

Méthodes psychométriques en qualité de vie

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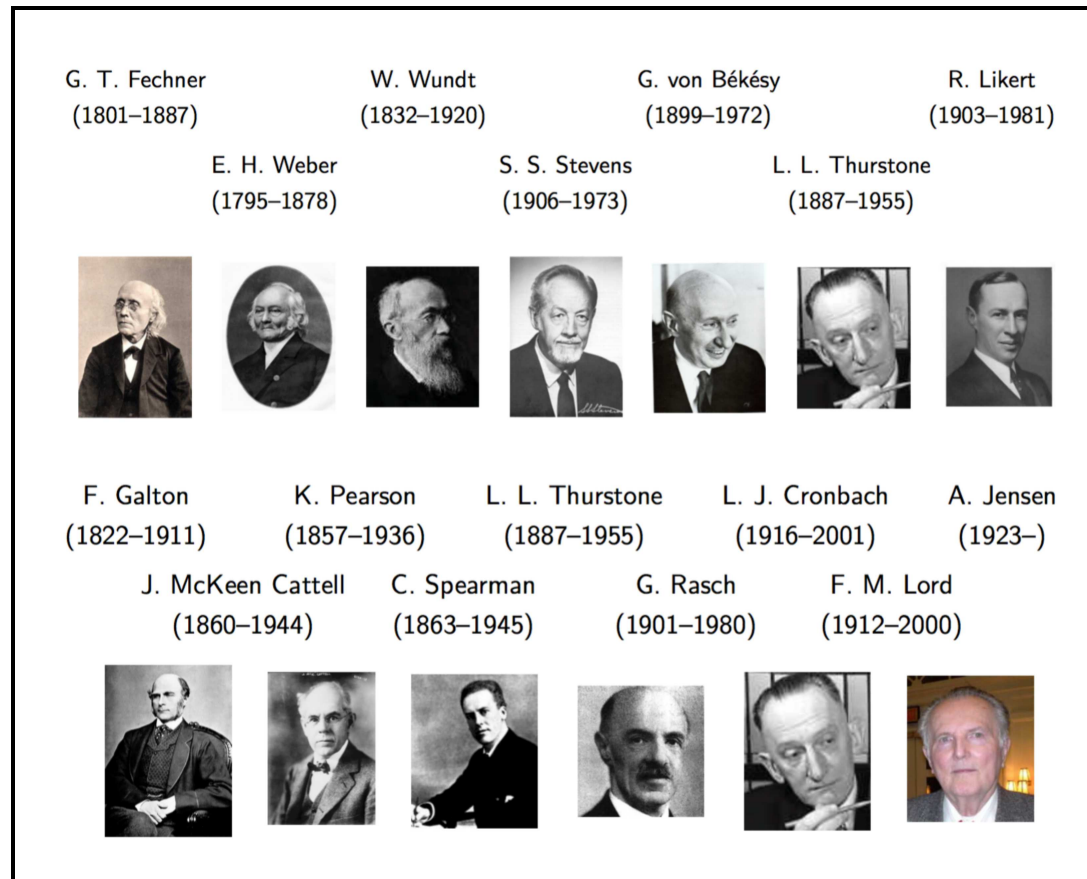
Analyses factorielles

- Analyse en composantes principales et analyse factorielle
- Analyse factorielle exploratoire
- Analyse factorielle confirmatoire

It is rather surprising that systematic studies of human abilities were not undertaken until the second half of the last century. . . . An accurate method was available for measuring the circumference of the earth 2,000 years before the first systematic measures of human ability were developed.¹

