Project 3

FYS4150 Computational Physics

Even S. Håland

Abstract

The motion of the planets in the solar system is governed by gravitational forces between the planets and the sun, and between the planets themselves. This can be expressed mathematically as a set of coupled differential equation. In this project we solve these equations, mainly by using the velocity Verlet algorithm. When doing so we use an object oriented approach, and two classes are developed; a *planet* class and a *solver* class. We find that the problem can be solved in a quite nice and compact way, and we get quite good estimates of the planetary orbits.

1 Introduction

2 Modelling planetary motion

Newtons law of gravitation states that the gravitational force between two objects, with mass M and m respectively, is given by

$$F_G = \frac{GMm}{r^2}, \tag{1}$$

where G is the gravitational constant and r is the distance between the two objects.

We start by considering a system with only two objects, namely the Sun (with mass $M_\odot$) and the Earth (with mass M_{Earth}), and we assume that the Sun is fixed, so the only motion we have to care about is that of the Earth. We also assume that the motion of the Earth is co-planar, and take this to be the xy-plane, with the Sun in the origin. When writing the programs and implementing the algorithm we actually work in $3D$, but extending from two to three dimensions is quite trivial.

The forces acting on the Earth in the x- and y-direction are then given by

$$F_{G,x} = -F_G \cos\theta = -\frac{GM_\odot M_{\text{Earth}}}{r^2}\cos\theta = -\frac{GM_\odot M_{\text{Earth}}}{r^3}x$$

and

$$F_{G,y} = -F_G \sin\theta = -\frac{GM_\odot M_{\text{Earth}}}{r^2}\sin\theta = -\frac{GM_\odot M_{\text{Earth}}}{r^3}y,$$

where we have used the relations $x = r\cos\theta$ and $y = r\sin\theta$. From Newtons second law we know that the accelerations, a_x and a_y, are given as

$$a_x = \frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_{\text{Earth}}} \quad \text{and} \quad a_y = \frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_{\text{Earth}}}.$$

We also know that the acceleration is the time derivative of the velocity, v, which again is the time derivative of the position, meaning that we can express the equations of motion as two coupled first order differential equations in each dimension:

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad a_x = \frac{dv_x}{dt} = -\frac{GM_\odot x}{r^3}, \quad a_y = \frac{dv_y}{dt} = -\frac{GM_\odot y}{r^3}.$$

When extending to three dimensions we simply add two more equations like those above, simply replacing x or y by z.

The next thing we want to do is to scale the equations appropriately. When working on an astronomical scale we prefer to work with years (yr) as the time unit and AU[1] as the length unit. If we assume the orbit to be circular (which is very close to the truth), the acceleration is given as

$$a = \frac{v^2}{r} = \frac{F_G}{M_{\text{Earth}}} = \frac{GM_\odot}{r^2} \quad \Rightarrow \quad v^2 r = GM_\odot.$$

[1] Astronomical units; 1 AU is defined as the mean distance between the Earth and the Sun.

Planet	Mass (kg)	Distance to Sun (AU)
Earth	6×10^{24}	
Jupiter	1.9×10^{27}	
Mars	6.6×10^{23}	
Venus	4.9×10^{24}	
Saturn	5.5×10^{26}	
Mercury	3.3×10^{23}	
Uranus	8.8×10^{25}	
Neptune	1.03×10^{26}	
Pluto	1.31×10^{22}	

where $r = 1$ AU, while the velocity is

$$v = \frac{2\pi r}{t} = 2\pi \ \mathrm{AU/yr},$$

which means that

$$GM_{\odot} = 4\pi^2 \ \mathrm{AU}^3/\mathrm{yr}^2.$$

3 Algorithms

4 Code

5 Results

6 Summary and conclusions