CPSC-354 Report

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Abstract

Consisting of CPSC 354 material

Contents

1 Introduction

This Report contains mainly consists of the Homework for each week, and the main project. In section 1, you can find the stuff done by Professor Kurz. Section 2 consists of Homework for each week. Section 3 consists of the project.

2 Homework

2.1 Week 1

Homework 1: Using Euclid's Elements Proposition 2 Algorithm on finding the Greatest Common Divisor amongst two numbers.

2.1.1 Python

In python, this algorithm can be written as:

```
a = 9
b = 33

while(a!=b):
    if(a>b):
         a = a-b
     else:
         b = b-a

print(a)
```

Using the sample given, 9 and 33, the Greatest Common Divisor is found in a simple way. Using the Euclid's Elements Proposition 2 Algorithm, we have two variables: **a** and **b**. To start, our samples are manually entered from the start of the program, and will begin the while loop. The while loop will continue until the **a** is the same value as **b**. The first line of code in the loop is an **if-statement**. This will compare **a** and **b** values, and will continue inside the **if-statement** if **a** is a large value than **b**. The Euclid's Elements Algorithm says: If

$$a > b$$

then replace **a** by

$$a - b$$

This is what happens in this first **if-statement**, as the variable **a** is replaced with a - b

If the **if-statement** is not executed, then the else statement will be preformed. Since our while loop tells us that it will continue until **a** is the same value as **b**, then we know that this else statement is:

$$b > a$$

This is the second part of the Euclid's Elements Proposition 2 Algorithm. It says that when:

$$b > a$$

to replace **b** with

$$b - a$$

This is what happens in this line of python code. The variable **b** is clearly replaced with b - a.

This while loop will continue until the values of **a** and **b** are the same. Once they are, the program will print the value of **a**. In this particular example, the value **3** would be printed.

2.1.2 C++

In C++, the algorithm can be written as:

```
#include <iostream>

using namespace std;

int main(int charc, char** argv){
    int a = 9;
    int b = 33;

    while(a!=b){
        if(a>b){
            a = a-b;
        }
        else{
            b = b-a;
        }
    }
    cout << a << endl;
}
```

Very similar code to the code in 2.1.1. The same process is being used, with two variables being created before the while loop. The while loop will continue until the values of the two variables are the same. Then the two conditions of

$$a > b$$

and

$$b > a$$

, are evaluated using an **if-statement** and **else-statement**. Once the correct condition is identified, the corresponding calculation done of each variable takes place. This will continue until the Greatest Common Divisor is found, and is printed to the screen. In this place, 3 is again printed.

2.2 Week 2

Homework 2: Create six Functions using Haskell: Select Evens, Select Odds, Member, Append, Revert, and Less Equal.

2.2.1 Select Evens and Select Odds

The task of these two functions could be used with each other. For Select Evens, the user would write the function name, then a list. The output would be the even element indices (Starting with 1 not 0). Here is the code for both.

```
select_evens [] = []
select_evens (x:xs) = select_odds xs
```

```
select_odds (x:xs) = x : select_evens xs
select_odds [] = []
```

These two functions are connected. Without one, the other will not work.

Going through the program, we will start with Select Odds. Using recursion, the head element will be split off first. Select Evens will be called with the other elements left in the list. Select Evens will then split the head and other elements again. This process will continue until the tail is an empty list. These methods will allow for only the odd indexed elements to be printed, or only the even indexed elements.

Here are a few outputs from the terminal:

```
ghci> select_evens ["a","b","c","d","e"]
["b","d"]
ghci> select_odds ["a","b","c"]
["a","c"]
ghci> select_odds ["a","b","c","d","e"]
["a","c","e"]
ghci> select_odds [1,2,3,4,5]
[1,3,5]
ghci> select_evens[432,34,543,2334,23]
[34,2334]
```

Here is the Task 2 Equational Reasoning for the Select Evens and Odds Function:

```
select_evens ["a","b","c","d","e"] =

select_evens ("a" : ["b","c","d","e"]]) = select_odds ["b","c","d","e"]

select_odds ("b" : ["c","d","e"]) = "b" : select_evens["c", "d", "e"]

select_evens ("c" : ["d", "e"]) = select_odds["d", "e"]

select_odds ("d" : ["e"]) = "b" : ("d" : select_evens ["e"])

select_evens ("e" : []) = select_odds []

select_odds [] =  "b" : ("d" : ([]))

["b", "d"]
```

Note that if Select Odds was called, the same procedure would occur, but would start with Select Odds first and would be the opposite of this output.

2.2.2 Member

In the member function, the user would ask for a "True" or "False" about if a list consisted of a particular element. If the list consisted of the element, then True would be returned. Otherwise, False is returned.

```
member y (x:xs) = if y==x
    then True
```

```
    else if len(xs) < 1
        then False
        else member y xs
```

The element that the user wants to know is the y. It starts off by comparing y to x, which is the head of the current list. If y is the same as x, then the element that the user is looking for is indeed in the list and True is returned. However, if it is not, then we will use the len function to see the size of the rest of the list. If it is not above 1, then False is returned. False is returned because there is nothing left to compare to the user element. However if it is above 1, then member is called again with the user element and the rest of the elements. This simulates the element being compared to each element in the list.

