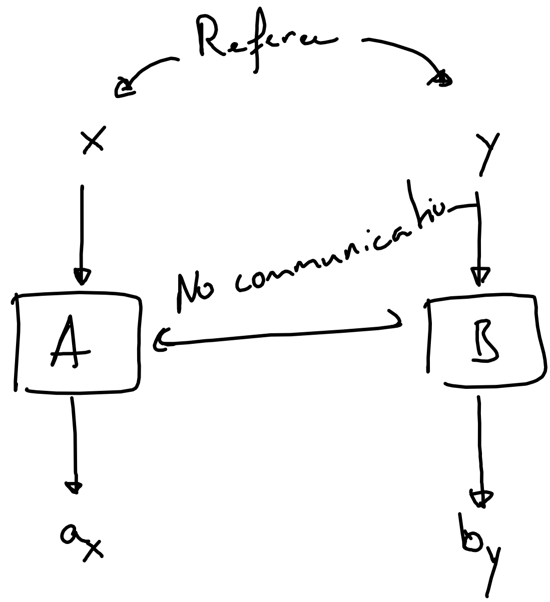
# ATTP Module 2 Lecture #4

→ Non-local Quantum Games
(CHSH, <u>Magic Square</u>)




CHSH (Clauser, Horne, Shimony, Holt)

$$a \oplus b = x \wedge y$$

sum mod 2

AND

| | | $x \wedge y$ | $(x, y)$ |
|---|---|---|---|
| $a_0 \oplus b_0$ | $=$ | $\underline{0}$ | 0 0 |
| $a_0 \oplus b_1$ | $=$ | $\underline{0}$ | 0 1 |
| $a_1 \oplus b_0$ | $=$ | $\underline{0}$ | 1 0 |
| $a_1 \oplus b_1$ | $=$ | $\underline{1}$ | 1 1 |

(Last time: GHZ game:

$$a \oplus b \oplus c = x \vee y \vee z$$

$\rightarrow$ there is a quantum strategy that leads to 100% win)

Q: CHSH: if players share a state $|\psi\rangle$, can they beat the 75% win-chance?

$\rightarrow$ $|\psi\rangle = |00\rangle - |11\rangle$

Both get to apply a unitary $U(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$U(\theta_A) \otimes U(\theta_B) |\psi\rangle$$

$$= \cos(\theta_A + \theta_B)\left(|00\rangle - |11\rangle\right) + \sin(\theta_A + \theta_B)\left(|01\rangle + |10\rangle\right)$$

Q: $P(a \oplus b = 0) = \cos^2(\theta_A + \theta_B)$

$\rightarrow$ if A gets $x = 0$, they choose $\theta_A = \ldots$

" " $x = 1$, " " $\theta_A = \ldots$

if: $x = y = 1$

$$\boxed{\begin{array}{lll} \theta_A + \theta_B = \frac{3\pi}{8} & & \text{if } x = y = 1 \\ \theta_A + \theta_B = \pm\frac{\pi}{8} & & \text{otherwise} \end{array}}$$

$$\boxed{\begin{array}{lll} \theta_A + \theta_B = \frac{3\pi}{8} & & \text{if } x = y = 1 \\ \theta_A + \theta_B = \pm\frac{\pi}{8} & & \text{otherwise} \end{array}}$$

$$|\psi\rangle = \cos(\theta_A + \theta_B)\left(|00\rangle - |11\rangle\right) - \sin(\theta_A + \theta_B)\left(|01\rangle + |10\rangle\right)$$

