

## Team Note of Powered by Zigui

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ALL BELOW HERE ARE USELESS IF YOU READ THE STATEMENT WRONG

## 0 Quotes and Prerequisites

evenharder : Mental Abuse To Humans  
 djkim0613 : 열심히 응원하겠습니다.  
 SoulTch : How much is this bus ticket?  
 \* This template is brought from that of 'Deobureo Minkyu Party'

### Run script

```
#!/bin/bash
g++ -fsanitize=undefined -std=c++14 -O2 -o /tmp/pow $1.cpp
time /tmp/pow < $1.in
# export PATH=~:$PATH
```

### Debug Code

```
#define setz(x) memset(x, 0, sizeof(x))
#define sz(x) ((int)(x).size())
#define rep(i, e) for (int i = 0, _##i = (e); i < _##i; i++)
#define repp(i, s, e) for (int i = (s), _##i = (e); i < _##i; i++)
#define repr(i, s, e) for (int i = (s)-1, _##i = (e); i >= _##i; i--)
#define repi(i, x) for (auto &i : (x))
// using namespace std;
using ll = long long;
using pii = pair<int, int>;
using pll = pair<ll, ll>;
template<typename T>
ostream &operator<<(ostream &os, const vector<T>& v) {
    cout << "[";
    for (auto p : v) cout << p << ", ";
    cout << "]";
    return os;
}
#ifdef __SOULTCH
#define debug(...) 0
#define endl '\n'
#else
#define debug(...) cout << " [-] ", _dbg(#__VA_ARGS__, __VA_ARGS__)
template<class TH> void _dbg(const char *sdbg, TH h){ cout << sdbg << '=' << h << endl; }
template<class TH, class... TA> void _dbg(const char *sdbg, TH h, TA... a) {
    while(*sdbg != ',') cout << *sdbg++;
    cout << '=' << (h) << ', ';
    _dbg(sdbg+1, a...);
}
#endif
```

## Reminders

Pre-submit	Wrong answer:
예제 작성해보기 (최소, 최대) 메모리, overflow 분석하기 올바른 문제에 제출하기	코드 + debug output 출력 다중 테케 문제에서 초기화 확인하기 알고리즘이 제한조건을 전부 다루는지 확인하기 지문 다시 읽어보기 corner case 찾아보기 초기화 안 된 지역변수 찾아보기 N, M, i, j 등 변수 확인하기 풀이 증명하기 STL 함수 다시 생각해보기 이 목록 다시 읽어보기 알고리즘 팀원에게 설명하기 팀원이랑 코드 보기 잠깐 일어나서 생각 재정비하고 오기 입출력 형식 확인하기 (whitespace 포함)
Runtime error:	Time limit exceeded: / Memory limit exceeded:
코너 케이스 처리해보기 초기화 안 된 변수 찾기 out-of-range 확인하기 assertion 넣어보기 무한 재귀 확인하기 null pointer 확인하기 메모리 사용량 확인하기	무한 루프 확인하기 알고리즘 시간 복잡도 확인하기 data copy 어느 정도 하는지 확인하기 (reference) 입출력 규모 생각하기 (scanf 고려해보기) vector, map 최소화하기 팀원에게 알고리즘 물어보기 최대 메모리 사용량 계산하기 다중 테케 문제에서 초기화하기

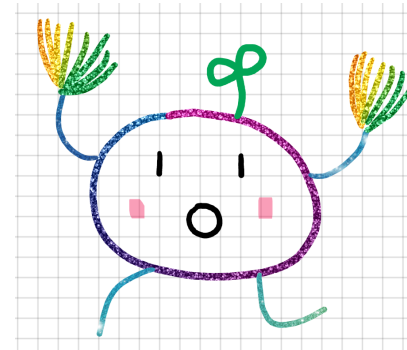


Figure 1: 풀다가 막힐 때는 이 그림을 봅시다. 아자아자 화이팅!

# 1 Math

## 1.1 Basic Mathematics

### 1.1.1 Trigonometry

- $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
- $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$
- $\sin 2\theta = 2 \sin \theta \cos \theta$
- $c^2 = a^2 + b^2 - 2ab \cos \gamma$

### 1.1.2 Generating Function

- $\sum_n (pn + q)x^n = \frac{p}{1-x} + \frac{q}{(1-x)^2}$  (Arithmetic progression)
- $\sum_n (rx)^n = (1 - rx)^{-1}$  (Geometric progression)
- $\sum_n \binom{m}{n} x^n = (1 + x)^m$  (Binomial coefficient)
- $\sum_n \binom{m+n-1}{n} x^n = (1 - x)^{-m}$  (Multiset coefficient)

### 1.1.3 Calculus

- $\int_a^b f(x) dx = \frac{b-a}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$  (Simpson's Rule, for cubic poly)
- $\int u'v dx = uv - \int uv' dx$  (Integration by parts)

## 1.2 Number Theory

### 1.2.1 Lattice Points under Line

```
ll calc(ll a, ll b, ll c, ll n) {
    if(!n) return 0;
    ll tmp = a/c*n*(n-1)/2;
    tmp += b/c*n;
    return tmp + calc(c, (a*n+b)%c, a%c, ((a%c)*n+b%c)/c);
} // 0 <= x < n, 0 < y <= (a/c)x + (b/c)
```

### 1.2.2 Shanks' Baby-step Giant-step

```
ll mexp(ll x, ll y, ll p) {
    if(!y) return 1;
    if(y & 1) return x * mexp(x*x%p, y>>1, p) % p;
    return mexp(x*x%p, y>>1, p);
}
vector<ll> get_factor(ll n) {
    vector<ll> v;
    for(ll i=2; i*i<=n; i++) {
```

```
        if(n % i == 0) {
            v.push_back(i);
            while(n % i == 0) n /= i;
        }
    }
    if(n > 1) v.push_back(n);
    return v;
}
ll get_primitive(ll n) {
    ll phi = n-1; // assume n is prime
    vector<ll> fact = get_factor(phi);
    for(ll x=2; x<=n; x++) {
        int yes = 1;
        for(ll y : fact) {
            yes &= (mexp(x, phi / y, n) != 1);
        }
        if(yes) return x;
    }
    return -1;
}
// find x s.t. x^k mod n = a -> (g^k)^y mod n = a, where x = g^y
ll bsgs(ll k, ll a, ll n) {
    ll g = get_primitive(n), phi = n-1; // assume n is prime
    if(g == -1) return -1;
    ll m = ceil(sqrt(n) + 1e-9);
    vector<pl> prec(m);
    for(ll j=0; j<m; j++) {
        prec[j] = {mexp(g, j * k % phi, n), j};
    }
    sort(prec.begin(), prec.end());
    ll cur = a, ncur = mexp(g, (phi - m) * k % phi, n);
    for(ll i=0; i<m; i++) {
        auto it = lower_bound(prec.begin(), prec.end(), pl(cur, 0));
        if(it->first == cur) {
            return ans = mexp(g, (i*m + it->second) % phi, n);
        }
        cur = cur * ncur % n;
    }
    return 0;
}
```

