Lover

0. $A \sim B$ og $B \sim C$ gir $A \sim C$
1. $\Delta E = Q + W$
2. $\Delta S \geq 0$ Isolert system
3. $\lim_{\mathrm{T}\to 0} \Rightarrow S = \text{konstant}$

Mye brukte formler

$F =$	$E - TS$	Helmholtz
$G =$	$E - TS + PV$	Gibbs

$$F = -kT\ln(Z)$$
$$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = NkT$$
$$\ln N! \approx N\ln N - N$$
$$S \equiv k\ln\Omega$$
$$\Delta S_{\text{mixing}} = k\ln\binom{N}{N_A}$$
$$W_{AB} = W_B - W_A = \int_{V_A}^{V_B} P(V)DV$$
$$E = -kT^2\left(\frac{\partial \ln Z}{\partial T}\right)$$

Ideell gass

$$PV = NkT$$
$$\Delta S = \frac{dE}{T} = \frac{Q}{T} = \frac{C_V dT}{T}$$
$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mE}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

Multiplisitet

$$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow! N_\downarrow}$$ Paramagnet

$$\Omega(N, q) = \binom{q + N - 1}{q} = \frac{(q + N - 1)!}{q!(N-1)!}$$ Einsteinkrystall

$$\Omega_N = \frac{1}{N!}\frac{V^N}{h^{3N}}\frac{2\pi^{3N/2}}{(3N/2 - 1)!}(\sqrt{2mE})^{3N-1}$$ Ideell gass

Varmeutveksling

$$e \equiv \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}$$ Effektivitet varmemaskin

$$e \equiv \frac{Q_c}{W} = \frac{1}{Q_h/Q_c - 1}$$ Effektivitet kjølemaskin

$$\frac{dP}{dT} = \frac{S_g - S_l}{V_g - V_l} = \frac{L}{T\Delta V}$$ Clausius-Clapeyron

Kanonisk

$$P = \frac{1}{Z}\exp(-\epsilon_i/kT)$$ Boltzmannsfordelingen

$$Z = \sum_i \exp(-\epsilon_i/kT)$$ Partisjonsfunksjonen

$$Z_N = \frac{Z_1^N}{N!}$$ Total partisjon

Grand-kanonisk

$$\mathcal{Z} = \sum_N \sum_i \exp -[\epsilon_i - N\mu]/kT$$ Gibbs sum

$$\bar{n}_{FD} = \frac{1}{\exp[(\epsilon - \mu)/kT] + 1}$$ Fermi-Dirac

$$\bar{n}_{BE} = \frac{1}{\exp[(\epsilon - \mu)/kT] - 1}$$ Bose-Einstein

$$\bar{n}_B = \exp[-(\epsilon - \mu)/kT]$$ Boltzmann

$$E = \int_0^\infty \epsilon\bar{n}(\epsilon, \mu, T)D(\epsilon)d\epsilon$$
$$N = \int_0^\infty \bar{n}(\epsilon, \mu, T)D(\epsilon)d\epsilon$$

Begreper

Isokor

Romlige dimensjoner (Volum, areal, lengde) holdes konstant

Isoterm

Temperaturen holdes konstant

Isobar

Trykket holdes konstant

Kvasistatisk

En prosess som skjer så sakte at systemet blir værende i likevekt.

Adiabatisk

Ingen endring i varme, i de fleste tilfeller heller ingen endring i entropi.

