Closed Form formula sor simple linear regression

Evert Jan Karman

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Documenting the closed form formula for simple linear regression

1 The gradient formula for RSS

$$\nabla RSS(w_0, w_1) = \left(\frac{\partial RSS}{\partial w_0}, \frac{\partial RSS}{\partial w_1}\right)$$

Where

$$\frac{\partial RSS}{\partial w_0} = -2 \cdot \sum_{i=0}^{N-1} (y_i - (w_0 + w_1 x_i))$$

And

$$\frac{\partial RSS}{\partial w_1} = -2 \cdot \sum_{i=0}^{N-1} ((y_i - (w_0 + w_1 x_i)) \cdot x_i)$$

2 Solving for gradient eq zero

When we solve for gradient = zero, we get

$$w_0 \cdot N = \sum_{i=0}^{N-1} y_i - w_1 \cdot \sum_{i=0}^{N-1} x_i$$

and

$$w_0 \cdot \sum_{i=0}^{N-1} x_i = \sum_{i=0}^{N-1} y_i \cdot x_i - w_1 \cdot \sum_{i=0}^{N-1} x_i^2$$

We are looking at a system of two equations:

$$w_0 \cdot N = A - w_1 \cdot B$$
$$w_0 \cdot B = C - w_1 \cdot D$$

With A, B, C, D expressions not dependent on w_0 or w_1

$$A = \sum_{i=0}^{N-1} y_i$$
$$B = \sum_{i=0}^{N-1} x_i$$
$$C = \sum_{i=0}^{N-1} x_i \cdot y_i$$
$$D = \sum_{i=0}^{N-1} x_i^2$$

So that

$$w_1 = \frac{C - \frac{A \cdot B}{N}}{D - \frac{B \cdot B}{N}}$$

And then

$$w_0 = \frac{A}{N} - w_1 \cdot \frac{B}{N}$$