Quantum Games

① Q. Game Theory

→ Crash course GT / Prisoner's Dilemma

→ EWL - construction
(Eisert Wilkens Lewenstein)

② Non-local games
(CHSH, Parity / GHZ, Magic Square)

③ Q. video game

Q. Game Theory

Game = A situation between 2 or more players, each with their own strategy (set of moves) and their own payoff.

Normal form game: (N, S, U)

#players $\rightarrow N$

U: payoffs, $U = U_1 \times U_2 \times \ldots \times U_N$

$S = S_1 \times S_2 \times \ldots \times S_N$

S_1: $(A, B, C, \ldots)$

<u>Best response</u>: Given other player's fixed strategies, best resp = strategy that maximizes payoff.

(Dominant strategy)

<u>Nash equilibrium</u>: A set of strategies s.t. no player wants to change their strategy.

Prisoner's Dilemma

2 prisoners, each can Cooperate or Defect

$$(N, S, U)$$

$$\Big(2, \underbrace{\underbrace{(C, D)}_{S_1} \times \underbrace{(C, D)}_{S_2}}_{S}, U\Big)$$

$$\begin{array}{cc}
A\ \begin{array}{c} C \\ D \end{array} & \left(\begin{array}{c|c} (3, 3) & (0, 5) \\ \hline (5, 0) & (1, 1) \end{array}\right) \\
& \begin{array}{cc} C & \qquad D \end{array} \\
& B
\end{array}$$

D = dominant

$\longrightarrow (D, D) =$ NE

EWL - construction for Q. Prisoner's Dilemma.

$$\hat{J}\,|00\rangle = |00\rangle + i|11\rangle \qquad \left(\frac{1}{\sqrt{2}} \cdots\right)$$

$$\left(\hat{J} = e^{i\frac{\pi}{4}\hat{X}\otimes\hat{X}}\right)$$

→ Exercise : show ↑

→ Change notation: $|CC\rangle, |DD\rangle, |CD\rangle, |DC\rangle$

$|00\rangle, |11\rangle, \ldots$

$$|\psi_f\rangle = \hat{J}^\dagger\left(\hat{U}_A \otimes \hat{U}_B\right)\hat{J}\,|CC\rangle$$

$$\hat{U}(\phi,\theta) = \begin{pmatrix} e^{i\phi}\cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & e^{-i\phi}\cos\frac{\theta}{2} \end{pmatrix}$$

$$\boxed{|\psi_f\rangle = \hat{J}^\dagger(\hat{U}_A \otimes \hat{U}_B)\hat{J}|00\rangle}$$

$\hat{U}_C = \hat{I} : |00\rangle$

Payoff:

$\$_A = r P_{DD} + sP_{\cdot} -$

$P_{DD} \uparrow |\langle DD|\psi_f\rangle|^2$

$\hat{U}_D = i\hat{Y} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(D, D)

$|\psi_f\rangle = J^\dagger(i\hat{Y} \otimes i\hat{Y})(|00\rangle + i|11\rangle)$

$= J^\dagger(\underbrace{i\hat{Y}|0\rangle \otimes i\hat{Y}|0\rangle} + i\underbrace{(i\hat{Y})|1\rangle}_{-|0\rangle} \otimes (i\hat{Y})|1\rangle)$

$= J^\dagger(|11\rangle + i|00\rangle)$

exercise

$= i|11\rangle$

$$\boxed{\begin{aligned} \hat{J}|00\rangle &= |00\rangle + i|11\rangle \\ \hat{J}|11\rangle &= |11\rangle + i|00\rangle \end{aligned}}$$

$\rightarrow$ (C, D) $\rightarrow \dots \quad -|01\rangle$

$\hat{U}_Q = i\hat{Z} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

(Q, Q):

$|\psi_f\rangle = J^\dagger(\underbrace{|00\rangle + i|11\rangle}_{J|00\rangle})$

$= -|00\rangle$

$J = e^{i\frac{\pi}{4}XX} = 1 + \quad \hat{X}\otimes\hat{X}$

$= \cos\frac{\pi}{4}\,\hat{1}\otimes\hat{1} + i\sin\frac{\pi}{4}\,\hat{X}\otimes\hat{X}$

$$\boxed{\hat{J} = \frac{1}{\sqrt{2}}(\hat{1}\otimes\hat{1} + i\hat{X}\otimes\hat{X})}$$