Here are some outputs for this function:

```
ghci> member "a" ["a", "b"]
True
ghci> member "a" ["c", "d", "e"]
False
ghci> member 4 [2,3,5]
False
ghci> member 4 [2,3,3,4,5]
True
ghci> member 4 [4,1,6,2]
True
```

Here is the Task 2 Equational Reasoning for the Member Function:

```
member 4 [2,3,4] =
    4 == 2
    else if len([3,4]) < 1

    len[3,4] = 3

    else if 3 < 1
        then False
        else member 4 [3,4]

member 4 [3,4]
    4 == 3
    else if len([4]) < 1

    len[4] = 2

    else if 2 < 1
        then False
        else member 4 [4]

member 4 [4]
    4 == 4
    then True
```

2.2.3 Append

The append function takes two lists from the user, and appends all of the second lists elements to the back of the first list. This function was difficult for me at first, but I realized that the first list stays at the front of the list, and that only the second list needed elements to "move".

Below is my code for the Append Function:

```
append [] (y:ys) = if len(ys) > 0
    then
        y : append [] ys
    else
        [y]
append (x:xs) (y:ys) = x : append xs (y:ys)
```

The last line will be the first line that executes. The user will have the two lists, but the function will have to iterate the entire (x:xs) list first. This is because these elements will be in the front of the new list regardless. After this line is recursively called, the first line will be called with an empty list and all of the second list. The len function is also used in this function as well. The if statement looks at the size of the tail elements of the second list. If it is greater than 0, then it will recursively add the elements to the list that already contains the first list elements. Once the len(ys) is not greater than 0, the else statement will have [y] which will basically not recall any of the append functions, and will not append anything to the list. This will let the recursion end.

Here are some outputs of the append function:

```
ghci> append [1,2] [3,4,5]
[1,2,3,4,5]
ghci> append [1,2,3,4,5] [7,8,9]
[1,2,3,4,5,7,8,9]
```

Here is the Task 2 Equational Reasoning for the Append Function:

```
append [1,2,3] [7,8,9] =
    1 : (append [2,3] [7,8,9])
    1 : (2 : (append [3] [7,8,9]))
    1 : (2 : (3 : (append [] [7,8,9])))
    1 : (2 : (3 : (7 : (append [8,9]))))
    1 : (2 : (3 : (7 : (8 : (append [9])))))
    1 : (2 : (3 : (7 : (8 : (9 : (append[])))))))
    1 : (2 : (3 : (7 : (8 : (9 : [])))))
```

2.2.4 Revert

The Revert function will take a list from the user, and output the list with the elements in reversed order.

Below is the code to the Revert Function:

```
revert [] = []
revert (x:xs) = append (revert xs) [x]
```

This function uses the previously created function Append. Revert will call the append method recursively. It will append the first element of the list to the back, then continue with the rest of the elements. This was designed with some thought of a Stack.
Here are some outputs for the Revert Function:

```
ghci> revert [1,2,3]
[3,2,1]
ghci> revert [8,1,2,4]
[4,2,1,8]
```

Here is the Task 2 Equational Reasoning for the Revert Function:

```
revert [1,2,3] =
    append (revert [2,3]) [1]
    append (append (revert([3]) [2])) [1]
    append (append (append (revert []) [3]) [2]) [1]
    append (append (append [] [3]) [2]) [1]
    append (append [3] [2]) [1]
    append [3,2] [1]
    [3,2,1]
```

2.2.5 Less Equal

The last function that I created was Less Equal. This simply compared the elements from two lists to see if first list was less than or equal to the same indexed element of the other list. For example [1,2] vs [5,6] would be true because 1 is less than or equal to 5 and 2 is less than or equal to 6.

Below is the code to the Less equal Function

```
less_equal [] [] = True
less_equal (x:xs) (y:ys) = if x<=y
    then
        less_equal xs ys
    else
        False
```

The beginning of this function will start with the second line. If compares the head elements of list 1 and list 2. If x is less than or equal to, the Less Equal function will be called again, but this time with the tail elements. If at any point the list 1 element "x" is greater than list 2 element "y", False will be returned. If all of the elements are compared, then base case is called. This is because there are no elements left, so they are empty lists. Since nothing was flagged, then True will be returned, meaning that all of the list 1 elements are either less than or equal to the list 2 elements at the same index.

Here are some outputs for the Less Equal Function:

```
ghci> less_equal [1,2,3] [2,3,4]
True
ghci> less_equal [1,2,3] [2,3,2]
False
ghci> less_equal [1,2,3] [2,3,3]
True
```

Here is the Task 2 Equational Reasoning for the Less Equal Function:

```
less_equal [1,2,3] [2,3,4] =
    1 <= 2
    then
        less_equal [2,3] [3,4]

    2 <= 3
    then
        less_equal [3] [4]

    3 <= 4
    then
        less_equal [] []

    True
```

If the left value was ever larger, than False would be the value.

2.2.6 Len

I did not create this function. I took this from Hackmd.io, thanks to Professor Kurz. Below is the code used that my other functions also used.

```
len [] = 0
len (x:xs) = 1 + len xs
```

2.3 Week 3

Week 3 Homework was about the Hanoi Tower. The task was to complete the execution from the dots down. Here that is:

```
hanoi 5 0 2
hanoi 4 0 1
hanoi 3 0 2
hanoi 2 0 1
hanoi 1 0 2 = move 0 2
move  0 1
hanoi 1 2 1 = move 2 1
move 0 2
hanoi 2 1 2
hanoi 1 1 0 = move 1 0
move  1 2
hanoi 1 0 2 = move 0 2
        move 0 1
        hanoi 3 2 1
            hanoi 2 2 0
                hanoi 1 2 1 = move 2 1
                move 2 0
                hanoi 1 1 0 = move 1 0
            move 2 1
```

```
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
    move 0 2
    hanoi 4 1 2
        hanoi 3 1 0
            hanoi 2 1 2
                hanoi 1 1 0 = move 1 0
                move 1 2
                hanoi 1 0 2 = move 0 2
            move 1 0
            hanoi 2 2 0
                hanoi 1 2 1 = move 2 1
                move 2 0
                hanoi 1 1 0 = move 1 0
        move 1 2
        hanoi 3 0 2
            hanoi 2 0 1
                hanoi 1 0 2 = move 0 2
                move 0 1
                hanoi 1 2 1 = move 2 1
            move 0 2
            hanoi 2 1 2
                hanoi 1 1 0 = move 1 0
                move 1 2
                hanoi 1 0 2 = move 0 2
```

I followed this exact sequence with the Tower of Hanoi website. It took 31 turns, which is the minimum moves possible for a Hanoi Tower of size 5.