1. J NUNNALLY et I BERNSTEIN. *Psychometric Theory*. 3rd. McGraw-Hill, 1994.

Avant Jan de Leeuw & Bengt Muthén



Multivariate Behavioral Research, 1987, 22, 267–305

A Brief History of the Philosophical Foundations of Exploratory Factor Analysis

Stanley A. Mulaik

Georgia Institute of Technology

Exploratory factor analysis derives its key ideas from many sources. From the Greek rationalists and atomists comes the idea that appearance is to be explained by something not observed. From Aristotle comes the idea of induction and seeking common features of things as explanations of them. From Francis Bacon comes the idea of an automatic algorithm for inductively discovering common causes. From Descartes come the ideas of analysis and synthesis that underlie the emphasis on analysis of variables into orthogonal or linearly independent factors and focus on reproducing (synthesizing) the correlation matrix from the factors. From empiricist statisticians like Pearson and Yule comes the idea of exploratory, descriptive statistics. Also from the empiricist heritage comes the false expectation some have that factor analysis yields unique and unambiguous knowledge without prior assumptions—the inductivist fallacy. This expectation founders on the indeterminacy of factors, even after their loadings are defined by rotation. Indeterminacy is unavoidable in the interpretation of common factors because the process of interpretation is inductive and inductive inferences are not uniquely determined by the data on which they are based. But from Kant we learn not to discard inductive inferences but to treat them as hypotheses that must be tested against additional data to establish their objectivity. And so the conclusions of exploratory factor analyses are never complete without a subsequent confirmatory analysis with additional variables and new data.

ACP et AF

▷ 01a-scores.pdf

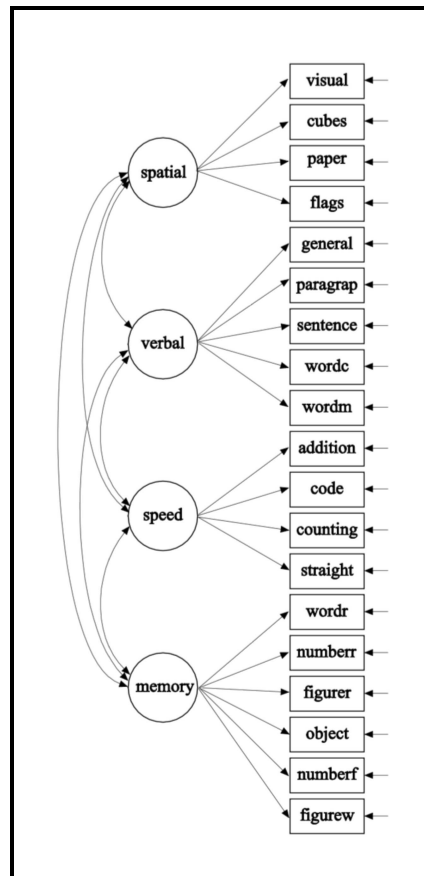
Les composantes C_i ($i = 1, \dots, p$) de l'analyse en composantes principales (ACP) sont construites comme de simples combinaisons linéaires des p variables d'origine : $C_i = \sum_{j=1}^p w_{ij} x_j$.

Dans le cadre de l'analyse factorielle, on considère au contraire des combinaisons linéaires de facteurs² :

$$x_i \approx \sum_{j=1}^k w_{ij} F_j.$$

2. W REVELLE. *An introduction to psychometric theory with applications in R*. <http://www.personality-project.org/r/book/>. 2016, chap. 6.

Modèle de Holzinger & Swineford



Comparaison ACP *versus* AF

```
library(psych)
principal(HS[,c("visual", "cubes", "paper")], nfactors = 3,
          rotate = "none")
```

Standardized loadings (pattern matrix) based upon correlation matrix

	PC1	PC2	PC3	h2	u2	com
visual	0.77	-0.41	0.48	1	0.0e+00	2.3
cubes	0.70	0.71	0.10	1	-4.4e-16	2.0 ❶
paper	0.80	-0.22	-0.56	1	-6.7e-16	2.0

	PC1	PC2	PC3
SS loadings	1.72	0.72	0.56
Proportion Var	0.57	0.24	0.19


```
fa(HS[,c("visual", "cubes", "paper")], nfactors = 1)
```

Standardized loadings (pattern matrix) based upon correlation matrix

	MR1	h2	u2	com
visual	0.62	0.39	0.61	1
cubes	0.48	0.23	0.77	1 ❶
paper	0.71	0.50	0.50	1

