- Semi-supervised Learning with Deep Generative Models
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Introduction

Traditional ways of self-supervised learning:

- Simplest: self-training, using predicted results as labels
- TSVM: make unlabeled data far from the margins
- Graph-based methods

This work. Models the problem with a generative model (including class labels). Naturally, variational inference is introduced.

Deep Generative Models for Semi-supervised Learning

The paper discussed two models for semi-supervised training, and how they can be combined to produce a uniform model.

Latent-feature discrimnative model (M1): the idea is simple. Use VAE to learn representations, and use any other classifier for discrimantive learning. In particular, we model

$$p(z) \sim \mathcal{N}(z|0, I)$$
$$p_\theta(x|z) \sim f(x|z; \theta)$$

where $f(x|z;\theta)$ represent likelihood function. The parameters are computed using a neural network.

Generative semi-supervised model (M2): in this case, a latent class variable y is introduce. The model is defined as

$$p(y) = Cat(y|\pi); \quad p(z) = \mathcal{N}(z|0, I); \quad p_\theta(x|y, z) = f(x; y; \theta)$$

We assume that y and z are marginally independent.

Stack generative semi-supervised model (M1 + M2): just use z in M1 as x in M2. This produces a model $p_\theta(x, y, z_1, z_2) = p(y)p(z_2)p_\theta(z_1|y, z_2)p_\theta(x|z_1)$.

It seems that these two models are trained in two different phases in this work.

Objective

First, we should clarify the approximate posteriors used here. For M1, this is

$$\text{M1}: q_\phi(\mathbf{z}|\mathbf{x}) = \mathcal{N}\left(\mathbf{z}|\boldsymbol{\mu}_\phi(\mathbf{x}), \operatorname{diag}\left(\boldsymbol{\sigma}^2_\phi(\mathbf{x})\right)\right)$$

For M2 it is $q_\phi(z, y|x)$. It is useful to split this into $q_\phi(z|y, x)$ and $q_\phi(y|x)$ though:

$$q_\phi(\mathbf{z}|y, \mathbf{x}) = \mathcal{N}\left(\mathbf{z}|\boldsymbol{\mu}_\phi(y, \mathbf{x}), \operatorname{diag}\left(\boldsymbol{\sigma}^2_\phi(\mathbf{x})\right)\right); \quad q_\phi(y|\mathbf{x}) = \operatorname{Cat}(y|\boldsymbol{\pi}_\phi(\mathbf{x}))$$

For M1, this is just the VAE bound:

$$\log p_\theta(x) \geq E_{q_\phi(z|x)}[\log p_\theta(x|z)] - KL[q_\phi(z|x)\|p_\theta(z)] = -\mathcal{J}(x)$$

For M2, our general goal is maximum likelihood estimation. For labeled dataset, our goal is to maximize $\log p_\theta(x, y)$. This is

$$\log p_\theta(\mathbf{x}, y) \geq \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x},y)}\left[\log p_\theta(\mathbf{x}|y, \mathbf{z}) + \log p_\theta(y) + \log p(\mathbf{z}) - \log q_\phi(\mathbf{z}|\mathbf{x}, y)\right] = -\mathcal{L}(\mathbf{x}, y)$$

Intuitively, this learning two things:

- Given z, y, how do we construct x?
- Given x, y, what is z?

For unlabelled data, we want to maximize $p_\theta(x)$. That is

$$\begin{aligned}\log p_\theta(\mathbf{x}) &\geq \mathbb{E}_{q_\phi(y,\mathbf{z}|\mathbf{x})}\left[\log p_\theta(\mathbf{x}|y, \mathbf{z}) + \log p_\theta(y) + \log p(\mathbf{z}) - \log q_\phi(y, \mathbf{z}|\mathbf{x})\right] \\ &= \sum_y q_\phi(y|\mathbf{x})(-\mathcal{L}(\mathbf{x}, y)) + \mathcal{H}\left(q_\phi(y|\mathbf{x})\right) = -\mathcal{U}(\mathbf{x})\end{aligned}$$

Intuitively, given x, we are

- Guess a y, use this to learn things about z, as in the first part
- Learn ϕ that maximizes the reconstruction probability. At the same time, the entropy should be large.

Then our objective is that

$$\mathcal{J} = E_{D_L}\left[-\mathcal{L}(x, y)\right] + E_{D_U}\left[-\mathcal{U}(x)\right]$$

One thing that we note that, the model is only learning $q_\phi(y|x)$ on unlabeled dataset, which is an undersirable property. Ideally, all model and variational parameters should learn in all cases. To remedy this, a classfication loss is added:

$$\mathcal{J}^\alpha = \mathcal{J} + \alpha E_{D_L}\left[-\log q_\phi(y|x)\right]$$

In practive, α is set to 0.1

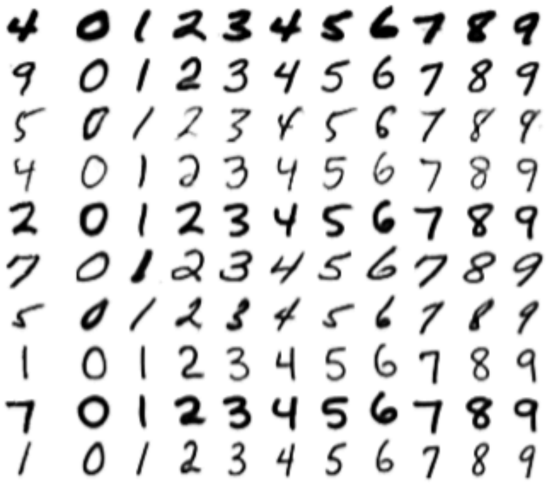
Computational Complexity

Unfortunately, this scales with the class number.

Experimental Results

- Train on MNIST with few labels. It works the best
- Conditional generation: set y (digit) or z (style) fixed. This generates very good results:

(a) Handwriting styles for MNIST obtained by fixing the class label and varying the 2D latent variable $\mathbf{z}$



(b) MNIST analogies

(c) SVHN analogies