### 1.2.3 Extended Euclidean Algorithm

```
// ax + by = gcd(a,b). x, y?
pll ext_gcd(ll a, ll b) {
    if(b) {
        auto tmp = ext_gcd(b, a%b);
        return {tmp.second, tmp.first - (a/b) * tmp.second};
    }
    else return {1, 0};
}
```

```
// ax = gcd(a, m) mod m. x?
ll mod_inv(ll a, ll m) {
    return (ext_gcd(a, m).first + m) % m;
}

1.2.4 Chinese Remainder Theorem

ll pos_rem(ll a, ll m) { // m > 0. a % m?
    ll res = abs(a) % m;
    return a > 0 ? res : (res ? m - res : 0);
}

// ax = c mod m, bx = d mod n. x?
ll solve(ll a, ll c, ll m, ll b, ll d, ll n) {
    a = pos_rem(a, m); c = pos_rem(c, m); // if a, c not in [0, m)
    b = pos_rem(b, n); d = pos_rem(d, n); // if b, d not in [0, n)
    ll g = _gcd(a, _gcd(c, m)); a /= g, c /= g, m /= g;
    g = _gcd(b, _gcd(d, n)); b /= g, d /= g, n /= g;
    if(c % _gcd(a, m) || d % _gcd(b, n)) return inf;
    ll t1 = (mod_inv(a, m) * c) % m;
    ll t2 = (mod_inv(b, n) * d) % n;
    g = _gcd(m, n);
    ll lc = m * n / g;
    if(abs(t1 - t2) % g) return inf;
    pl p = ext_gcd(m, n);
    ll q = (t1 * p.second * n / g + t2 * p.first * m / g);
    return pos_rem(q, lc);
}
```

### 1.2.5 Möbius Inversion Formula

$$\forall n \in \mathbb{N} \ g(n) = \sum_{d|n} f(d) \implies f(n) = \sum_{d|n} \mu(d)g(n/d)$$

### 1.3 FFT

$$\text{FFT} : (a_0, a_1, \dots, a_{n-1}) \mapsto (\sum_{j=0}^{n-1} a_0(\omega^0)^j, \sum_{j=0}^{n-1} a_1(\omega^1)^j, \dots, \sum_{j=0}^{n-1} a_{n-1}(\omega^{n-1})^j)$$

```
void fft(vector<base>& a, bool inv) {
    int n = a.size(), j = 0;
    vector<ll> roots(n/2);
    for(int i=1; i<n; i++) {
        int bit = (n >> 1);
        while(j >= bit) {
            j -= bit;
            bit >>= 1;
        }
        j += bit;
        if(i < j) swap(a[i], a[j]);
    }

    double ang = 2 * acos(-1) / n * (inv ? -1 : 1);
    for(int i=0; i<n/2; i++)
        roots[i] = base(cos(ang * i), sin(ang * i));
```

```
/* In NTT, let prr = primitive root. Then,
int ang = mexp(prr, (mod - 1) / n);
if(inv) ang = mexp(ang, mod - 2);
for(int i=0; i<n/2; i++){
    roots[i] = (i ? (1ll * roots[i-1] * ang % mod) : 1);
}
also, make sure to apply modulus under here
*/
for(int i=2; i<=n; i<=1) {
    int step = n / i;
    for(int j=0; j<n; j+=i) {
        for(int k=0; k<i/2; k++) {
            ll u = a[j+k], v = a[j+k+i/2] * roots[step * k];
            a[j+k] = u+v;
            a[j+k+i/2] = u-v;
        }
    }
}
if(inv) for(int i=0; i<n; i++) a[i] /= n;
}

void conv(vector<base>& x, vector<base>& y) {
    int n = 2; while(n < max(x.size(), y.size())) n <<= 1;
    n <<= 1;
    x.resize(n); y.resize(n);
    fft(x, false); fft(y, false);
    for(int i=0; i<n; i++) x[i] *= y[i];
    fft(x, true); // access (ll)round(x[i].real())
}
```

### 1.4 Miller-Rabin + Pollard-Rho

//Prove By Solving - <https://www.acmicpc.net/problem/4149>

```
namespace miller_rabin{
    lint mul(lint x, lint y, lint mod){ return (__int128) x * y % mod; }
    lint ipow(lint x, lint y, lint p){
        lint ret = 1, piv = x % p;
        while(y){
            if(y&1) ret = mul(ret, piv, p);
            piv = mul(piv, piv, p);
            y >>= 1;
        }
        return ret;
    }
    bool miller_rabin(lint x, lint a){
        if(x % a == 0) return 0;
        lint d = x - 1;
        while(1){
            lint tmp = ipow(a, d, x);
            if(d&1) return (tmp != 1 && tmp != x-1);
            else if(tmp == x-1) return 0;
        }
    }
}
```

```

        d >>= 1;
    }
}
bool isprime(lint x){
    for(auto &i : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}){
        if(x == i) return 1;
        if(x > 40 && miller_rabin(x, i)) return 0;
    }
    if(x <= 40) return 0;
    return 1;
}

namespace pollard_rho{
    lint f(lint x, lint n, lint c){
        return (c + miller_rabin::mul(x, x, n)) % n;
    }
    void rec(lint n, vector<lint> &v){
        if(n == 1) return;
        if(n % 2 == 0){
            v.push_back(2);
            rec(n/2, v);
            return;
        }
        if(miller_rabin::isprime(n)){
            v.push_back(n);
            return;
        }
        lint a, b, c;
        while(1){
            a = rand() % (n-2) + 2;
            b = a;
            c = rand() % 20 + 1;
            do{
                a = f(a, n, c);
                b = f(f(b, n, c), n, c);
            }while(gcd(abs(a-b), n) == 1);
            if(a != b) break;
        }
        lint x = gcd(abs(a-b), n);
        rec(x, v);
        rec(n/x, v);
    }
    vector<lint> factorize(lint n){
        vector<lint> ret;
        rec(n, ret);
        sort(ret.begin(), ret.end());
        return ret;
    }
};

```

## 1.5 Black Box Linear Algebra + Kitamasa

```

vector<int> berlekamp_massey(vector<int> x){
    vector<int> ls, cur;
    int lf, ld;
    for(int i=0; i<x.size(); i++){
        lint t = 0;
        for(int j=0; j<cur.size(); j++){
            t = (t + 1ll * x[i-j-1] * cur[j]) % mod;
        }
        if((t - x[i]) % mod == 0) continue;
        if(cur.empty()){
            cur.resize(i+1);
            lf = i;
            ld = (t - x[i]) % mod;
            continue;
        }
        lint k = -(x[i] - t) * ipow(ld, mod - 2) % mod;
        vector<int> c(i-lf-1);
        c.push_back(k);
        for(auto &j : ls) c.push_back(-j * k % mod);
        if(c.size() < cur.size()) c.resize(cur.size());
        for(int j=0; j<cur.size(); j++){
            c[j] = (c[j] + cur[j]) % mod;
        }
        if(i-lf+(int)ls.size() >= (int)cur.size()){
            tie(ls, lf, ld) = make_tuple(cur, i, (t - x[i]) % mod);
        }
        cur = c;
    }
    for(auto &i : cur) i = (i % mod + mod) % mod;
    return cur;
}

int get_nth(vector<int> rec, vector<int> dp, lint n){
    int m = rec.size();
    vector<int> s(m), t(m);
    s[0] = 1;
    if(m != 1) t[1] = 1;
    else t[0] = rec[0];
    auto mul = [&rec](vector<int> v, vector<int> w){
        int m = v.size();
        vector<int> t(2 * m);
        for(int j=0; j<m; j++){
            for(int k=0; k<m; k++){
                t[j+k] += 1ll * v[j] * w[k] % mod;
                if(t[j+k] >= mod) t[j+k] -= mod;
            }
        }
    };
    for(int j=2*m-1; j>=m; j--){
        for(int k=1; k<=m; k++){
            t[j-k] += 1ll * t[j] * rec[k-1] % mod;

