- β-VAE: Learning Basic Visual Concepts with a Constrainted Variational Framework
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Motivation

The idea is simple. And I think the key contribution is to point out that β controls the degree of disentanglement.

The objective is to learn a disentangle representation for data. Disentangle means, variation in one generative factor corresponds to variation in a **single** latent unit.

Current approaches often assumes prior knowledge about the generative factor. This is not always true.

InfoGAN does a good job. However, there are some disadvantages:

- Training instability, reduced sample diversity (GAN's)
- Lacks a principled inference network.

So the idea is to use VAE. The only change is to add more weight on the KL term. **The key is the $p(z)$ is an isotropic unit Gaussian**, implying different component are independent. So this constraint also makes the learnt factor independent.

β-VAE Framework

We assume that each image is generated as follows:

- Some factors v, w are drawn from $p(v), p(w)$
- x is generated from $p(x|v, w) = Sim(v, w)$, where Sim is some real world simulator (like a graphical engine).
- **Besides, each component of v is conditionally independent**. This means $\log p(v|x) = \sum_k \log p(v_k|x)$.

Our goal is to latent a inference network describing $q_\phi(z|x)$, such that $q_\phi(z|x)$ capture the generative factors v in a disentangled manner. This can be achieve by matching it to a isotropic unit Gaussian $p(z)$. Combining reconstruction loss in VAE, we then have the constrained optimization problem

$$\max_{\phi,\theta} \mathbb{E}_{x\sim\mathbf{D}} \left[\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})]\right] \quad \text{subject to } D_{KL}\left(q_\phi(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z})\right) < \epsilon$$

The generalized Lagrangian:

$$\mathcal{F}(\theta, \phi, \beta; \mathbf{x}, \mathbf{z}) = \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] - \beta\left(D_{KL}\left(q_\phi(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z})\right) - \epsilon\right)$$

This can be rewritten into

$$\mathcal{F}(\theta, \phi, \beta; \mathbf{x}, \mathbf{z}) \geq \mathcal{L}(\theta, \phi; \mathbf{x}, \mathbf{z}, \beta) = \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})] - \beta D_{KL}\left(q_\phi(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z})\right)$$

The important thing to note here is, for a particular problem, optimzing $\mathcal{L}$ with a particular β is equivalent to optimizing the constrained problem with a particular ϵ.

Interpretation: β controls the trade-off between information capacity of z and the degree of disentanglement.

Disentanglemnt metric

First, a new data set is generated as follows.

Let's assume that $v \in \mathbb{R}^K$. To generate a single dataset,

1. $y \in \{1, \ldots, K\}$ is randomlly chosen
2. For $l = 1, \ldots, L$
    1. $(v_{li}, w_{li}), (v_{lj}, w_{lj})$ is generated from $p(v)$ and $p(w)$.
    2. **Set** $v_{liy} = v_{ljy}$. This means that we restrict one factor to be unchanged. So we will predict what is this factor is in this task.
    3. Generate x_{li}, x_{lj}.
    4. Generate z_{li}, z_{lj}.
3. Compute $z_{diff} = \frac{1}{L} \sum_{l=1}^{L} |z_{li} - z_{lj}|$

Then one data point will look like z_{diff}, y. This is just a classification problem and we can measure the accuracy.

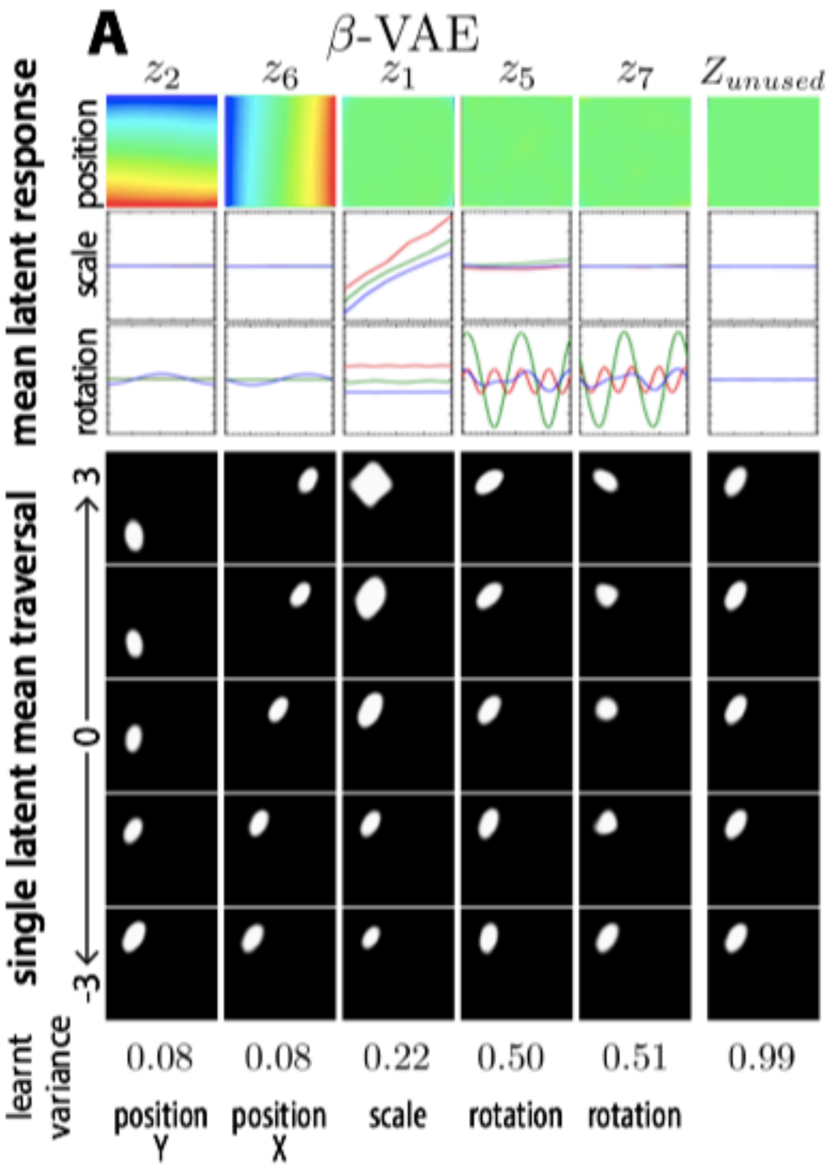
Experiments

Compared with InfoGAN, DC-IGN, VAE

- Qualitatively
    - Better disentanglement.
    - Learns extra unlabelled data generative factors that DC-IGN cannot learn (it is semi-supervised)
- Quantitative benchmarks
    - Just better.

Some words about this figure. So the dataset they use is generated, so they have fine-grained control over factors.

- First, how the dimensions are selected: given an image, find sigma $q_\phi(z|x)$. Sigma will imply the information contained.
- The first three row: we vary the **ground truth** factors (v), and see how the predicted mean (μ_z) changes
- The last five rows: we vary the mean μ_z, and watch how the generated image changes.



Understanding the effects of β

We try to figure out what is optimal β under different latent size z. Let the size of data by M and latent size be N, we define $\beta_{normalized} = \frac{\beta M}{N}$.

It is found that

- There is a positive correlation between β_{norm} and n.
- Given n, the relationship between disentanglement performance and β is like a U-curve.
- VAE reconstruction quality is a poor indicator of learnt disentanglement.