The word "Hanoi" appears 31 times. The word "move" also appears 31 times.
A simple equation can be expressed for the number of blocks in the Hanoi Tower. Assuming n is the number of blocks/rings in the tower, the minimum number of moves can be expressed as

$$2^n - 1$$

In this case, n was 5. Thus,

$$2^5 = 32$$

and then

$$32 - 1 = 31$$

Here are some other minimum moves required for other ring heights.

$$2^1 - 1 = 1$$

$$2^2 - 1 = 3$$

$$2^3 - 1 = 7$$

$$2^4 - 1 = 15$$

$$2^6 - 1 = 63$$

2.4 Week 4

Week 4 consisted of making Parse Tree and Abstract Syntax Tree of math 5 problems.

2.4.1 Parse Tree Diagrams

This is Part 1 of Week 4's Homework. The task was to write out the derivation trees for the following:

$$2 + 1$$

$$1 + 2 * 3$$

$$1 + (2 * 3)$$

$$(1 + 2) * 3$$

$$1 + 2 * 3 + 4 * 5 + 6$$

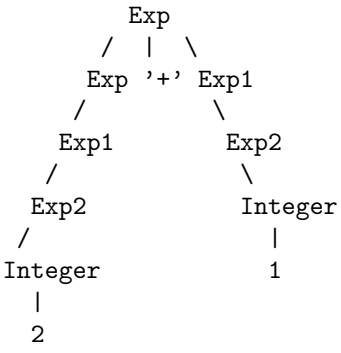
while using this Context-Free Grammar:

```
Exp -> Exp '+' Exp1
Exp1 -> Exp1 '*' Exp2
Exp2 -> Integer
Exp2 -> '(' Exp ')'
Exp -> Exp1
Exp1 -> Exp2
```

For

$$2 + 1$$

I got the following Parse Tree:

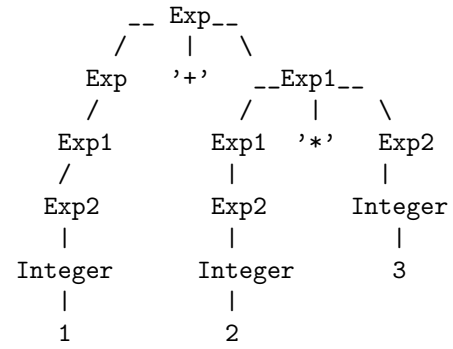


Here this tree starts with Exp. It expands for addition with Exp and Exp1. After this, I simply converted Exp to Exp1 to Exp2 then the number 2, and Exp1 to Exp2 then the number 1.

For

$$1 + 2 * 3$$

I got the following Parse Tree:

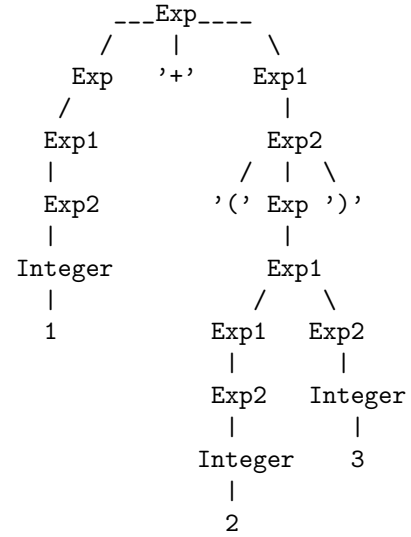


This Tree starts similar to last time with Exp. Exp will expand to Exp and Exp1. Exp will expand directly until it is the integer 1. Exp1 will expand to Exp1 and Exp2 for multiplication. Exp1 and Exp2 from this will become 2 and 3. This will allow the tree to calculate 2 times 3 to become 6. Then the root will have the two nodes of 1 and 6 to add to become 7.

For

$$1 + (2 * 3)$$

I got the following Parse Tree:

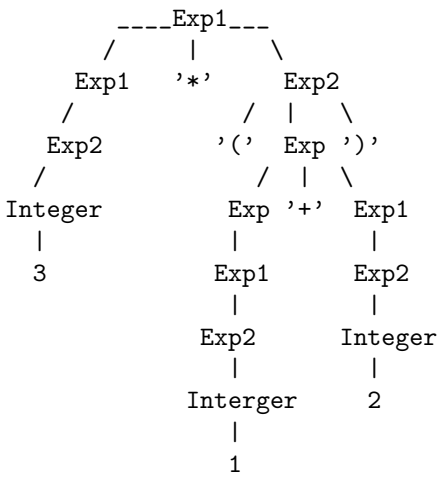


This is a little more complex than the prior example, but will end with the same answer of 7. Exp will expand to Exp and Exp1 for addition. Exp will expand until it becomes the integer of 1. Exp1 will become Exp2 then Exp because this had parenthesis. Exp will expand to Exp1, then Exp1 will expand to Exp1 and Exp2. These will become 2 and 3 so it will be multiplied together. Then this will finally become 7 if calculated.