```

```

        if(t[j-k] >= mod) t[j-k] -= mod;
    }
}
t.resize(m);
return t;
};
while(n){
    if(n & 1) s = mul(s, t);
    t = mul(t, t);
    n >>= 1;
}
lint ret = 0;
for(int i=0; i<m; i++) ret += 111 * s[i] * dp[i] % mod;
return ret % mod;
}
int guess_nth_term(vector<int> x, lint n){ // init with > 3k, O(1^2 lg n)
    if(n < x.size()) return x[n];
    vector<int> v = berlekamp_massey(x);
    if(v.empty()) return 0;
    return get_nth(v, x, n);
}
struct elem{int x, y, v;}; // A_(x, y) <- v, 0-based. no duplicate please..
vector<int> get_min_poly(int n, vector<elem> M){
    // smallest poly P such that A^i = sum_{j < i} {A^j \times P_j}
    vector<int> rnd1, rnd2;
    mt19937 rng(0x14004);
    auto randint = [&rng](int lb, int ub){
        return uniform_int_distribution<int>(lb, ub)(rng);
    };
    for(int i=0; i<n; i++){
        rnd1.push_back(randint(1, mod - 1));
        rnd2.push_back(randint(1, mod - 1));
    }
    vector<int> gobs;
    for(int i=0; i<2*n+2; i++){
        int tmp = 0;
        for(int j=0; j<n; j++){
            tmp += 111 * rnd2[j] * rnd1[j] % mod;
            if(tmp >= mod) tmp -= mod;
        }
        gobs.push_back(tmp);
        vector<int> nxt(n);
        for(auto &i : M){ // sparse matrix * vector
            nxt[i.x] += 111 * i.v * rnd1[i.y] % mod;
            if(nxt[i.x] >= mod) nxt[i.x] -= mod;
        }
        rnd1 = nxt;
    }
    auto sol = berlekamp_massey(gobs);
    reverse(sol.begin(), sol.end());
}

```

```

    return sol;
}
lint det(int n, vector<elem> M){
    vector<int> rnd;
    mt19937 rng(0x14004);
    auto randint = [&rng](int lb, int ub){
        return uniform_int_distribution<int>(lb, ub)(rng);
    };
    for(int i=0; i<n; i++) rnd.push_back(randint(1, mod - 1));
    for(auto &i : M){
        i.v = 111 * i.v * rnd[i.y] % mod;
    }
    auto sol = get_min_poly(n, M)[0];
    if(n % 2 == 0) sol = mod - sol;
    for(auto &i : rnd) sol = 111 * sol * ipow(i, mod - 2) % mod;
    return sol;
}

```

## 2 Geometry

### 2.1 struct Point

```

const double eps = 1e-10;
template <class T>
struct point{
    typedef point P;
    T x, y;
    point(T x=0, T y=0) : x(x), y(y) {}
    bool operator< (P a) const {return fabs(x-a.x) > eps ? x<a.x : y<a.y;}
    bool operator== (P a) const {return max(fabs(x-a.x), fabs(y-a.y)) < eps;}
    P operator+ (P a) const {return P(x+a.x, y+a.y);}
    P operator- (P a) const {return P(x-a.x, y-a.y);}
    P operator- () const {return P(-x, -y);};
    T operator* (P a) const {return x*a.x + y*a.y;} // inner prod
    T operator/ (P a) const {return x*a.y - y*a.x;} // outer prod
    T dist2() const {return x*x + y*y;}
    double dist() const {return sqrt(double(dist2()));}
    P perp() const {return P(-y, x);}; // rotate 90 deg ccw
    P mult(T t) const {return P(x*t, y*t);}
    P unit() const {return P(x/dist(), y/dist());}
    P rotate(double a){
        return P(x*cos(a)-y*sin(a), x*sin(a)+y*cos(a));
    }
};

int sgn(double x) {return (x > eps) - (x < -eps);}
typedef point<double> P;

```

## 2.2 Distance, Intersection

### 2.2.1 Point-to-Line

```
double lineDist(P a, P b, P p) {
    return ((b-a)/(p-a))/(b-a).dist(); // a->b : left (+), right : (-);
}
```

### 2.2.2 Point-to-Segment

```
double segDist(P s, P e, P p) {
    if(s == e) return (p-s).dist(); // mind the eps
    double d = (e-s).dist2(), t = min(d, max(.0, (p-s)*(e-s)));
    return ((p-s).mult(d)-(e-s).mult(t)).dist() / d;
}
```

### 2.2.3 Line intersection

```
template<class P>
pair<int, P> lineInter(P a, P b, P c, P d){
    if((b-a)/(d-c) == 0) // parallel, mind the eps
        return {-(b-a)/(c-a) == 0, a};
    double oa = (d-c)/(a-c), ob = (d-c)/(b-c);
    return {(a.mult(ob) - b.mult(oa)).mult(1/(ob-oa))};
} // 1,0,-1(inf) : inter
```

### 2.2.4 Segment Intersection

```
bool onSegment(P s, P e, P p) {return segDist(s, e, p) < eps;}
```

```
template<class P> vector<P> segInter(P a, P b, P c, P d){
    double oa = (d-c)/(a-c), ob = (d-c)/(b-c),
           oc = (b-a)/(c-a), od = (b-a)/(d-a);
    if(sgn(oa)*sgn(ob) < 0 && sgn(oc)*sgn(od) < 0)
        return {(a.mult(ob) - b.mult(oa)).mult(1/(ob-oa))};
    set<P> S;
    if(onSegment(c, d, a)) S.insert(a);
    if(onSegment(c, d, b)) S.insert(b);
    if(onSegment(a, b, c)) S.insert(c);
    if(onSegment(a, b, d)) S.insert(d);
    return vector<P>(S.begin(), S.end());
}
```

### 2.2.5 Circle-Line Intersection

```
vector<P> circLine(P A, P B, P O, double r){
    vector<P> v;
    P X = O-A, D = B-A;
    double rat = 1.0 * (X*D) / (D*D);
    double det = (X*D)*(X*D) - (D*D)*(X*X - r*r);
    if(det < 0) return {};
    else if(det < eps) return {P(A + D.mult(rat))};
    det = sqrt(det);
```

```
        return {P(A + D.mult(rat + det/(D*D))),
                P(A + D.mult(rat - det/(D*D)))};
}
```

### 2.2.6 Circle-Line Tangent

```
vector<P> circTangent(P A, P O, double r){
    if((O-A).dist2() < r*r + eps) return {};
    double th = asin(r/(O-A).dist());
    return {A + (O-A).rotate(th).mult(cos(th)),
            A + (O-A).rotate(-th).mult(cos(th))};
}
```

## 2.3 Convex Hull

```
vector<pll> get_CV(vector<pll> V){
    sort(V.begin(), V.end());
    sort(V.begin() + 1, V.end(), [&](pll x, pll y){
        pll xx = x - V[0];
        pll yy = y - V[0];
        ll res = xx / yy;
        if(res != 0) return res > 0;
        if(xx.first != yy.first) return xx.first < yy.first;
        return xx.second < yy.second;
    });

    vector<pll> ret;
    for(auto val : V){
        while(ret.size() > 1){
            pll xx = ret[ret.size() - 2] - val;
            pll yy = ret[ret.size() - 1] - val;

            if(xx / yy <= 0) ret.pop_back();
            else break;
        }
        ret.push_back(val);
    }

    return ret;
}
```

## 2.4 Sorting Points by Angle

```
// credit : http://koosaga.com/97
auto angle_sort = [&](P &a, P &b){
    if((a < P(0, 0)) ^ (b < P(0, 0))) return b < a;
    if(a / b != 0) return a / b > 0;
    return a.dist2() < b.dist2(); // norm
}; // counter-clockwise sort
```

## 2.5 Rotating Calipers

```
void rotating_calipers(vector<pll> CV){
    int pos = 0;
    for(int i = 0 ; i < CV.size() ; i++) if(CV[pos] < CV[i]) pos = i;

    int ind1 = 0, ind2 = pos;
    ll dist = (CV[ind1] - CV[ind2]) * (CV[ind1] - CV[ind2]);

    auto get_v = [&](int x) { return CV[(x + 1) % CV.size()] - CV[x];};
    for(int i = 0 ; i < CV.size() ; i++){
        pll v = get_v(i);
        while((-v) / get_v(pos) < 0) pos = (pos + 1) % CV.size();
        ll tmp_dist = (CV[pos] - CV[i]) * (CV[pos] - CV[i]);
        if(dist < tmp_dist) {
            dist = tmp_dist;
            ind1 = i;    ind2 = pos;
        }
    }

    printf("%lld %lld %lld %lld\n", CV[ind1].first, CV[ind1].second, CV[ind2].first,
    CV[ind2].second);
}
```