$\theta_{A,0} = \ldots \qquad \theta_{A,1} = \ldots$

$\theta_{B,0} = \ldots \qquad \theta_{B,1} = \ldots$

$\to \theta_A + \theta_B = \frac{3\pi}{8} \qquad x = y = 1$

$\theta_A + \theta_B = \pm\frac{\pi}{8} \qquad$ otherwise

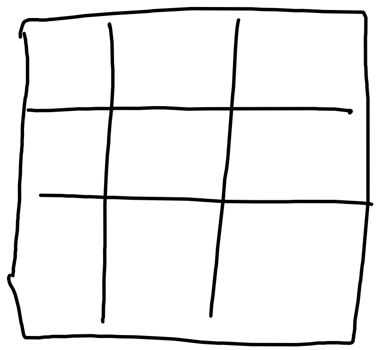
$\to$

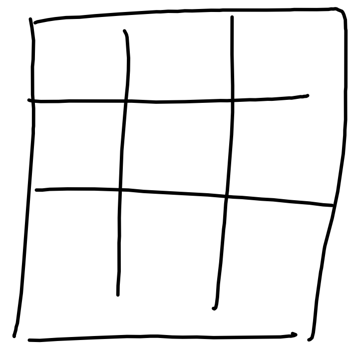
$\boxed{\cos^2\left(\frac{3\pi}{8}\right) \sim (0.3)^2 \qquad \sin^2\left(\frac{3\pi}{8}\right) \sim (0.9)^2}$ $\quad x = y = 1$

$\boxed{\cos^2\left(\pm\frac{\pi}{8}\right) \sim (0.9)^2 \qquad \sin^2\left(\pm\frac{\pi}{8}\right) \sim (0.3)^2}$ $\quad$ otherwise

$\to$ win chance $(0.9)^2$

# (Quantum) Magic Square
## (Mermin - Peres square)

A: 

B: 

- Fill in square, each square has either 0 or 1
  - A: all rows must sum to even parity
  - B: all col$^{s}$ " " " odd parity

- Referee: pick a random row & column:
  - → We win if:
    1) Our constraints satisfied (valid filling)
    2) The digit in the intersecting square we have the same digit.

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | ✓ |
| 1 | 0 | 1 | ✓ |
| 0 | 1 | ~~0~~ 1 | ✗ ✓ |
| ✓ | ✓ | ✓ ✗ | |

row even
col odd

8/9

→ $\times 2 - 1$
translate $(0,1) \to (-1,1)$

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## Quantum version:

$XY = iZ$
$YZ = iX$
$ZX = iY$

| | | |
|---|---|---|
| $X \otimes X$ | $Y \otimes Z$ | $Z \otimes Y$ |
| $Y \otimes Y$ | $Z \otimes X$ | $X \otimes Z$ |
| $Z \otimes Z$ | $X \otimes Y$ | $Y \otimes X$ |

→

$$(|01\rangle - |10\rangle) \otimes (|01\rangle - |10\rangle)$$

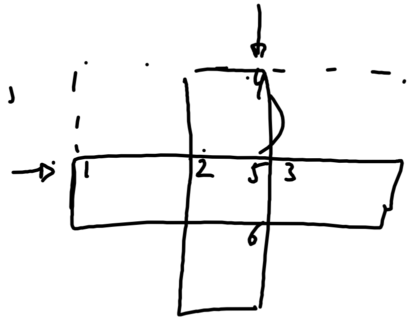
$$(Z \otimes Z) \cdot (X \otimes Y) \cdot (Y \otimes X) = (\underbrace{ZX}Y \otimes \overbrace{ZY}^{-iX}X)$$
$$= i \otimes -i = \hat{1} \otimes \hat{1}$$

$$(Y \otimes Z) \cdot (Z \otimes X) \cdot (X \otimes Y) = (YZX \otimes ZXY)$$
$$= i \otimes i = -\hat{1} \otimes \hat{1}$$

1) Intersecting square matches

2) Row = even

3) Col = odd.




Assume: question = (2, 6)

$$|0\rangle_2|0\rangle_5 + |1\rangle_2|1\rangle_5$$

2) $|0\rangle_1|0\rangle_3 + |1\rangle_1|1\rangle_3$ $\quad (|0\rangle_1|1\rangle_3 + |1\rangle_1|0\rangle_3)$

$$\Big((|0\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1|0\rangle_2|1\rangle_3)|0\rangle_5 + (|0\rangle_1|1\rangle_2|1\rangle_3 + |1\rangle_1|1\rangle_2|0\rangle_3)|1\rangle_5\Big)$$

3)

$$\Big(\underbrace{|100\rangle}_{4\,5\,6} + |001\rangle\Big) +$$

row col

$|000\rangle|100\rangle +$

if max ent

$$(U_A \otimes I)|\psi\rangle = (I \otimes U_A^{\dagger})|\psi\rangle$$

Concurrence