	MR1
SS loadings	1.12
Proportion Var	0.37

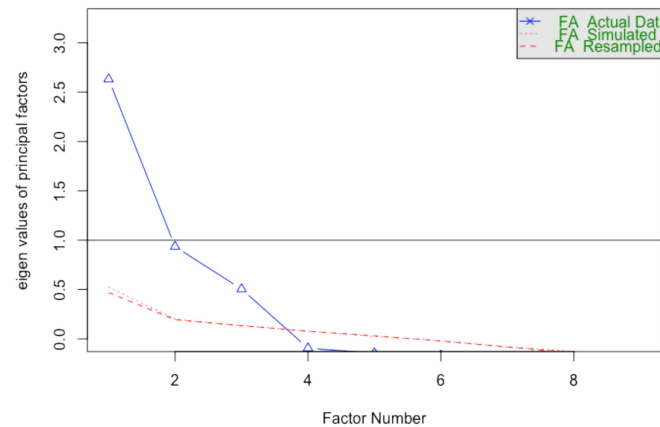
Sélection de modèle

- Sélection des variables à inclure : analyse d'items ou hypothèses *a priori*
- Sélection du nombre de facteurs : méthode exploratoire, hypothèses *a priori*, analyse parallèle
- Type de rotation : en fonction des hypothèses théoriques
- Méthode d'estimation (OLS, ML, WLS et PA)
- Matrice de corrélation (Pearson, tétra- ou polychorique)
- Nombre de sujets nécessaires³

3. Rouquette A et Falissard B. « Sample size requirements for the internal validation of psychiatric scales ». In : *International Journal of Methods in Psychiatric Research* 20.4 (2011), p. 235–249.

Analyse parallèle

```
d <- HS[,7:15]
describe(d)
fa.parallel(d, fm = "pa", fa = "fa", main = "")
```



Solution factorielle à 1 facteur

	PA1	PA2	PA3	h2	u2
visual	0,559			0,313	0,687
cubes	0,301			0,090	0,910
paper	0,365			0,133	0,867
paragrap	0,761			0,580	0,420
sentence	0,724			0,525	0,475
wordm	0,768			0,590	0,410
addition	0,259			0,067	0,933
counting	0,339			0,115	0,885
straight	0,468			0,219	0,781

```
1 m1 <- fa(d, nfactors = 1, fm = "pa")  
2 m2 <- fa(d, nfactors = 2, fm = "pa")  
3 m3 <- fa(d, nfactors = 3, fm = "pa")
```

Solution factorielle à 2 facteurs

	PA1	PA2	PA3	h2	u2
visual	0,275	0,430		0,341	0,659
cubes	0,134	0,244		0,100	0,900
paper	0,060	0,449		0,223	0,777
paragrap	0,851	0,005		0,727	0,273
sentence	0,854	-0,038		0,709	0,291
wordm	0,828	0,033		0,705	0,295
addition	-0,034	0,434		0,180	0,820
counting	-0,083	0,642		0,383	0,617
straight	0,007	0,734		0,542	0,458

```
1 m1 <- fa(d, nfactors = 1, fm = "pa")
2 m2 <- fa(d, nfactors = 2, fm = "pa")
3 m3 <- fa(d, nfactors = 3, fm = "pa")
```

Solution factorielle à 3 facteurs

	PA1	PA2	PA3	h2	u2
visual	0,196	0,591	0,032	0,477	0,523
cubes	0,043	0,510	-0,121	0,256	0,744
paper	-0,062	0,685	0,020	0,453	0,547
paragrap	0,846	0,016	0,007	0,728	0,272
sentence	0,885	-0,065	0,007	0,753	0,247
wordm	0,805	0,080	-0,013	0,692	0,308
addition	0,045	-0,154	0,732	0,512	0,488
counting	-0,034	0,121	0,691	0,524	0,476
straight	0,032	0,380	0,458	0,461	0,539

```
1 m1 <- fa(d, nfactors = 1, fm = "pa")  
2 m2 <- fa(d, nfactors = 2, fm = "pa")  
3 m3 <- fa(d, nfactors = 3, fm = "pa")
```

Analyse exploratoire et confirmatoire (CFA)

La CFA revient à imposer une structure particulière, c.a.d. à contraindre certains paramètres du modèle, et à tester l'adéquation du modèle avec les données.

	PA1	PA2	PA3	h2	u2
visual		0,591		0,477	0,523
cubes	○	0,510		0,256	0,744
paper		0,685		0,453	0,547
paragrap	0,846		○	0,728	0,272
sentence	0,885			0,753	0,247
wordm	0,805	○		0,692	0,308
addition			0,732	0,512	0,488
counting	○		0,691	0,524	0,476
straight			0,458	0,461	0,539

Utilisation de lavaan

Le package lavaan⁴ dispose de 4 commandes essentielles : lavaan, cfa, sem, growth.