For

$$(1 + 2) * 3$$

I got the following Parse Tree:

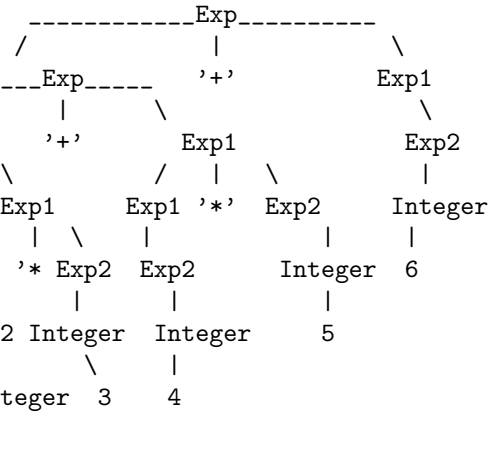


This time the parenthesis is with the addition of 1 and 2. I started with multiplication (Exp1) because it is 3 being multiplied with the entire parenthesis. Exp1 will expand to Exp1 and Exp2. Exp1 will become the integer 3. Exp2 will first go back to Exp because this is a parenthesis and our rule allows to go back to Exp when Parenthesis is present. Exp will then expand to Exp and Exp1 for the addition. These will expand until the integers 1 and 2 are present. Then 1 added to 2 will become 3. 3 multiple with 3 will become 9. This answer is 9.

For

$$1 + 2 * 3 + 4 * 5 + 6$$

I got the following Parse Tree:



This was by far the most complicated of the problems. To start, I expanded with the addition of 1 with everything else. This means Exp would be the root with Exp and Exp1 being the child nodes. Exp1 will become Exp2 then the integer 1. Now I must account for the other 5 numbers in the equation. Starting with Exp again I decided for the sum of 2 multiple with 3 being added to entire sum of 4 multiplied with 5

and then added with 1. Exp will expand to Exp and Exp1 for this addition. Exp1 will become the 2 times 3 side of the equation. Exp1 will expand to Exp1 and Exp2. Exp1 will expand until integer 2 is reached, and Exp2 is expanded until integer 3 is reached. Now back to the Exp that is being added with 2 times 3. This Exp will be expanded to Exp and Exp1. Exp will be expanded until integer 6 is reached. Exp1 is the multiplication formula. Exp1 will expand to Exp1 and Exp2. These two nodes will then be converted until integer 4 and 5 are reached. Now the tree is complete. First 1 will be added with the rest of the equation. The rest of the equation will have 2 multiplied with 3 become 6. 4 is multipled by 5 to become 20 then added with 6 to become 26. This 26 is added with 6 from 2 times 3. Then our 1 is added to become 33.

2.4.2 Abstract Syntax Tree

The same 5 problems (Equations) that were used for the Parse Tree's are now used to be made into Abstract Syntax Trees.

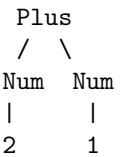
These AST are to be made using this BNFC Grammar:

```
Plus.   Exp ::= Exp "+" Exp1 ;
Times.  Exp1 ::= Exp1 "*" Exp2 ;
Num.    Exp2 ::= Integer ;

coercions Exp 2 ;
```

For

$$2 + 1$$

I got the following Abstract Syntax Tree:

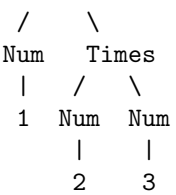


This Abstract Syntax Tree is easy to read. The only math being used is an addition sign. Simply Plus is used then expanded to its two terms of 2 and 1.

For

$$1 + 2 * 3$$

I got the following Abstract Syntax Tree:

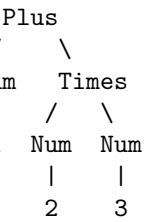


This Abstract Syntax Tree is bigger than the prior equation/problem. I once again started with Plus then expanded 1 and the product of 2 times 3. This works with our Grammar Rules.

For

$$1 + (2 * 3)$$

I got the following Abstract Syntax Tree:

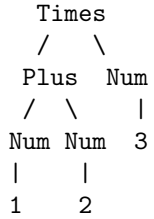


This is the same exact AST as the prior example. Since the parenthesis is around the multiplication, which would be first in this instance anyways, nothing changes. Other than the parenthesis being added, nothing changes, so the rest of the AST is the same

For

$$(1 + 2) * 3$$

I got the following Abstract Syntax Tree:

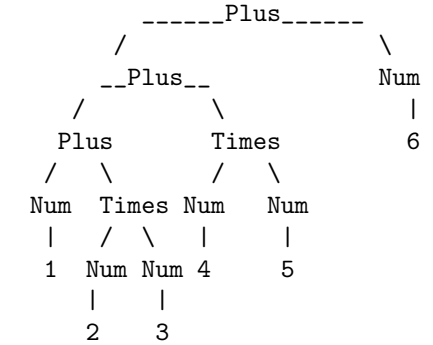


This AST is different than the prior examples. Since the parenthesis are around the 1 plus 2, then this part must be done first in order to be multiplied with the three. I had Times be the root with the expansion of integer 3, and Plus. The Plus would then expand to 1 and 2. By this tree, 1 Plus 2 is 3 then 3 times 3 is 9.