## 2.6 Smallest Enclosing Circle

```
//Prove By Solving - https://www.acmicpc.net/problem/11930
int main(){
    scanf("%d", &N);
    for(int i = 1 ; i <= N ; i++) scanf("%lf%lf%lf", &A[i].x, &A[i].y, &A[i].z);

    int t = 70000;
    double rate = 1.0;
    point cur = (point){0, 0, 0};
    for(int i = 1 ; i <= t; i++){
        int ind = 1;
        for(int j = 1 ; j <= N ; j++)
            if( (A[j] - cur) * (A[j] - cur) >
                (A[ind] - cur) * (A[ind] - cur)) ind = j;
        cur = cur + (A[ind] - cur) * rate;
        rate *= 0.99;
    }
    double r = 0;
    for(int i = 1 ; i <= N ; i++) r = max(r, (A[i] - cur) * (A[i] - cur));
    cout << sqrt(r);
    return 0;
} // Non-deterministic, deterministic O(n lg n) requires Voronoi diagram
```

## 2.7 Circumcircle

```
double cc_radius(P& A, P& B, P& C){
    return (B-A).dist() * (C-B).dist() * (A-C).dist() / fabs((A-B) / (B-C)) / 2;
}

P cc_center(P& A, P& B, P& C){
    P b = C-A, c = B-A;
    return A + (b.mult(c.dist2()) - c.mult(b.dist2())).perp().mult(0.5/(b/c));
}
```

## 2.8 Polygon Area

```
double ans = 0; // ans : double area
for(int i=0;i<points.size();i++)
    ans += points[i] / points[(i+1 == points.size() ? 0 : i+1)];
```

## 3 Strings

### 3.1 Aho-Corasick Algorithm

```
namespace aho_corasick {
    const int MAXN = 100000, MAXC = 26;
    int trans[MAXN+1][MAXC];
    int fail[MAXN+1];
    bool term[MAXN+1];

    void build(const vector<string> &v) {
        setz(trans), setz(fail), setz(term);
        int cnode = 1;

        rep(i, s, v) {
            int p = 0;
            rep(j, s, v[i]) {
                char c = j-'a';
                if (!trans[p][c]) trans[p][c] = cnode++;
                p = trans[p][c];
            }
            term[p] = true;
        }

        queue<int> q; rep(i, MAXC) if (trans[0][i]) q.push(trans[0][i]);
        while(!q.empty()) {
            int t = q.front(); q.pop();
            rep(i, MAXC) {
                if (trans[t][i]) {
                    int p = fail[t];
                    while(p and not trans[p][i]) p = fail[p];
                    p = trans[p][i];
                    fail[trans[t][i]] = p;
                    if (term[p]) term[trans[t][i]] = true;
                }
            }
        }
    }
}
```



```

        q.push(trans[t][i]);
    }
}
}

bool query(string &t) {
    int p = 0;
    rep(i, t) {
        char c = i-'a';
        while(p and not trans[p][c]) p = fail[p];
        p = trans[p][c];
        if (term[p]) return true;
    }
    return false;
}
}

```

### 3.2 Lexicographically Smallest String Rotation

```

int min_rotation(string s) {
    int a=0, N=s.size(); s += s;
    rep(b,N) rep(i,N) {
        if (a+i == b || s[a+i] < s[b+i]) {b += max(0, i-1); break;}
        if (s[a+i] > s[b+i]) {a = b; break;}
    }
    return a; // rotate(v.begin(), v.begin()+min_rotation(v), v.end());
}

```

### 3.3 Suffix Array

```

// str : abracadabra
// SA : 10 7 0 3 5 8 1 4 6 9 2
// LCP : 1 4 1 1 0 3 0 0 0 2
vector<int> make_sa(const string& s) {
    int n = s.length();
    int lim = max(128, n+1);
    vector<int> sa(n), g(n+1), ng(n+1), cnt(lim), ind(lim+1);
    rep(i, n) sa[i] = i, g[i] = s[i];
    g[n] = 0;
    for(int t=1; t<s.length(); t<=<=1)
    {
        auto cmp = [&] (int a, int b) {
            return g[a] != g[b] ? g[a] < g[b] : g[a+t] < g[b+t];
        };
        rep (i, n) cnt[g[min(i+t, n)]]++;
        repp(i, 1, lim) cnt[i] += cnt[i-1];
        repr(i, n, 0) ind[--cnt[g[min(i+t, n)]]] = i;
        rep (i, lim) cnt[i] = 0;
        rep (i, n) cnt[g[i]]++; // same as cnt[g[ind[i]]]++
    }
}

```

```

        repp(i, 1, lim) cnt[i] += cnt[i-1];
        repr(i, n, 0) sa[--cnt[g[ind[i]]]] = ind[i];
        ng[sa[0]] = 1;
        repp(i, 1, n) ng[sa[i]] = ng[sa[i-1]] + cmp(sa[i-1], sa[i]);
        g = ng;

        fill(cnt.begin(), cnt.end(), 0);
        fill(ind.begin(), ind.end(), 0);
    }
    return sa;
}

```

```

vector<int> make_lcp(const string& s, const vector<int>& sa) {
    int n = s.length(), len = 0;
    vector<int> lcp(n-1), rank(n);
    for(int i=0; i<n; i++)
        rank[sa[i]] = i;
    for(int i=0; i<n; i++) {
        if(rank[i]) {
            int j = sa[rank[i]-1];
            int lc = n - max(i, j);
            while(len < lc && s[i+len] == s[j+len]) len++;
            lcp[rank[i]-1] = len;
        }
        if(len) len--;
    }
    return lcp;
}

```

### 3.4 Manacher's Algorithm

```

// 0-based
// s = # h # e # l # l # o #
// ret = 0 1 0 1 0 1 2 1 0 1 0
vector<int> manacher(const string& s) {
    int n = s.size(), r = -1, k = -1;
    vector<int> p(n);
    for (int i=0; i<n; i++) {
        if (i<=r) p[i] = min(r-i, p[2*k-i]);
        while (i-p[i]-1>=0 and i+p[i]+1<n and s[i-p[i]-1] == s[i+p[i]+1]) p[i]++;
        if (r < i+p[i]) r = i+p[i], k = i;
    }
    return p;
}

```

### 3.5 Z Algorithm

```

// 0-based
// s = a b c a b a b c a
// ret = 9 0 0 2 0 4 0 0 1

```

```

vector<int> z_algo(const string &s) {
    int l = 0, r = 0, N = sz(s);
    vector<int> Z(N);
    Z[0] = N;
    repp(i, 1, N) {
        if (i > r) {
            l = r = i;
            while(r < N and s[r] == s[r-1]) r++;
            r--;
            Z[i] = r-l+1;
        } else {
            int k = i-1;
            if (Z[k] < r-i+1) Z[i] = Z[k];
            else {
                l = i;
                while(r < N and s[r] == s[r-1]) r++;
                r--;
                Z[i] = r-l+1;
            }
        }
    }
    return Z;
}

```

## 4 Graph Theory

### 4.1 Strongly Connected Component

```

const int MAXN = 2e5 + 10; // > 2*N
int N, M;
int dfsn[MAXN], low[MAXN], finished[MAXN], cnt;
vector<int> ADJ[MAXN];
vector<vector<int>> G;
stack<int> S;
int f(int x){ // 0 1 2 3 4 5... -> f(1) f(-1) f(2) f(-2) f(3) f(-3)...
    return 2 * (abs(x) - 1) + (x < 0);
}

void add_edge(int x, int y){ // call by f(x), f(y)
    ADJ[x ^ 1].push_back(y);
    ADJ[y ^ 1].push_back(x);
}

// memset(finished, -1, sizeof(finished));
int scc(int here){
    static vector<int> tmp;
    S.push(here);
    dfsn[here] = low[here] = ++cnt;
    int &ret = low[here];

