

# Closed Form formula for simple linear regression

Evert Jan Karman

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Documenting the closed form formula for simple linear regression

## 1 The gradient formula for RSS

$$\nabla RSS(w_0, w_1) = \left( \frac{\partial RSS}{\partial w_0}, \frac{\partial RSS}{\partial w_1} \right)$$

Where

$$\frac{\partial RSS}{\partial w_0} = -2 \cdot \sum_{i=0}^{N-1} (y_i - (w_0 + w_1 x_i))$$

And

$$\frac{\partial RSS}{\partial w_1} = -2 \cdot \sum_{i=0}^{N-1} ((y_i - (w_0 + w_1 x_i)) \cdot x_i)$$

## 2 Solving for gradient eq zero

When we solve for gradient = zero, we get

$$w_0 \cdot N = \sum_{i=0}^{N-1} y_i - w_1 \cdot \sum_{i=0}^{N-1} x_i$$

and

$$w_0 \cdot \sum_{i=0}^{N-1} x_i = \sum_{i=0}^{N-1} y_i \cdot x_i - w_1 \cdot \sum_{i=0}^{N-1} x_i^2$$

We are looking at a system of two equations:

$$\begin{aligned}w_0 \cdot N &= A - w_1 \cdot B \\w_0 \cdot B &= C - w_1 \cdot D\end{aligned}$$

With A, B, C, D expressions not dependent on  $w_0$  or  $w_1$

$$\begin{aligned}A &= \sum_{i=0}^{N-1} y_i \\B &= \sum_{i=0}^{N-1} x_i \\C &= \sum_{i=0}^{N-1} x_i \cdot y_i \\D &= \sum_{i=0}^{N-1} x_i^2\end{aligned}$$

So that

$$w_1 = \frac{C - \frac{A \cdot B}{N}}{D - \frac{B \cdot B}{N}}$$

And then

$$w_0 = \frac{A}{N} - w_1 \cdot \frac{B}{N}$$