For

$$1 + 2 * 3 + 4 * 5 + 6$$

I got the following Abstract Syntax Tree:



This AST is much more complex than the other examples. To begin, I had Plus become the root, as 1 Plus other side of equation will be the child nodes. The nodes will be integer 1 and Plus. Plus will expand to Plus and Times. The Times will expand to the first part of the other equation of 2 Times 3. The Plus will expand to Times and integer 6. The Times will become 4 and 5. Doing this AST from bottom up, will have 4 Times 5 become 20. Then 20 Plus 6 becoming 26. 2 Times 3 becoming 6. 6 plus 26 becoming 32. Then finally 32 will be added with 1 to have 33 as the grand total, the same as the Parse Tree example.

Is the abstract syntax tree of 1+2+3 identical to the one of (1+2)+3 or the one of 1+(2+3)? Yes, the AST are the same for these three equations.

2.5 Week 5

For this week's homework, the task was to create Abstract Syntax Trees and evaluate lambda expressions.

2.5.1 Abstract Syntax Trees of Lambda Calculus

The following expressions were given to create 2-D Abstract Syntax Trees and Linearized Abstract Syntax Trees.

```
x
x x
x y
x y z
\ x.x
\ x.x x
(\ x . (\ y . x y)) (\ x.x) z
(\ x . \ y . x y z) a b c
```

Here is the Linearized and 2-D AST for x:

```
EVar(Ident "x")

        EVar
         |
         x
```

Here is the Linearized and 2-D AST for x x:

```
EApp(EVar(Ident "x") EVar(Ident "x"))
         EApp
        /    \
       EVar  EVar
        |     |
        x     x
```

Here is the Linearized and 2-D AST for x y:

```
EApp(EVar(Ident "x") EVar(Ident "y"))
        EApp
       /    \
```

```
 EVar  EVar
  |     |
  x     y
```

Here is the Linearized and 2-D AST for x y z:

```
EApp (EApp (EVar (Ident "x")) (EVar (Ident "y"))) (EVar (Ident "z"))
       EApp
      /    \
    EVar   EApp
     |    /    \
     z   EVar  EVar
          |     |
          y     x
```

Here is the Linearized and 2-D AST for

$$\backslash x.x$$

```
EAbs (Ident "x") (EVar (Ident "x"))
            EAbs
          /     \
         x      EVar
                 |
                 x
```

Here is the Linearized and 2-D AST for

$$\backslash x.xx$$

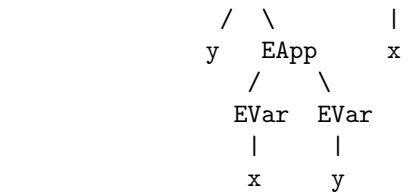
```
EAbs (Ident "x") (EApp (EVar (Ident "x")) (EVar (Ident "x")))
             EAbs
           /     \
        EApp     EVar
        /   \      |
    EVar    EVar   x
      |       |
      x       x
```

Here is the Linearized and 2-D AST for

$$(\backslash x.(\backslash y.xy))(\backslash x.x)z$$

```
EApp (EApp (EAbs (Ident "x") (EAbs (Ident "y") (EApp (EVar (Ident "x")) (EVar (Ident "y"))))) (EAbs (Id
```

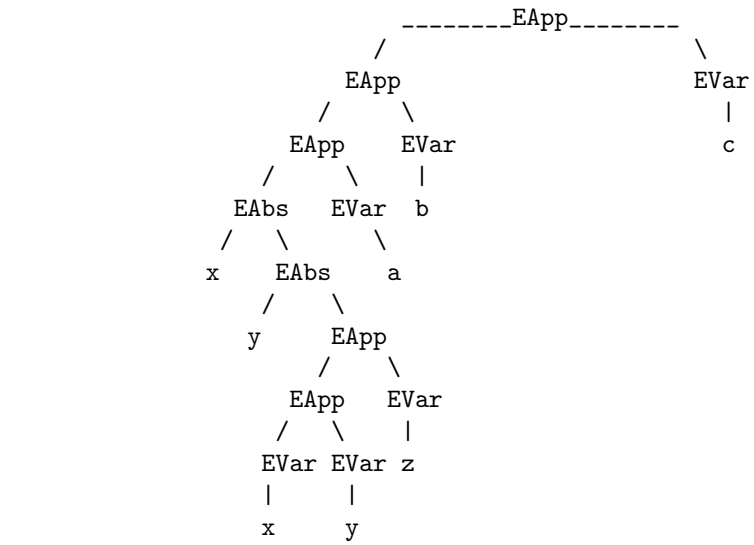
```
           EApp
         /      \
       EApp      EVar
    /      \       |
  EAbs     EAbs    z
  /  \     /  \
 x   EAbs  x   EVar
```



Here is the Linearized and 2-D AST for

$$(\backslash x.\backslash y.xyz)abc$$

```
EApp (EApp (EApp (EAbs (Ident "x") (EAbs (Ident "y") (EApp (EApp (EVar (Ident "x")) (EVar (Ident "y")))
```



These expressions start easy with simple free variables such as x. The problems gradually got harder to include bound variables, and would need to be changed if we were evaluating these expressions.