```

```

    for(int there : ADJ[here]){
        if(dfsn[there] == 0) ret = min(ret, scc(there));
        else if(finished[there] == -1) ret = min(ret, dfsn[there]);
    }

    if(dfsn[here] == low[here]){
        while(1){
            int x = S.top(); S.pop();
            finished[x] = G.size();
            tmp.push_back(x);
            if(x == here) break;
        }
        G.push_back(tmp);
        tmp.clear();
    }
    return ret;
}

```

#### 4.1.1 2-SAT

- scc를 실행시켜  $f(i)$  와  $f(-i)$ 가 같은 component에 있다면, 모순.
- $f(i)$  와  $f(-i)$  중 finished 배열의 수가 작은 것이 참이다.
  - SCC numbering의 역순이 위상정렬이기에,  $F \rightarrow T$ 를 유지하기 위함

### 4.2 Biconnected Component

```

// https://gist.github.com/koosaga/6f6fd50dd7067901f1b1
void dfs(int x, int p){
    dfn[x] = low[x] = ++piv;
    par[x] = p;
    for(int i=0; i<graph[x].size(); i++){
        int w = graph[x][i];
        if(w == p) continue;
        if(!dfn[w]){
            dfs(w, x);
            low[x] = min(low[x], low[w]);
        }
        else low[x] = min(low[x], dfn[w]);
    }
}

void color(int x, int c){
    if(c > 0) bcc[x].push_back(c); // c == 0 : first component
    vis[x] = 1;
    for(int i=0; i<graph[x].size(); i++){
        int w = graph[x][i];
        if(vis[w]) continue;
        if(dfn[x] <= low[w]){
            bcc[x].push_back(++cpiv);

```

```

        color(w, cpiv);
    }
    else color(w, c);
}
}

```

### 4.3 Euler Tour

```

struct Edge{
    int to, cnt; // to: 인접한 정점, cnt: 남은 사용 횟수
    Edge *dual; // dual: 역방향 간선을 가리키는 포인터
    Edge(): Edge(-1, 0){}
    Edge(int to1, int cnt1): to(to1), cnt(cnt1), dual(nullptr){}
};

void Eulerian(int curr){
    for(Edge *e: adj[curr]){
        if(e->cnt > 0){
            e->cnt--;
            e->dual->cnt--;
            Eulerian(e->to); // dfs
        }
    }
    cout << curr << '\n';
}

```

### 4.4 Heavy-Light Decomposition

```

int N, M;
vector<int> ADJ[MAXN];
int S[MAXN];
int hld_head[MAXN], color[MAXN], dfsn[MAXN], dcnt, hcnt;
int P[MAXN];

void dfs1(int here, int par){
    S[here] = 1; P[here] = par;
    for(int there : ADJ[here])
        if(there != par) dfs1(there, here), S[here] += S[there];
}

void dfs2(int here, int c){ // dfs reordering
    if(hld_head[c] == 0) hld_head[c] = here;
    dfsn[here] = ++dcnt; color[here] = c;

    sort(ADJ[here].begin(), ADJ[here].end(), [&](int x, int y){
        return S[x] > S[y];
    });

    int cnt = 0;
    for(int there : ADJ[here]) if(there != P[here]){
        if(++cnt == 1) dfs2(there, c);
        else dfs2(there, ++hcnt);
    }
}

```

```

    }
}

```

### 4.5 Dominator Tree

```

namespace Dtree {
    const int MAXN = 250001;
    vector<int> E[MAXN], RE[MAXN], rdom[MAXN];

    int S[MAXN], RS[MAXN], cs;
    int par[MAXN], val[MAXN];
    int sdom[MAXN], rp[MAXN];
    int dom[MAXN];

    int Find(int x, int c = 0) {
        if (par[x] == x) return c?-1:x;
        int p = Find(par[x], 1);
        if (p == -1) return c?par[x]:val[x];
        if (sdom[val[x]] > sdom[val[par[x]]]) val[x] = val[par[x]];
        par[x] = p;
        return c?p:val[x];
    }

    void Union(int x, int y) {
        par[x] = y;
    }

    void dfs(int x) {
        RS[S[x] = ++cs] = x;
        par[cs] = sdom[cs] = val[cs] = cs;
        for(int e : E[x]) {
            if (S[e] == 0) dfs(e), rp[S[e]] = S[x];
            RE[S[e]].pb(S[x]);
        }
    }

    int Do(int s, int *up) {
        dfs(s);
        for (int i = cs; i--; ) {
            for (int e : RE[i]) sdom[i] = min(sdom[i], sdom[Find(e)]);
            if (i > 1) rdom[sdom[i]].pb(i);
            for (int e: rdom[i]) {
                int p = Find(e);
                if (sdom[p] == i) dom[e] = i;
                else dom[e] = p;
            }
            if (i > 1) Union(i, rp[i]);
        }
        for (int i = 2; i <= cs; i++) if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
        for (int i = 2; i <= cs; i++) {
            up[RS[i]] = RS[dom[i]];
        }
    }
}

```

```

    return cs;
}

void addE(int x, int y) {E[x].pb(y);}
}

```

#### 4.6 Global Min Cut

```

// Stoer-Wagner Algorithm, O(VE lg E)
int minimum_cut_phase(int n, int &s, int &t,
vector<vector<int>> &adj, vector<int> vis){
vector<int> dist(n);
int mincut = 1e9;
while(true){
    int pos = -1, cur = -1e9;
    for(int i=0; i<n; i++){
        if(!vis[i] && dist[i] > cur){
            cur = dist[i];
            pos = i;
        }
    }
    if(pos == -1) break;
    s = t;
    t = pos;
    mincut = cur;
    vis[pos] = 1;
    for(int i=0; i<n; i++){
        if(!vis[i]) dist[i] += adj[pos][i];
    }
}
return mincut; // optimal s-t cut here is, {t} and V \ {t}
}

int solve(int n, vector<vector<int>> adj){
    if(n <= 1) return 0;
    vector<int> vis(n);
    int ans = 1e9;
    for(int i=0; i<n-1; i++){
        int s, t;
        ans = min(ans, minimum_cut_phase(n, s, t, adj, vis));
        vis[t] = 1;
        for(int j=0; j<n; j++){
            if(!vis[j]){
                adj[s][j] += adj[t][j];
                adj[j][s] += adj[j][t];
            }
        }
        adj[s][s] = 0;
    }
    return ans;
}

```

## 5 Network Flow

### 5.1 Theorems

**Max-flow Min-cut theorem** : 정점  $s$ 에서 정점  $t$ 까지 흐를 수 있는 최대 유량(max-flow)은 정점  $s$ 와 정점  $t$ 를 분리하는 간선들의 가중치 합(min-cut)과 같다.

**Vertex cover** : 어떤 그래프의 정점의 집합  $S$ 에 대해 그래프의 모든 간선이  $S$ 의 원소 중 최소 하나와 연결되어 있을 때,  $S$ 를 해당 그래프의 vertex cover라고 하며, minimum vertex cover는 최소 개수의 정점을 사용한 vertex cover이다.

**Independent set** : 어떤 그래프의 정점의 집합  $S$ 에 대해  $S$ 의 서로 다른 두 정점을 연결하는 간선이 없을 때,  $S$ 를 해당 그래프의 independent set이라고 하며, maximum independent set은 최대 개수의 정점을 사용한 independent set이다.

**Matching (independent edge set)** : 어떤 그래프의 간선의 집합  $S$ 에 대해  $S$ 의 서로 다른 두 간선이 공통된 정점을 가지지 않을 때,  $S$ 를 해당 그래프의 matching이라고 하며, maximum matching은 최대 개수의 간선을 사용한 matching이다.