Isentropisk

Adiabatisk og kvasistatisk

Likevekt

- Mekanisk likevekt - $dP = 0$
- Termisk likevekt - $dT = 0$
- Diffusiv likevekt - $d\mu = 0$
- Kjemisk likevekt - $dG = 0$

Maxwellrelasjoner

Relasjoner bygget på Schwarz' teorem:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

Eksempler:

$$\begin{aligned}
+\left(\frac{\partial T}{\partial V}\right)_S &= -\left(\frac{\partial P}{\partial S}\right)_V \\
+\left(\frac{\partial T}{\partial P}\right)_S &= +\left(\frac{\partial V}{\partial S}\right)_P \\
+\left(\frac{\partial S}{\partial V}\right)_T &= +\left(\frac{\partial P}{\partial T}\right)_V \\
-\left(\frac{\partial S}{\partial P}\right)_T &= +\left(\frac{\partial V}{\partial T}\right)_P
\end{aligned}$$

Nyttige sammenhenger:

- Konstant energi og volum
  $\Rightarrow \Delta S > 0$
- Konstant temperatur og volum
  $\Rightarrow \Delta F < 0$
- Konstant temperatur og trykk
  $\Rightarrow \Delta G < 0$

Differensialer

$$\begin{aligned}
\frac{1}{T} &\equiv \left(\frac{\partial S}{\partial E}\right)_{N,V} \\
C_V &\equiv \left(\frac{\partial E}{\partial T}\right)_{N,V} \\
P &= +T\left(\frac{\partial S}{\partial V}\right)_{E,N} = -\left(\frac{\partial F}{\partial V}\right)_{T,N} \\
\mu &= -T\left(\frac{\partial S}{\partial N}\right)_{E,V} = +\left(\frac{\partial F}{\partial N}\right)_{T,V} = \left(\frac{\partial G}{\partial N}\right)_{T,P} \\
S &= -\left(\frac{\partial F}{\partial V}\right)_{T,N} = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = -\left(\frac{\partial G}{\partial T}\right)_{P,N} \\
V &= \left(\frac{\partial G}{\partial P}\right)_{T,N}
\end{aligned}$$

Identiteter

$$\begin{aligned} dE &= \quad TdS - PdV + \mu dN \\ dF &= -SdT - PdV + \mu dN \\ dG &= -SdT + VdP + \mu dN \end{aligned}$$

Utledninger

Den termodynamiske identitet

1. Tar utgangspunkt i $dQ = TdS$
2. T1L: $dE = dQ + dW$
3. $dW = PdV = \sigma dA = KdX$
4. Kan legge til μdN

$\Rightarrow TdS = dE - PdV + \mu dN$

Energi rett fra Z

Utgangspunkt:

$$Z(N, V, T) = \sum_i^N \exp(-\epsilon_i/kT)$$

$$P = (1/Z)\exp(-\epsilon_i/kT)$$

$$E = \sum_i^N \epsilon_i P(\epsilon_i) = (1/Z)\sum_i^N \epsilon_i \exp(-\epsilon_i\beta)$$

$$= \frac{1}{Z}\sum_i^N -\frac{\partial}{\partial\beta}\exp(-\epsilon_i\beta) = -\frac{1}{Z}\frac{\partial Z}{\partial\beta}$$

$$= -\frac{d\ln Z}{d\beta} = kT^2\left(\frac{\partial \ln Z}{\partial T}\right)$$

Adiabat for ideell gass

1. Ta utgangspunkt i Sackur-Tetrode
2. Adiabat - $dS = 0 \Rightarrow S_1 = S_2$
3. Bruk ekvipartisjonsprinsippet

$$\Rightarrow V_1^{2/3}T_1 = V_2^{2/3}T_2$$

Clausius-Clapeyron

1. $G_l = G_g$
2. $G_l(P_0 + dP, T_0 + dT) = G_g(P_0 + dP, T_0 + dT)$
3. Taylor:

$$G_l(P_0+dP, T_0+dT) \approx G_l(P_0, T_0)$$

$$+\left(\frac{\partial G_l}{\partial P}\right)dP + \left(\frac{\partial G_l}{\partial T}\right)dT$$

4. Bruker differensialene

$$\left(\frac{\partial G_l}{\partial P}\right)_T = V_l, \quad \left(\frac{\partial G_l}{\partial T}\right)_P = -S_l$$

$$\Rightarrow V_l dP - S_l dT = V_g dP - S_g dT$$

$$\Rightarrow \frac{dP}{dT} = \frac{S_l - S_g}{V_l - V_g} = \frac{L(T)}{T\Delta V}$$