2.5.2 Lambda Expression Evaluation

This section is to evaluate the following Lambda Expressions and solve:

```
1.  (\x.x) a
2.  \x.x a
3. (\x.\y.x) a b
4. (\x.\y.y) a b
5. (\x.\y.x) a b c
6. (\x.\y.y) a b c
7. (\x.\y.x) a (b c)
8. (\x.\y.y) a (b c)
9. (\x.\y.x) (a b) c
10. (\x.\y.y) (a b) c
11. (\x.\y.x) (a b c)
12. (\x.\y.y) (a b c)
```

Here is the evaluation for number 1:

```
(\x.x) a = a
```

Here is the evaluation for number 2:

```
\x.x a = a
```

Here is the evaluation for number 3:

```
(\x.\y.x) a b
(\y.a) b = a
```

Here is the evaluation for number 4:

```
(\x.\y.y) a b
(\y.y) b = b
```

Here is the evaluation for number 5:

```
(\x.\y.x) a b c
(\y.a) b c
(\y.a) c = a c
```

Here is the evaluation for number 6:

```
(\x.\y.y) a b c
(\y.y) b c
(\y.b) c = b c
```

Here is the evaluation for number 7:

```
(\x.\y.x) a (b c)
(\y.a) (b c) = a
```

Here is the evaluation for number 8:

```
(\x.\y.y) a (b c)
(\y.y) (b c) = b c
```

Here is the evaluation for number 9:

```
(\x.\y.x) (a b) c
(\y.a b) c = a b
```

Here is the evaluation for number 10:

```
(\x.\y.y) (a b) c
(\y.y) c = c
```

Here is the evaluation for number 11:

```
(\x.\y.x) (a b c)
(\y.a b c) = a b c
```

Here is the evaluation for number 12:

```
(\x.\y.y) (a b c)
(\y.y) = \y.y
```

2.5.3 evalCBN calculation

The task her was to write by hand the calculation of

```
(\x.x)((\y.y)a)
```

using evalCBN in interpreter.hs.
Clearly the answer here would simply be "a", but in terms of what the computer sees will be different

```
evalCBN (EApp (EAbs (Ident "x") (EVar (Ident "x"))) (EApp (EAbs (Ident "y") (EVar (Ident "y"))) (EVar (
evalCBN (subst (Ident "x") (EApp (EAbs (Ident "y") (EVar (Ident "y"))) (EVar (Ident "a"))) EVar (Ident "
evalCBN (EApp (EAbs (Ident "y") (EVar (Ident "y"))) (EVar (Ident "a"))) = Line 47
evalCBN (subst (Ident "y") (EVar (Ident "a")) (EVar (Ident "x"))) = Line 27
evalCBN (EVar (Ident "a")) = Line 47
EVar (Ident "a") = Line 32
```

2.6 Week 6

2.6.1 Evaluation

The goal this week was to Evaluate

```
(\exp . \two . \three . exp two three)
(\m.\n. m n)
(\f.\x. f (f x))
(\f.\x. f (f (f x)))
```

Here is what I got:

```
(\exp . \two . \three . exp two three)
(\m.\n. m n)
(\f.\x. f (f x))
(\f.\x. f (f (f x)))
=
((\m.\n. m n) (\f.\x. f (f x)) (\f2.\x2. f2 (f2 (f2 x2))))
=
((\n. (\f.\x. f (f x)) n) (\f2.\x2. f2 (f2 (f2 x2))))
=
(((\f.\x. f (f x)) (\f2.\x2. f2 (f2 (f2 x2)))))
=
(((\x. (\f2.\x2. f2 (f2 (f2 x2))) ((\f2.\x2. f2 (f2 (f2 x2))) x))))
=
(((\x. (\f2.\x2. ((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x
=
(((\x. (\f2.\x2. ((\x2. x (x (x x2)))) (((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x2))) x) x
=
(((\x. (\f2.\x2. ((x (x (x (((\f2.\x2. f2 (f2 (f2 x2))) x) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))))
=
(((\x. (\f2.\x2. ((x (x (x (((\x2. x (x (x x2)))) (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))))
=
(((\x. (\f2.\x2. ((x (x (x (((x (x (x (((\f2.\x2. f2 (f2 (f2 x2))) x) x2)))))))))))))
=
(((\x. (\f2.\x2. ((x (x (x (((x (x (x (((\x2. x (x (x x2)))) x2)))))))))))))
```

```
=
(((\x. (\f2.\x2. ((x (x (x (((x (x (x (((x (x (x x2))))))))))))))))))
=
(((\x. (\f2.\x2. ((x (x (x (((x (x (x (((x (x (x x2))))))))))))))))))
```

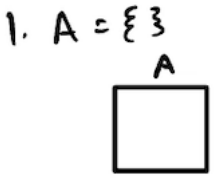
2.7 Week 7

For week 7, there were a variety of tasks. The first was to draw a picture for a number of ARSs. Next was to find an example of an ARS for specific cases. Another task was to the variable for certain lines. evalCBN for hw5 if not done, but in my case is done in Week 5 section. Lastly to use evalCBN for another lambda term.

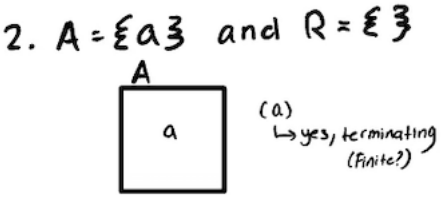
2.7.1 ARS Picture/Determination

The task in this section was to

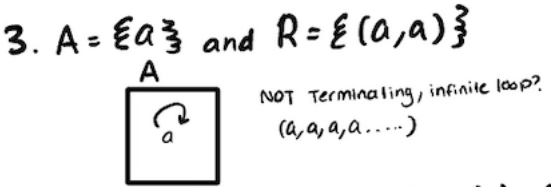
```
Consider the following list of ARSs.
1. A = {}
2. A = {a} and R = {}
3. A= {a} and R = {(a,a)}
4. A = {a,b,c} and R = {(a,b),(a,c)}
5. A = {a,b} and R = {(a,a),(a,b)}
6. A = {a,b,c} and R = {(a,b),(b,b),(a,c)}
7. A = {a,b,c} and R = {(a,b),(b,b),(a,c),(c,c)}

Draw a picture for each of the ARSs.

Is the ARS terminating? Is it confluent? Does it have unique normal forms?
```



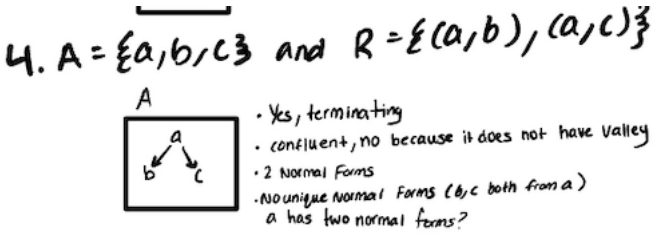
Picture 1:
It is terminating, it is confluent, and it has a unique normal form.