**König's theorem** : 이분 그래프의 maximum matching의 크기는 minimum vertex cover의 것과 같다.

**Dinic's Algorithm** : 시간 복잡도  $O(V^2E)$ , unit capacity에서는  $\min(V^{2/3}E, E^{3/2})$ .

**Circulation Problem** : 새로운 source/sink  $s_n, t_n$ 를 만들어서 다음과 같이 간선을 추가하고  $\max flow(s_n \rightarrow t_n) = \sum l_i$ 인지 확인, 이후  $s \rightarrow t$ 로 maxflow

- $s_n \rightarrow b(l), a \rightarrow t_n(l), a \rightarrow b(r-l), t \rightarrow s(\infty)$

### 5.2 Dinic's Algorithm

```
const int INF = 1e9;
struct Dinic{
    int N;
    struct edge{
        int index, cap, rev;
        edge() : index(0), cap(0), rev(0) {}
        edge(int index, int cap, int rev) : index(index), cap(cap), rev(rev) {}
    };

    vector<vector<edge>> ADJ;
    vector<int> R, W;

    Dinic() {}
    Dinic(int N) : N(N){
        ADJ.resize(N); R.resize(N); W.resize(N);
    }

    void CE(int node1, int node2, int cap){
        ADJ[node1].push_back(edge(node2, cap, ADJ[node2].size()));
        ADJ[node2].push_back(edge(node1, 0, ADJ[node1].size() - 1));
    }

    bool bfs(int src, int sink){
        fill(R.begin(), R.end(), -1);
        R[src] = 0;
        queue<int> Q; Q.push(src);
        while(Q.size()){
            int here = Q.front(); Q.pop();
```

```
                for(auto e : ADJ[here]){
                    if(e.cap > 0 && R[e.index] == -1)
                        R[e.index] = R[here] + 1, Q.push(e.index);
                }
            }
        }
        return R[sink] != -1;
    }

    int dfs(int here, int sink, int f){
        if(here == sink) return f;
        for(int &i = W[here] ; i < ADJ[here].size() ; i++){
            auto &e = ADJ[here][i];
            if(e.cap > 0 && R[here] < R[e.index]){
                int res = dfs(e.index, sink, min(f, e.cap));
                if(res) {
                    e.cap -= res;
                    ADJ[e.index][e.rev].cap += res;
                    return res;
                }
            }
        }
        return 0;
    }

    int solve(int src, int sink){
        int ret = 0;
        while(bfs(src, sink)){
            fill(W.begin(), W.end(), 0);
            int res;
            while((res = dfs(src, sink, INF))) ret += res;
        }
        return ret;
    }
};
```

### 5.3 MCMF with SPFA

```
const int INF = 1e9;
struct MCMF {
    struct edge {
        int there, cap, cost, rev;

        edge() : there(0), cap(0), cost(0), rev(0) {}
        edge(int there, int cap, int cost, int rev) : there(there), cap(cap),
            cost(cost), rev(rev) {}
    };

    int N;
    vector<vector<edge>> ADJ;
    vector<int> R, INQ, C, I;
```

```

MCMF() : N(0) {}
MCMF(int N) : N(N) { ADJ.resize(N + 1); R.resize(N + 1); INQ.resize(N + 1);
    C.resize(N + 1); I.resize(N + 1); }

void CE(int i, int j, int cap, int cost) {
    ADJ[i].push_back(edge(j, cap, cost, ADJ[j].size()));
    ADJ[j].push_back(edge(i, 0, -cost, ADJ[i].size() - 1));
}

bool SPFA(int src, int sink) {
    queue<int> Q;    Q.push(src);
    fill(R.begin(), R.end(), -1);    R[src] = 0;
    fill(C.begin(), C.end(), -1);    C[src] = 0;
    fill(INQ.begin(), INQ.end(), 0);    INQ[src] = 1;
    while (Q.size()) {
        int here = Q.front();    Q.pop();
        INQ[here] = 0;
        for (int i = 0; i < ADJ[here].size(); i++) {
            auto e = ADJ[here][i];
            if ((C[e.there] == -1 || C[e.there] > C[here] + e.cost)
                && e.cap > 0) {
                C[e.there] = C[here] + e.cost;
                R[e.there] = here;
                I[e.there] = i;
                if (!INQ[e.there]) INQ[e.there] = 1, Q.push(e.there);
            }
        }
    }
    if (C[sink] == -1) return false;
    return true;
}

pii mcmf(int src, int sink) {
    pii ret = { 0, 0 };
    while (SPFA(src, sink)) {
        int flow = INF, cost = 0;
        for (int here = sink; here != src; here = R[here])
            flow = min(flow, ADJ[R[here]][I[here]].cap);
        for (int here = sink; here != src; here = R[here]) {
            auto &e = ADJ[R[here]][I[here]];
            cost += e.cost * flow;
            e.cap -= flow;
            ADJ[e.there][e.rev].cap += flow;
        }
        ret.first += flow, ret.second += cost;
    }
    return ret;
}
};

```

## 5.4 Hungarian Method

```

namespace Hung {
    const int MX = 2000;
    // IMPORTANT : n <= m, 1-based
    using T = long double;

    T maxv = 1e200;
    T a[MX][MX], n, m;

    void init(int nn, int mm) { n = nn; m = mm; }
    void set_value(int x, int y, T val) { a[x][y] = val; }
    T solve(vector<int> &ans) {
        vector<T> v(m+1), u(n+1);
        vector<int> p(m+1), way(m+1);
        for (int i=1; i<=n; ++i) {
            p[0] = i;
            int j0 = 0;
            vector<T> minv(m+1, maxv);
            vector<char> used(m+1, false);
            do {
                used[j0] = true;
                T delta = maxv;
                int i0 = p[j0], j1;
                for (int j=1; j<=m; ++j) if (!used[j]) {
                    T cur = a[i0][j] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta) delta = minv[j], j1 = j;
                }
                for (int j=0; j<=m; ++j) {
                    if (used[j]) u[p[j]] += delta, v[j] -= delta;
                    else minv[j] -= delta;
                }
                j0 = j1;
            } while (p[j0] != 0);
            do {
                int j1 = way[j0];
                p[j0] = p[j1];
                j0 = j1;
            } while (j0);
        }
        ans.resize(n + 1);
        for (int j=1; j<=m; ++j) {
            ans[p[j]] = j;
        }
        return -v[0];
    }
}

```

## 5.5 Hopcroft-Karp Algorithm

```

struct hopcroft_karp{
    int N;
    vector<vector<int>> ADJ;
    vector<int> L, rev, used;

    hopcroft_karp() {}
    hopcroft_karp(int N) : N(N) {
        ADJ.resize(N);
        L.resize(N), rev.resize(N, -1), used.resize(N, 0);
    }

    void CE(int here, int there){
        ADJ[here].push_back(there);
    }

    void bfs(){
        queue<int> Q;
        for(int i = 0 ; i < N ; i++) {
            if(used[i]) L[i] = -1;
            else L[i] = 0, Q.push(i);
        }

        while(Q.size()){
            int here = Q.front(); Q.pop();
            for(int there : ADJ[here]){
                if(rev[there] != -1 && L[rev[there]] == -1) {
                    L[rev[there]] = L[here] + 1;
                    Q.push(rev[there]);
                }
            }
        }
    }

    bool dfs(int here){
        for(int there : ADJ[here]){
            if(rev[there] == -1 || (L[here] < L[rev[there]] && dfs(rev[there]))){
                rev[there] = here;
                used[here] = 1;
                return true;
            }
        }
        return false;
    }

    int solve(){
        int ret = 0;
        while(1){
            bfs();
            int res = 0;

```

```

                for(int i = 0 ; i < N ; i++) {
                    if(used[i]) continue;
                    res += dfs(i);
                }
                if(res == 0) break;
                ret += res;
            }
        }
        return ret;
    }
};