Bose-Einstein-fordelingen

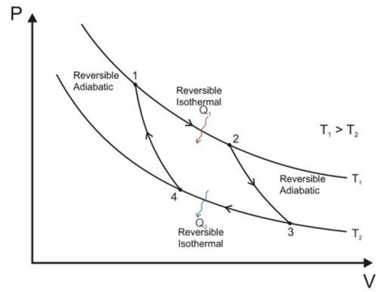
1. Tar utgangspunkt i

$$\mathcal{Z} = \sum_i^N \exp[-(n\epsilon - n\mu)/kT]$$

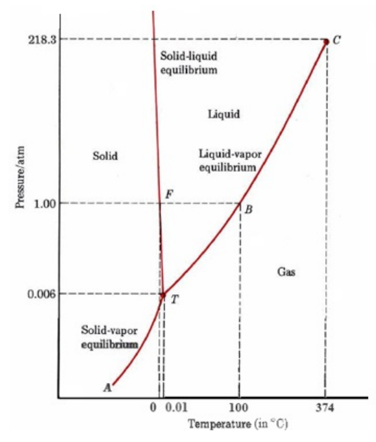
2. Bruker at $n = 1, 2, 3...$
3. Bruker $\sum_n q^n = 1/(1-q)$, $q < 1$
4. $\bar{n} = \sum nP(n)$

$$\Rightarrow \frac{1}{\mathcal{Z}}\sum_n n\exp(-nx) = \frac{1}{\mathcal{Z}}\sum_n \frac{\partial}{\partial x}\exp(-nx)$$

$$\Rightarrow \frac{1}{\mathcal{Z}}\frac{\partial \mathcal{Z}}{\partial x} = \frac{1}{\exp[(\epsilon - \mu)/kT] - 1}$$



(a) Carnotsyklusen



(b) Faseoverganger for rene substanser

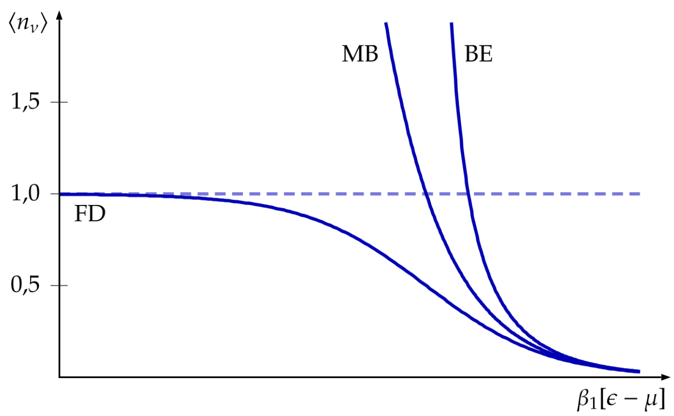


Figure 1: Fordelingsfunksjoner (Fermi-Dirac, Boltzmann, Bose-Einstein)

Tilstandstettheten og fermienergi

3D:

$$D(n)dn = N_T \cdot A = 2 \cdot (1/8) \cdot 4\pi n^2 dn = \pi n^2 dn \qquad N_T = 2 \text{ for fermioner}$$

$$\Rightarrow D(n)dn = D(\epsilon)d\epsilon \Rightarrow D(\epsilon) = \pi n^2 (1/2an) = \frac{\pi}{2a} n = \frac{\pi}{2a}\sqrt{\frac{\epsilon}{a}}$$

$$N = N_T \cdot V(n_{max}) = 2 \cdot \frac{1}{8} \cdot \frac{4}{3}\pi n_{max}^3 \quad \Rightarrow n_{max}^2 = \left(\frac{3N}{\pi}\right)^{2/3}$$

$$\epsilon_F = \epsilon(n_{max}) = a n_{max}^2 = a\left(\frac{3N}{\pi}\right)^{2/3}$$