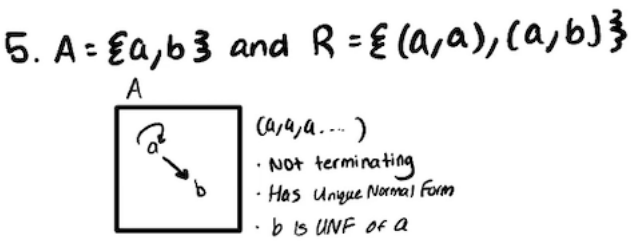
Picture 2:
It is terminating, it is confluent, and it has a unique normal form.



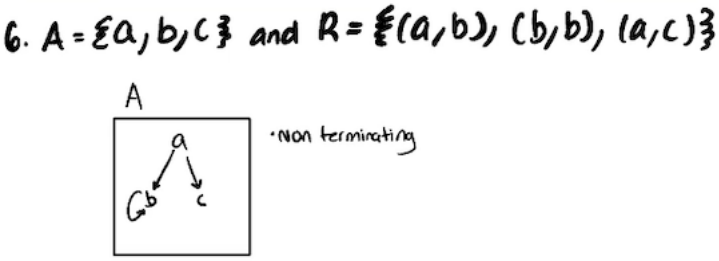
Picture 3:
It is not terminating, it is confluent, and it does not have unique normal forms.



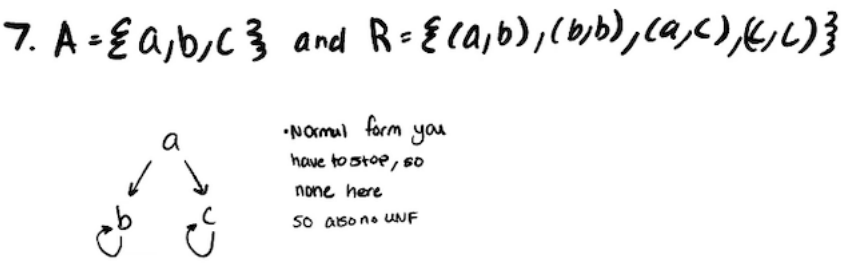
Picture 4:
It is terminating, it is not confluent, and it does not have unique normal forms.



Picture 5:
It is not terminating, it is confluent, and it does have unique normal forms. b is the UNF of a.



Picture 6:
It is not terminating, it is not confluent, and it does not have unique normal forms.



Picture 7:

It is not terminating, it is not confluent, and it does not have unique normal forms.

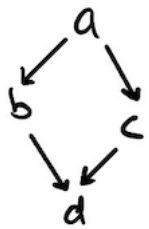
2.7.2 ARS examples

In this section of the homework, the task was to try find an example of an ARS for specific combinations, and to draw the picture of that example.

Here are the combinations that are being tested:

confluent	terminating	has unique normal forms	example
True	True	True	
True	True	False	
True	False	True	
True	False	False	
False	True	True	
False	True	False	
False	False	True	
False	False	False	

For number 1, confluent = True, terminating = True, and Unique Normal Form = True, I got the following ARS:

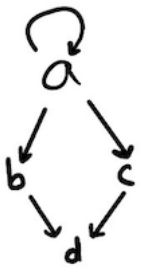


Here, confluence, terminating, and Unique Normal Form are all true. The a is the peak, and b and c reduce into d which is the valley. There is no possibility of having an infinite loop. Lastly, d is the UNF.

For number 2, confluent = True, terminating = True, and Unique Normal Form = False, I got the following ARS:

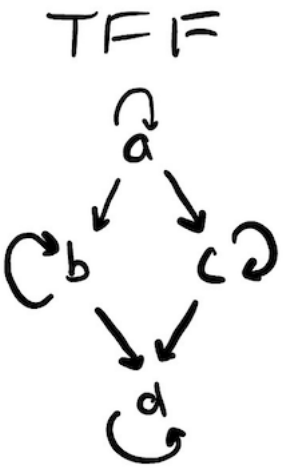
There is not an example for this situation. When Confluence and Terminating are True, that means that UNF must be true. There will not be any situation or diagram where this is not true.

For number 3, confluent = True, terminating = False, and Unique Normal Form = True, I got the following ARS:



Similar to my example in number 1, Confluence is true due to the peak existing with a valley. d would be the UNF in this case. Lastly, it is not terminating because of the possibility of an endless loop on a.

For number 4, confluent = True, terminating = False, and Unique Normal Form = False, I got the following ARS:

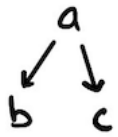


Here confluence is true due to the peak and valley being present. Terminating would be false because of the possibility of having an endless loop. Lastly a,b,c, and d all have a possibility of an endless loop, which means that none of them are normal forms, meaning no UNF could be possible.

For number 5, confluent = False, terminating = True, and Unique Normal Form = True, I got the following ARS:

There is no scenario that exists for this situation. When Confluence is false, UNF will never be true.