```

## 6 Optimization Tricks

### 6.1 Knuth Optimization

- Recurrence :  $D[i][j] = \min_{i < k < j} (D[i][k] + D[k][j]) + C[i][j]$
- Quadrangle Inequality :  $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$ ,  $a \leq b \leq c \leq d$
- Monotonicity :  $C[b][c] \leq C[a][d]$ ,  $a \leq b \leq c \leq d$
- $A[i][j] = (\min. k \text{ s.t. } D[i][j] \text{ is min.})$ . Then  $A[i][j-1] \leq A[i][j] \leq A[i+1][j]$
- $O(N^2)$  time complexity

```

// opt[i-1][i] = i
for(int d=2;d<=n;d++) {
    for(int i=1;i+d<=n+1;i++) {
        for(int k=opt[i][j-1], j=i+d; k<=opt[i+1][j]; k++) {
            int v = dp[i][k] + dp[k][j] + c[i][j];
            if(dp[i][j] > v) dp[i][j] = v, opt[i][j] = k;
        }
    }
}

```

### 6.2 Divide and Conquer Optimization

- Recurrence :  $D[t][i] = \min_{k < i} (D[t-1][k] + C[k][i])$
- Min index :  $A[t][i] \leq A[t][i+1]$  ( $A[t][i] = (\min. k \text{ s.t. } D[t][i] \text{ is min.})$ )
- Quadrangle Inequality :  $C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$ ,  $a \leq b \leq c \leq d$
- Able to Divide and Conquer base on calculating  $D[t][i]$
- $O(TN \lg N)$  time complexity

```

// range of index : [l,r]
// range of dp : [s,e]
void dnc(int t, int l, int r, int s, int e)
{
    if(s > e) return;
    int m = (s+e)/2;

```

```

D[t][m] = 2e9;
for(int k=1;k<m&& k<=r;k++)
{
    int tmp = D[t-1][k] + C[k][m];
    if(D[t][m] > tmp)
        D[t][m] = tmp, A[t][m] = k;
}
dnc(t, 1, A[t][m], s, D[t][m]-1);
dnc(t, A[t][m], r, D[t][m]+1, e);
}

```

### 6.3 Convex Hull Trick

- Recurrence :  $dp[i] = \min_{j < i} (dp[j] + a[i]b[j])$ ,  $b[i-1] \leq b[i]$
- Think as  $dp[x] = a[i] = \min_{j < i} (b[j] \cdot x + dp[j])$
- Thus push lines and find minimum (by binary search)
- If  $a[i] \leq a[i+1]$  sweeping is possible
- Intersection of  $y = a_i x + b_i$  and  $y = a_{i+1} x + b_{i+1}$  :  $x = \frac{b_{i+1} - b_i}{a_i - a_{i+1}}$

### 6.4 Centroid Decomposition

```

// credit : https://gist.github.com/igorcarpanese/75162f3253bd230abd0f32f9950bf384
int dfs(int u, int p) {
    sub[u] = 1;
    for (auto v : tree[u])
        if (v != p and !chk[v]) sub[u] += dfs(v, u);
    return sub[u] + 1;
} // calc subtree size, chk means that node was a centroid

// each tree has at most two centroids
int centroid(int u, int p, int r) { // r : root
    for (auto v : tree[u])
        if (v != p and !chk[v] and sub[v] > sub[r]/2) return centroid(v, u);
    return u;
}

```

## 7 Data Structure

### 7.1 Persistent Segment Tree

```

const MAXN = 1e5 + 10;
struct node{
    node *l, *r;
    int cnt;
    node () {}
} pool[(1 << 17) * 17], *tree_head[MAXN];

```

```

int tcnt;
node* alloc(){
    memset(pool + tcnt, 0, sizeof(node));
    return pool + tcnt++;
}

node * init(int l, int r){
    node *ret = alloc();
    if(l != r) {
        int mid = (l + r) / 2;
        ret->l = init(l, mid);
        ret->r = init(mid + 1, r);
    }
    return ret;
}

void update(node * here, node *par, int l, int r, int val){
    if(l == r) {
        here->cnt = par->cnt + 1;
        return;
    }

    int mid = (l + r) / 2;
    if(val <= mid){
        here->l = alloc();
        here->r = par->r;
        update(here->l, par->l, l, mid, val);
    }
    else {
        here->l = par->l;
        here->r = alloc();
        update(here->r, par->r, mid + 1, r, val);
    }
    here->cnt = here->l->cnt + here->r->cnt;
}

int query(node *node1, node *node2, int l, int r, int k){
    if(l == r) return l;
    int ccc = node1->l->cnt - node2->l->cnt;
    int mid = (l + r) / 2;
    if(k <= ccc) return query(node1->l, node2->l, l, mid, k);
    else return query(node1->r, node2->r, mid + 1, r, k - ccc);
}

```

### 7.2 Link-Cut Tree

```

struct node{
    node *pp, *p, *l, *r;
    int val;
    node(){ p = 0, l = 0, r = 0;}
    node(int val) : val(val) { p = 0, l = 0, r = 0;}
}

```



```

};

void push(node *x){}
void pull(node *x){}

void rotate(node *x){
    if(!x->p) return;
    push(x->p); // if there's lazy stuff
    push(x);
    node *p = x->p;
    bool is_left = (p->l == x);
    node *b = (is_left ? x->r : x->l);
    x->p = p->p;
    if(x->p && x->p->l == p) x->p->l = x;
    if(x->p && x->p->r == p) x->p->r = x;
    if(is_left){
        if(b) b->p = p;
        p->l = b;
        p->p = x;
        x->r = p;
    }
    else{
        if(b) b->p = p;
        p->r = b;
        p->p = x;
        x->l = p;
    }
    pull(p); // if there's something to pull up
    pull(x);
    //if(!x->p) root = x; // IF YOU ARE SPLAY TREE
    if(p->pp){ // IF YOU ARE LINK CUT TREE
        x->pp = p->pp;
        p->pp = nullptr;
    }
}

void splay(node *x){
    while(x->p){
        node *p = x->p;
        node *g = p->p;
        if(g){
            if((p->l == x) ^ (g->l == p)) rotate(x);
            else rotate(p);
        }
        rotate(x);
    }
}

void access(node *x){
    splay(x);
    push(x);
    if(x->r){

```

```

        x->r->pp = x;
        x->r->p = nullptr;
        x->r = nullptr;
    }
    pull(x);
    while(x->pp){
        node *nxt = x->pp;
        splay(nxt);
        push(nxt);
        if(nxt->r){
            nxt->r->pp = nxt;
            nxt->r->p = nullptr;
            nxt->r = nullptr;
        }
        nxt->r = x;
        x->p = nxt;
        x->pp = nullptr;
        pull(nxt);
        splay(x);
    }
}

node *root(node *x){
    access(x);
    while(x->l){
        push(x);
        x = x->l;
    }
    access(x);
    return x;
}

node *par(node *x){
    access(x);
    if(!x->l) return nullptr;
    push(x);
    x = x->l;
    while(x->r){
        push(x);
        x = x->r;
    }
    access(x);
    return x;
}

node *lca(node *s, node *t){
    access(s);
    access(t);
    splay(s);
    if(s->pp == nullptr) return s;
    return s->pp;
}

void link(node *par, node *son){

```

```

    access(par);
    access(son);
    //son->rev ^= 1; // remove if needed
    push(son);
    son->l = par;
    par->p = son;
    pull(son);
}
void cut(node *p){
    access(p);
    push(p);
    if(p->l){
        p->l->p = nullptr;
        p->l = nullptr;
    }
    pull(p);
}

```

### 7.3 Dynamic Convex Hull

```

// https://github.com/niklasb/contest-algos/blob/master/convex_hull/dynamic.cpp
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};
struct HullDynamic : public multiset<Line> { // will maintain upper hull for maximum
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
    }
    void insert_line(ll m, ll b) {
        auto y = insert({m, b});
        y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
        if (bad(y)) { erase(y); return; }
        while (next(y) != end() && bad(next(y))) erase(next(y));
        while (y != begin() && bad(prev(y))) erase(prev(y));
    }
}

