \end{cases}$$

where:

m – initial concentration in the injection point
x_i – location of the injection point

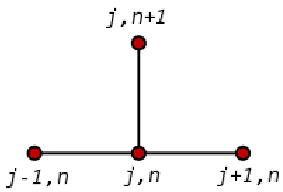
$C_a = \frac{U\Delta t}{\Delta x}$- advective Courant number

$C_d = \frac{D\Delta t}{\Delta x^2}$- diffusive Courant number

Input data:

dx – spatial resolution
dt – time step
D – dispersion coefficient
U – advection coefficient
nt – number of time steps
nx – number of computational nodes
f(x) – function describing the initial distribution of the tracer

Explicit methods allow to calculate the function value in n+1 time step based on values assigned to n time step only.



Iteration formula (QUICKEST method):

$$c_j^{n+1} = c_j^n + \left[C_d(1-C_a) - \frac{C_a}{6}(C_a^2 - 3C_a + 2)\right]c_{j+1}^n - \left[C_d(2-3C_a) - \frac{C_a}{2}(C_a^2 - 2C_a - 1)\right]c_j^n$$
$$+ \left[C_d(1-3C_a) - \frac{C_a}{2}(C_a^2 - C_a - 2)\right]c_{j-1}^n + \left[C_d C_a + \frac{C_a}{6}(C_a^2 - 1)\right]c_{j-2}^n$$

Laboratory outline:

1. Writing the computer programme solving the Taylor model using the QUICKEST method.
2. Calculation of the temporal and spatial evolution of tracer concentration in the river for a specified initial and boundary conditions.
3. Testing of the numerical stability for different combinations of U and D parameters.
4. Checking of the mass conservation law (if the total mass of the tracer present in the river is constant in time)
5. Computer programme can be written in any programming language or software environment. Recommended environment is MATLAB.
6. Programme code supplemented with appropriate comments should be included as a part of a report prepared in pdf format.
7. The report must include the conclusion.