$\hat{J}^\dagger = \frac{1}{\sqrt{2}}(\hat{1}\otimes\hat{1} - i\hat{X}\otimes\hat{X})$

$$
\begin{array}{ccc}
(\theta,\phi) = (0,0) & (\theta,\phi) \ldots & (\theta,\phi)\ldots \\
\downarrow & \downarrow & \downarrow \\
U_B = I & U_B = i\hat{y} & U_B = \\
\downarrow & \downarrow & \downarrow \\
C & D & Q
\end{array}
$$

$$
\begin{array}{c}
C \\ D \\ Q
\end{array}
\begin{pmatrix}
(3,3) & (0,5) & (1,1) \\
(5,0) & (1,1) & (0,5) \\
(1,1) & (5,0) & (3,3)
\end{pmatrix}
$$

$$
(U_C, U_Q) = J^{\dagger}\Big(\overset{I|0\rangle}{|0\rangle} \otimes \overset{iZ|0\rangle}{i|0\rangle} + i\big(\overset{I|1\rangle}{|1\rangle} \otimes \overset{iZ|1\rangle}{-i|1\rangle}\big)\Big)
$$

$$
= J^{\dagger}\left(i|00\rangle + |11\rangle\right)
$$

$$
= i\left(|00\rangle - i|11\rangle\right) + \left(|11\rangle - i|00\rangle\right)
$$

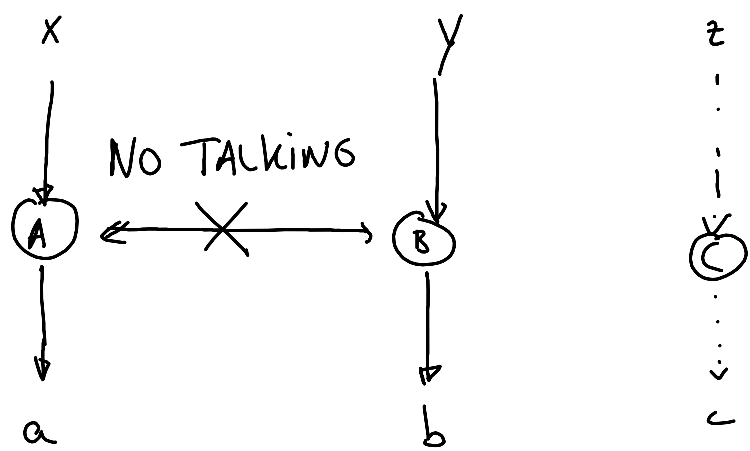
$$
\simeq |11\rangle
$$

$$
(U_D, U_Q) \longrightarrow (C, D) \to (0,5)
$$

GW (Gutoshi-Watrous)

Non-local games

A non-local game is a cooperative game between two or more players, each of which receives an input based on which they produce an output; they win if their outputs satisfy a condition.



CHSH: $a \oplus b = x \wedge y$ $\quad (x, y, a, b, \in \{0,1\})$

sum mod 2 $\qquad$ AND

Parity: $a \oplus b \oplus c = x \vee y \vee z$

Parity: $a \oplus b \oplus c = x \vee y \vee z$

Reference: $(0,0,0), (1,1,0), (1,0,1), (0,1,1)$

$x \quad y \quad z$

$a_0 \oplus b_0 \oplus c_0 = 0$

$a_1 \oplus b_1 \oplus c_0 = 1$

$\vdots \qquad = 1$

$= 1$

output if A gets $x=0$ as input

GHZ: $\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$

if input $= 0$, measure in $\hat{X}$ (apply $\hat{H}$)

$= 1$, measure in $\hat{Y}$ (apply $\hat{H}\hat{S}$)

$\hat{H}|0\rangle = |0\rangle + |1\rangle$

$|000\rangle \longrightarrow (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \cdots$

$= |000\rangle + |001\rangle + |010\rangle + \cdots + |111\rangle$

$\hat{H}|1\rangle = |0\rangle - |1\rangle$

$|111\rangle \longrightarrow = (|0\rangle - |1\rangle) \otimes (|0\rangle - |1\rangle) \cdots$

$= |000\rangle - |001\rangle + |011\rangle + \cdots - |111\rangle$

GHZ in X-basis: $|000\rangle + |110\rangle + |101\rangle + |011\rangle$