

# FYS4411 - Computational Physics II

## Project 1

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### Abstract

Write the abstract here

- Github repository containing programs and results are in: <https://github.com/evenmn/FYS4411/tree/master/Project%201>

## 1 Introduction

Introduction

## 2 Theory

We study a system of  $N$  bosons trapped in a harmonic oscillator with the Hamiltonian given by

$$\hat{H} = \sum_i^N \left( -\frac{\hbar^2}{2m} \nabla_i^2 + V_{ext}(\vec{r}_i) \right) + \sum_{i<j}^N V_{int}(\vec{r}_i, \vec{r}_j) \quad (1)$$

with  $V_{ext}$  as the external potential, which is the harmonic oscillator potential, and  $V_{int}$  as the interaction term, which can be ignored when developing the benchmarks.

The wavefunction is on the form

$$\Psi_T(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, \alpha, \beta) = \prod_i^N g(\alpha, \beta, \vec{r}_i) \prod_{i<j} f(a, r_{ij}) \quad (2)$$

where  $r_{ij} = |\vec{r}_i - \vec{r}_j|$  and  $g$  is assumed to be an exponential

$$g(\alpha, \beta, \vec{r}_i) = \exp[-\alpha(x_i^2 + y_i^2 + \beta z_i^2)] \quad (3)$$

which is practical since

$$\prod_i^N g(\alpha, \beta, \vec{r}_i) = \exp[-\alpha(x_1^2 + y_1^2 + \beta z_1^2 + \cdots x_N^2 + y_N^2 + \beta z_N^2)]. \quad (4)$$

$\alpha$  is a variational parameter that we later use to find the energy minimum, and  $\beta$  is a constant. The  $f$  presented above is the correlation wave function, which is

$$f(a, r_{ij}) = \begin{cases} 0 & r_{ij} \leq a \\ \left(1 - \frac{a}{r_{ij}}\right) & r_{ij} > a. \end{cases} \quad (5)$$

The first case we will take into account, is when  $a = 0$ , and one might observe that  $f = 1$  then. Anyway, ...

We want to calculate the local energy as a function of  $\alpha$ , and then use Variational Monte Carlo (VMC) described in section 3.1. For the non-interacting case, the analytical expression is well-known and given by

$$E = \hbar\omega(n + 1/2) \quad (6)$$

where  $n$  is the total number of free dimensions, which gonna be an useful benchmark. The local energy is

$$E_L(\vec{r}) = \frac{1}{\Psi_T(\vec{r})} \hat{H} \Psi_T(\vec{r}) \quad (7)$$

which gives the following results considering  $a = 0$ :

INSERT ANALYTICAL EXPRESSIONS FROM A

For  $a \neq 0$  it gets rather more complicated, because we need to deal with the correction wave function as well. By defining

$$f(a, r_{ij}) = \exp\left(\sum_{i < j} u(r_{ij})\right) \quad (8)$$

and doing a change of variables

$$\frac{\partial}{\partial r_k} = \frac{\partial}{\partial r_k} \frac{\partial r_{kj}}{\partial r_{kj}} = \frac{\partial r_{kj}}{\partial r_k} \frac{\partial}{\partial r_{kj}} = \frac{(r_k - r_j)}{r_{kj}} \frac{\partial}{\partial r_{kj}} \quad (9)$$

one will end up with

$$\begin{aligned}
E_L = \sum_k \bigg( & -\frac{1}{2} \left( 4\alpha^2 (x_k^2 + y_k^2 + \beta^2 z_k^2 - \frac{1}{\alpha} - \frac{\beta}{2\alpha}) \right. \\
& - 4\alpha \sum_{j \neq k} (x_k, y_k, \beta z_k) \frac{(\vec{r}_k - \vec{r}_j)}{r_{kj}} u'(r_{kj}) \\
& + \sum_{ij \neq k} \frac{(\vec{r}_k - \vec{r}_j)(\vec{r}_k - \vec{r}_i)}{r_{ki} r_{kj}} u'(r_{ki}) u'(r_{kj}) \\
& \left. + \sum_{j \neq k} \left( u''(r_{kj}) + \frac{2}{r_{kj}} u'(r_{kj}) \right) \right) + V_{ext}(\vec{r}_k) \bigg). \tag{10}
\end{aligned}$$

This is not a pretty expression, but .. We could also split up the local energy expression

$$E_{L,i} = -\frac{\hbar^2}{2m} \frac{\nabla_i^2 \Psi_T}{\Psi_T} + V_{ext}(\vec{r}_i) = E_{k,i} + E_{p,i} \tag{11}$$

and calculate the local energy with a numerical approach where the second derivative can be approximated by the three-point formula:

$$f''(x) \simeq \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}. \tag{12}$$

In our case the position is a three dimensional vector, so we need to handle each dimension separately.

## 3 Methods

### 3.1 Variational Monte Carlo

### 3.2 Metropolis algorithm

#### 3.2.1 Brute force

#### 3.2.2 Importance sampling

## 4 Results

## 5 Discussion

## 6 Conclusion