Les commandes inclut entre autres des procédures d'estimation par intervalles (bootstrap), de simulation et de transfert de données/modèles avec Mplus.⁵

<http://lavaan.ugent.be>

4. Y ROSSEEL. « lavaan, An R Package for Structural Equation Modeling ». In : *Journal of Statistical Software* 48.2 (2012), p. 1–36.

5. AA BEAUJEAN. « Factor Analysis Using R ». In : *Practical Assessment, Research & Evaluation* 18.4 (2013), p. 1–11; AA BEAUJEAN. *Latent Variable Modeling Using R, A Step-by-Step Guide*. New York : Routledge, 2014.

Modèle en traits corrélés

```
library(lavaan)
library(semPlot)
m <- 'Visual =~ visual + cubes + paper
      Verbal =~ paragraf + sentence + wordm
      Speed  =~ addition + counting + straight'
r <- cfa(m, data = d)
summary(r, fit.measures = TRUE)
semPaths(r, whatLabels = "est")

> summary(r, fit.measures = TRUE)
lavaan (0.5-20) converged normally after 35 iterations
```

①

②

③

④

Number of observations

301

Estimator	ML
Minimum Function Test Statistic	85.306
Degrees of freedom	24
P-value (Chi-square)	0.000

Model test baseline model:

Minimum Function Test Statistic	918.852
Degrees of freedom	36
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.931	❶
Tucker-Lewis Index (TLI)	0.896	

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3737.745
Loglikelihood unrestricted model (H1)	-3695.092
Number of free parameters	21
Akaike (AIC)	7517.490
Bayesian (BIC)	7595.339
Sample-size adjusted Bayesian (BIC)	7528.739

Root Mean Square Error of Approximation:

RMSEA	0.092	②
90 Percent Confidence Interval	0.071 0.114	
P-value RMSEA \leq 0.05	0.001	

Standardized Root Mean Square Residual:

SRMR	0.065	③
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Parameter Estimates:

Information
Standard Errors

Expected
Standard

Latent Variables:

	Estimate	Std.Err	Z-value	P(> z)
Visual =~				
visual	1.000			
cubes	0.554	0.100	5.554	0.000
paper	0.729	0.109	6.685	0.000
Verbal =~				
paragrap	1.000			
sentence	1.113	0.065	17.014	0.000
wordm	0.926	0.055	16.703	0.000
Speed =~				
addition	1.000			

4

counting	1.180	0.165	7.152	0.000
straight	1.082	0.151	7.155	0.000

Covariances:

	Estimate	Std.Err	Z-value	P(> z)
Visual ~~				
Verbal	0.408	0.074	5.552	0.000
Speed	0.262	0.056	4.660	0.000
Verbal ~~				
Speed	0.173	0.049	3.518	0.000

5

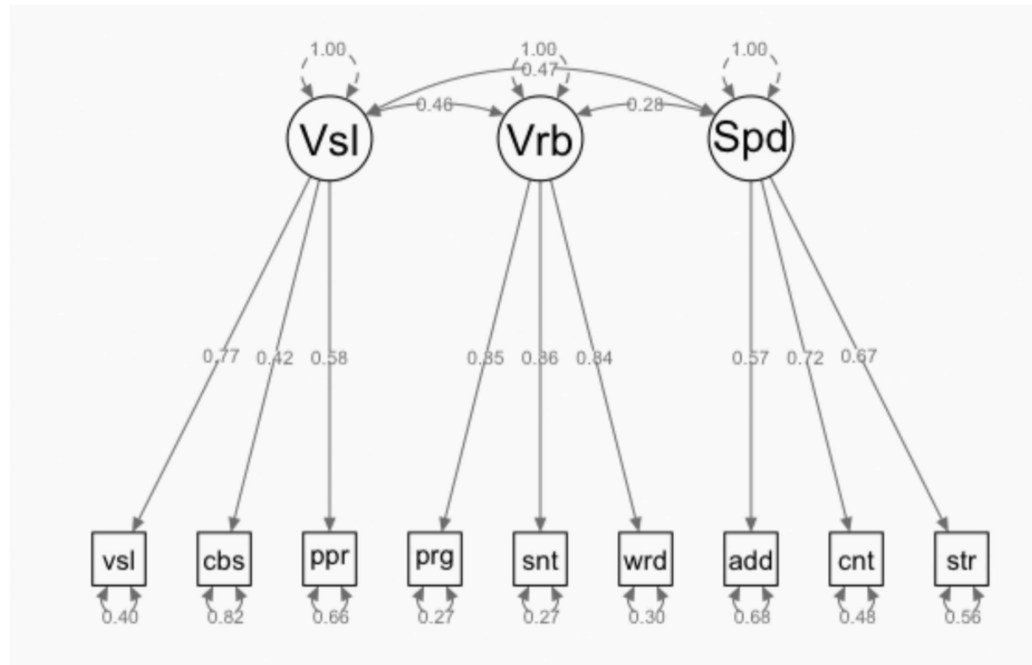
Variances:

	Estimate	Std.Err	Z-value	P(> z)
visual	0.549	0.114	4.833	0.000
cubes	1.134	0.102	11.146	0.000
paper	0.844	0.091	9.317	0.000
paragrap	0.371	0.048	7.779	0.000
sentence	0.446	0.058	7.642	0.000

wordm	0.356	0.043	8.277	0.000
addition	0.799	0.081	9.823	0.000
counting	0.488	0.074	6.573	0.000
straight	0.566	0.071	8.003	0.000
Visual	0.809	0.145	5.564	0.000
Verbal	0.979	0.112	8.737	0.000
Speed	0.384	0.086	4.451	0.000

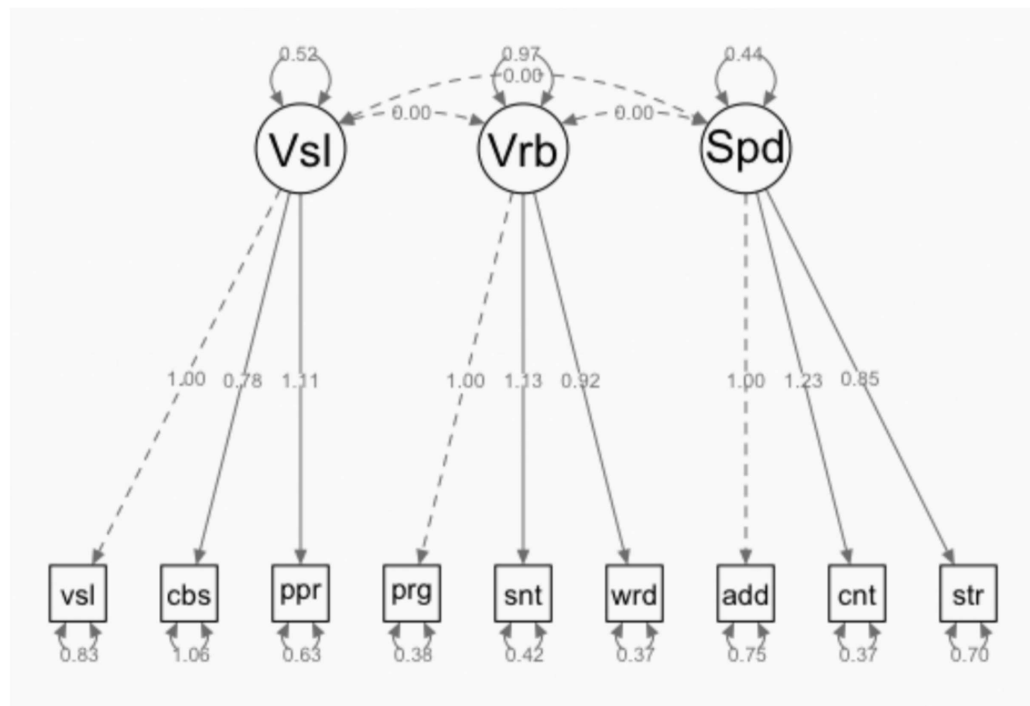
Modèle en traits corrélés (paramétrisation alternative)

```
r <- cfa(m, data = d, std.lv = TRUE)
```