For number 6, confluent = False, terminating = True, and Unique Normal Form = False, I got the following ARS:

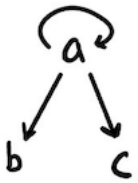


Again, a is the peak, but there is no valley. This means that confluent is false for this ARS. This is terminating because a, b, and c, do not have opportunity to be part of an endless loop anywhere. This ARS would simply start at a then go to b or c. This ARS does not have a UNF because b and c are both normal forms, but a must only have one normal form to have a UNF.

For number 7, confluent = False, terminating = False, and Unique Normal Form = True, I got the following ARS:

There is not an example for this scenario. When confluence is false, UNF will always never be True.

For number 8, confluent = False, terminating = False, and Unique Normal Form = False, I got the following ARS:



Similar to number 7, but instead of just b from a, there is b and c. That means that both are normal forms, but a must only have one normal form to have an unique normal form. In this case, all 3 are False.

2.7.3 Bound and Free Variables

In this section of Week 7, the task was to

```
in lines 5-7 and also in lines 18-22 explain for each variable

- whether it is bound or free
- if it is bound say what  the binder and the scope of the variable are
```

Here are lines 5-7 that are referred to:

```
evalCBN (EApp e1 e2) = case (evalCBN e1) of
    (EAbs i e3) -> evalCBN (subst i e2 e3)
    e3 -> EApp e3 e2
```

The variables e1 and e2 are both bound variables. Their scope goes from the line 5 to line 7. The reason they are bound variables is because the rest of the this function uses e1 and e2 because of there instance in

```
evalCBN (EApp e1 e2)
```

which tells us that the other lines are bound by the occurrence of e1 and e2.

The binder of the variables e1 and e2 is

```
(EApp e3 e2)
```

The variable i and e3 (on line 6) is a bound variable. The scope is just the line it is on, which is line 6,

```
evalCBN (subst i e2 e3)
```

The binder is

```
(EAbs i e3).
```

The second instance of e3, which is different than the one mentioned on line 6, has a scope of

```
EApp e3 e2
```

and a binder of

```
e3
```

Here are the lines 18-22 that are referred to:

```
subst id s (EAbs id1 e1) =
    let f = fresh (EAbs id1 e1)
        e2 = subst id1 (EVar f) e1 in
        EAbs f (subst id s e2)
```

For the variable id, s, id1, and e1, they are all bound variables. The scope would be lines 18-22, and the binder for s would be

```
id s (EAbs id1 e1)
```

The variable f would also be a bound variable. The scope would be from lines 19 to 22. The binder would be

```
let f
```

the variable e2 would also be a bound variable. The scope would be from line 20-22. The binder would be

```
e2
```

The binder for id1 and e1 would be

```
 (EAbs id1 e1)
```

2.7.4 evalCBN calculation, part 2

The task here was to use evalCBN and subst from Interpreter.hs, and solve by hand showing how to computer processes this information and calculations.

```
(\x.\y.x) y z
```

This was the lambda expression given. If calculating here, the answer would simply be the second y, which would have to be substituted.

```
evalCBN (EApp (EApp (EAbs (Ident "x") (EAbs (Ident "y") (EVar (Ident "x"))))
EVar (Ident "y"))) (EVar (Ident "z"))) = From Parser

evalCBN ((EAbs (Ident "x") (EAbs (Ident "y") (EVar (Ident "x")))) (EVar (Ident "z"))
(EVar (Ident "y"))) = Line 27

evalCBN (EVar (Ident "y")) = Line 48

EVar(Ident "y") = Line 32
```

2.8 Week 8

This week the task was to examine the rewrite rules of

```
aa -> a
bb -> b
ba -> ab
ab -> ba
```

and to answer the following questions:

```
1. Why does the ARS not terminate?
2. What are the normal forms?
3. Can you change the rules so that the new ARS has unique normal forms (but still has
the same equivalence relation)?
4. What do the normal forms mean? Describe the function implemented by the ARS.
```

For **number 1**, the ARS as currently constructed would **not terminate** because the rules

```
ba -> ab
ab -> ba
```

These two rules are doing the opposite of each other. Every time ba transforms into ab, ab will transform back to ba. This will continue to happen, and will be an endless loop, meaning that these rewrite rules are not terminating.

For **number 2**, the normal forms would be a and b. Since aa reduces to a, and bb reduces to b, but a and b do not reduce to anything, these would be the normal forms.

number 3: Here are the rules I changed so that the new ARS has unique normal forms, and still has the same equivalence relation.

```
aa -> a
bb -> b
ba -> ab
```

I simply got rid of the last rule. a is the UNF for aa, and b is the UNF for bb. Since the rewriting rules are terminating now, this will allow for UNF to exist.

number 4: The normal forms mean that it does not reduce into anything else.

3 Project

Swift, created in 2014 by Apple, is a language predominantly to build applications for computers, phones, and more. This programming language allowed for better User-Interface and easier understanding on how to build specific ideas in the application.

3.1 Specification

This project will be learning the programming language Swift, then creating an easy-to-follow tutorial, and finally implement an interesting project using what I learned. The final coding project has not yet been decided, but I will first download the necessary applications, packages, and more. Then I will follow tutorials and watch videos about how to program in Swift.

3.2 Prototype

3.3 Documentation

3.4 Critical Appraisal

...

4 Conclusions

(approx 400 words)

In the conclusion, I want a critical reflection on the content of the course. Step back from the technical details. How does the course fit into the wider world of programming languages and software engineering?

References

[PL] Programming Languages 2022, Chapman University, 2022.

Hackmd.io Professor Kurz, Chapman University 2022

Tower of Hanoi