```

```

ll eval(ll x) {
    auto l = *lower_bound((Line) { x, is_query });
    return l.m * x + l.b;
}
};

```

### 7.4 Stern-Brocot Tree

```

// __int128 is recommended
bool test(ll a, ll b) { // for testing directions, vary by prob
    // return true if (true value) >= a/b
    ll n = 0, m = 1;

    rep(i, N) {
        if (n < m*A[i].fi) n = A[i].fi, m = 1;

        ll c = b*n+m*a, d = m*b;
        ll g = gcd(c, d);
        n = c/g;
        m = d/g;

        if (n > m*A[i].se) return false;
    }
    return true;
}

pair<ll, ll> stern_brocot(ll M, ll N) {
    // M : max value
    // N : max divisor
    // if result is a/b, return as {a, b}
    ll a = 0, b = 1; // l
    ll c = 1, d = 0; // r
    int l, r;
    bool chg = true;

    while(chg) {
        chg = false;

        // to left
        l = 0, r = (N-d-1)/b+1;
        while(l < r) {
            int mid = (l+r+1)/2;
            if (test(a*mid+c, b*mid+d)) r = mid-1;
            else l = mid;
        }

        c += a*1;
        d += b*1;
        chg |= (l > 0);

        // to right

```

```

    l = 0, r = (d?(N-b-1)/d+1:M);
    while(l < r) {
        int mid = (l+r+1)/2;
        if (test(a+mid*c, b+mid*d)) l = mid;
        else r = mid-1;
    }

    a += c*l;
    b += d*l;
    chg |= (l > 0);
}

return {a, b};
}

```

## 7.5 Rope

```

#include <bits/stdc++.h>
#include <ext/rope>
using namespace std;
using namespace __gnu_cxx;

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);

    crope rp; // rope<char>
    string s("Lorem-ipsum");
    int n = s.length();
    rp.append(s.c_str()); // add element

    int x = 3, y = 8; // split and merge below
    rp = rp.substr(x, y-x) + rp.substr(0, x) + rp.substr(y, n);
    cout << rp.at(0) << '\n'; // get element, 'e'
    cout << rp << '\n'; // print, "em-ip|Lor|sum"
}

```

## 7.6 Bitset

```

#include <bitset>
#include <iostream>
using namespace std;

int main() {
    bitset<8> b1(13);           // 00001101
    bitset<8> b2("10111");     // 00010111

    cout << b1.count() << endl; // 3
    cout << b1.test(6) << endl; // 0, since 2^6-th bit is 0
    b1.set(6);                  // set to 1, 1-fill if no param
}

```

```

b2.reset(2);                    // set to 0, 0-fill if no param
// use 'flip' for flipping

cout << "b1:" << b1 << endl;   // b1:01001101
cout << "b2:" << b2 << endl;   // b2:00010011
// use any, none, all (c++11) for bit checking
// supported operators : &, |, ^, <<, >>, ~, ==, !=
// these operators must match size (given to template)

cout << "&:" << (b1 & b2) << endl; // &: 00000001
cout << "^:" << (b1 ^ b2) << endl; // ^: 01011110
cout << "|:" << (b1 | b2) << endl; // |: 01011111
cout << "~:" << (~b1) << endl;    // ~: 10110010
cout << "<<:" << (b1 << 3) << endl; // <<:01101000
cout << b2.to_ulong() << endl;   // 19, c++11 supports ullong
}

```

## 7.7 Policy Based Data Structure

```

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef tree<
    int,
    null_type,
    less<int>,
    rb_tree_tag,
    tree_order_statistics_node_update >
    ordered_set;

// less<int> : not allow for duplicate
// less_equal<int> : allow for duplicate
// use upper_bound when you erase from set used less_equal

int N;

int main(void) {
    iosstream::sync_with_stdio(false);
    cin.tie(nullptr);

    ordered_set X;

    X.insert(1);
    X.insert(2);
    X.insert(4);
    X.insert(8);
}

```



```

16 8086598962041600      41472 8 3 2 2 1 1 1 1 1 1
17 74801040398884800      64512 6 3 2 2 1 1 1 1 1 1
18 897612484786617600     103680 8 4 2 2 1 1 1 1 1 1

```

	< 10 <sup>k</sup> prime	> 10 <sup>k</sup> prime	# of prime
1	7	11	4
2	97	101	25
3	997	1009	168
4	9973	10007	1229
5	99991	100003	9592
6	999983	1000003	78498
7	9999991	10000019	664579
8	99999989	100000007	5761455
9	999999937	1000000007	50847534

	< 10 <sup>k</sup> prime	> 10 <sup>k</sup> prime
10	9999999967	100000000019
11	99999999977	1000000000003
12	999999999989	10000000000039
13	9999999999971	100000000000037
14	99999999999973	1000000000000031
15	999999999999989	10000000000000037
16	9999999999999937	100000000000000061
17	9999999999999997	100000000000000003
18	99999999999999989	1000000000000000003

NTT Prime:

$469762049 = 7 \times 2^{26} + 1$ . Primitive root : 3.  
 $998244353 = 119 \times 2^{23} + 1$ . Primitive root: 3.  
 $985661441 = 235 \times 2^{22} + 1$ . Primitive root: 3.  
 $1012924417 = 483 \times 2^{21} + 1$ . Primitive root: 5.

Primes near  $10^9$ :  $10^9 + [7, 9, 21, 33, 87]$

### 8.3 Fast Integer IO

```

// credit : https://github.com/koosaga/DeobureoMinkyuParty/blob/master/teamnote.tex
static char buf[1 << 19]; // size : any number geq than 1024
static int idx = 0;
static int bytes = 0;
static inline int _read() {
    if (!bytes || idx == bytes) {
        bytes = (int)fread(buf, sizeof(buf[0]), sizeof(buf), stdin);
        idx = 0;
    }
    return buf[idx++];
}
static inline int _readInt() {
    int x = 0, s = 1;
    int c = _read();

```

```

while (c <= 32) c = _read();
if (c == '-') s = -1, c = _read();
while (c > 32) x = 10 * x + (c - '0'), c = _read();
if (s < 0) x = -x;
return x;
}

```

### 8.4 C++ Tips / Environments

```

#include <bits/stdc++.h> // magic header
using namespace std;    // magic namespace

struct StupidGCCantEvenCompileThisSimpleCode{
    pair<int, int> array[1000000];
}; // https://gcc.gnu.org/bugzilla/show_bug.cgi?id=68203

// how to use rand (in 2017)
mt19937 rng(0xdeadbeef);
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
int randint(int lb, int ub){ return uniform_int_distribution<int>(lb, ub)(rng); }
shuffle(permutation.begin(), permutation.end(), rng);
mt19937_64 _R(chrono::steady_clock::now().time_since_epoch().count()); // _R()

// comparator overload
auto cmp = [](seg a, seg b){return a.func() < b.func(); };
set<seg, decltype(cmp)> s(cmp);
map<seg, int, decltype(cmp)> mp(cmp);
priority_queue<seg, vector<seg>, decltype(cmp)> pq(cmp); // max heap

// hash func overload
struct point{
    int x, y;
    bool operator==(const point &p)const{ return x == p.x && y == p.y; }
};
struct hasher {
    size_t operator()(const point &p)const{ return p.x * 2 + p.y * 3; }
};
unordered_map<point, int, hasher> hsh;

// c++ setprecision example
#include <iostream> // std::cout, std::fixed
#include <iomanip> // std::setprecision

int main () {
    double f = 3.14159;
    std::cout << std::setprecision(5) << f << '\n'; // 3.1416
    std::cout << std::setprecision(9) << f << '\n'; // 3.14159
    std::cout << std::fixed;
    std::cout << std::setprecision(5) << f << '\n'; // 3.14159
    std::cout << std::setprecision(9) << f << '\n'; // 3.141590000
}

```