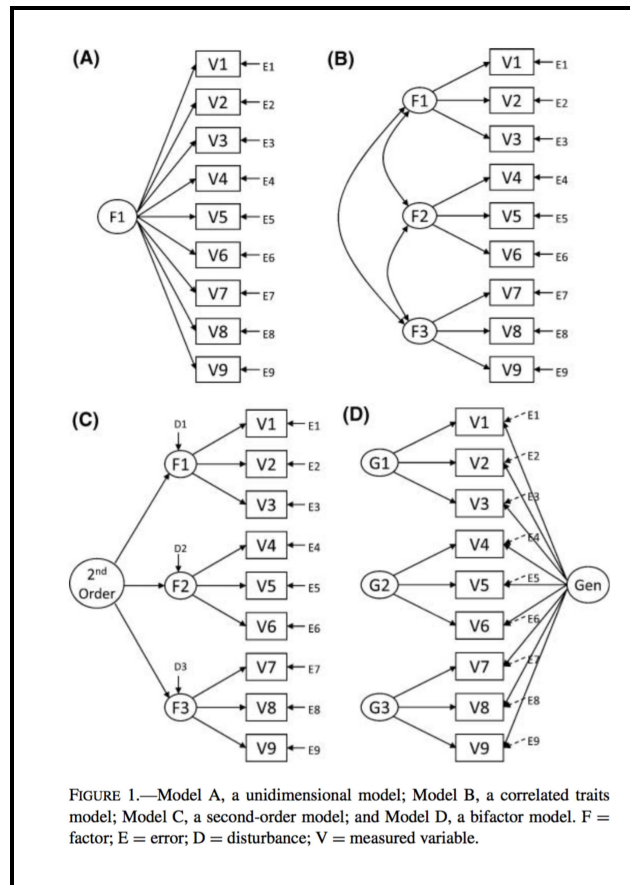


Modèle en traits orthogonaux

```
r <- cfa(m, data = d, orthogonal = TRUE)
```



Modèles de mesure en analyse factorielle



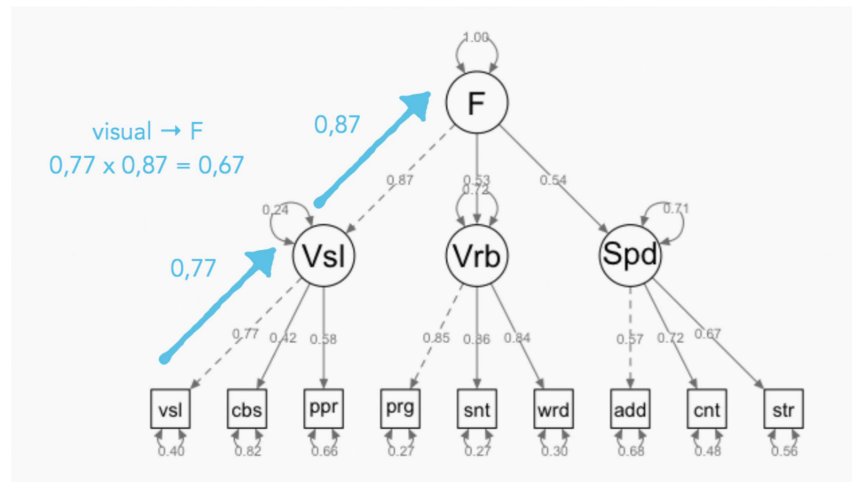
Modèles alternatifs pour les données HS

- **Modèle de second ordre** : modèle de mesure placé directement au niveau de la corrélation entre les facteurs spécifiques : les facteurs sont corrélés car ils « partagent une cause commune ». L'effet facteur primaire est appelé effet indirect.
- **Modèle bifactoriel** : tous les items sont associés à un même facteur général, et ce modèle inclut des facteurs spécifiques orthogonaux, appelés facteurs communs, qui résument la variance non expliquée par le facteur général.⁶

6. S.P. REISE, T.M. MOORE et M.G. HAVILAND. « Bifactor Models and Rotations : Exploring the Extent to Which Multidimensional Data Yield Univocal Scale Scores ». In : *Journal of Personality Assessment* 92.6 (2010), p. 544–559.

Modèle de second ordre

```
m3 <- 'Visual =~ visual + cubes + paper
Verbal =~ paragrap + sentence + wordm
Speed =~ addition + counting + straight
F =~ Visual + Verbal + Speed'
```



Fichier de données et scripts R disponibles à l'adresse suivante :
<https://bitbucket.org/chlallanne/eespe11>

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