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# Asynchronous Track-to-Track Association Algorithm Based on Dynamic Time Warping Distance

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**Abstract:** In the distributed multi-target tracking system, the local sensors often begin working at different time and provide tracks at different rates with different communication delays. As a result, the local tracks from different sensors are usually asynchronous. The current solution is time registration before track association which leads to track synchronization. However, when synchronizing, the estimation error increases. This affects performance of track-to-track association. In this paper, tracks are treated as time series, and using dynamic time warping method (DTW) measures the distance between any two tracks. DTW is a much more robust distance measure for time series, allowing similar shapes to match even if they are out of phase in the time axis. Considering track-to-track association problems, and confining the search area of DTW when optimizing, a fast algorithm is obtained. This is a post-processing technique of tracks. In order to make track-to-track association more accurately after obtaining the track data from sensors, the algorithm is proposed so that track fusion can be implemented next. Simulation results show that the presented method can effectively solve the asynchronous track-to-track association problem.

**Key Words:** Asynchronous, track-to-track association, dynamic time warping, time series

## 1 Introduction

In the distributed multi-target tracking system, because the local sensors usually begin working at different time and provide tracks at different rates with different communication delays, the received tracks of local nodes for the fusion center are often asynchronous. Therefore, concentrating on track-to-track association problems in asynchronous cases becomes more and more important for distributed multi-target tracking fusion in [1]. At present, the general method is registering in the time domain, aligning tracks to the unified time, then acquiring track sequences with the same length by using interpolation and extrapolation, and realizing tracks matching based on Euclidean distance at last[2]. An asynchronous track-to-track association algorithm based on the least squares has been proposed in [3]. In [4], mutation ant colony algorithm for asynchronous track-to-track association has also been put forward. However, these algorithms require time registration to complete. In the synchronization, the error of track estimation will spread, and this spread has a relationship with the error of the filtering equation. It is difficult to describe. Therefore, in such a way, the performance of track-to-track association has been affected greatly.

For asynchronous track-to-track association problems, considering tracks as time series, thus, dynamic time warping (DTW) can be used here. DTW is a much more robust distance measure for time series. It is widely used in science, medicine, industry, finance and so on, especially in artificial intelligence in [5]-[15]. DTW distance defines the best alignment matching relationship between sequences. It supports the similarity measure of time series with different lengths. And also, it supports the timeline scaling and bending in [16]. Compared to general Minkowski distance, DTW distance has better robustness. Based on DTW, time alignment for asynchronous track-to-track association doesn't need, and in this way, estimation error for track correlation cannot be increased greatly by using direct correlation algorithms. Moreover, the proposed algorithm is a post-processing technique of tracks by using DTW. After obtaining the track data from sensors, track-to-track association is carried out more accurately based on the proposed algorithm, in this way, track fusion can be implemented effectively next .

## 2 Asynchronous track-to-track association algorithm based on DTW

Tracks can be expressed as time series (one-dimensional or multi-dimensional). A one-dimensional time series is a sequence in the form of $x_1, x_2, \cdots x_i, \cdots, x_m$, where $x_i$ is a real number, $i = 1, 2, \cdots, m$. Without any loss of generality, time is discrete. The number $m$ in a time series is called its length. Define multi-dimensional time series as follows:

$$\begin{array}{l} x_{11}, x_{12}, \cdots x_{1i}, \cdots, x_{1m} \\ x_{21}, x_{22}, \cdots, x_{2i}, \cdots, x_{2m} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{q1}, x_{q2}, \cdots, x_{qi}, \cdots, x_{qm} \end{array},$$

where each $x_{lj}$ is a one-dimensional time series of length $m$, $j = 1, 2, \cdots, m$. The number $q$ is the dimensionality of the series, and the number $m$ is its length.

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Firstly, suppose we have two one-dimensional time series with the length $m$ and $n$, respectively,

$$x = x_1, x_2, \cdots, x_m,$$
$$y = y_1, y_2, \cdots, y_n.$$

We construct an $m$ by $n$ matrix $D = (d_{ij})_{m \times n}$, where $d_{ij}$ is the distance $d(x_i, y_j)$ between the two points $x_i$ and $y_j$. In standard DTW[6], $d_{ij}$ is usually a distance between two real values( $d_{ij} = (x_i - y_j)^2$ ). Each matrix element $(i, j)$ corresponds to the alignment between the points $x_i$ and $y_j$. In [5]-[7], we construct a warping path of matrix elements $(i, j)$ that defines a mapping between $x$ and $y$, where

$$W = \{w_1, w_2, \cdots, w_k\}, \max(m, n) \le k < m + n - 1.$$

The warping path $W$ must satisfy three conditions:

(a) Boundary conditions: $w_1 = (1,1)$, $w_k = (m, n)$;

(b) Continuity: Given $w_k = (i_k, j_k)$, $w_{k+1} = (i_{k+1}, j_{k+1})$, $i_{k+1} - i_k \le 1$, $j_{k+1} - j_k \le 1$;

(c) Monotonicity: Given $w_k = (i_k, j_k)$, $w_{k+1} = (i_{k+1}, j_{k+1})$, $i_{k+1} - i_k \ge 0$, $j_{k+1} - j_k \ge 0$.

There are exponentially many warping paths that satisfy the above conditions. However, we are only interested in the path that minimizes the warping cost:

$$DTW(x, y) = \min_W \{\sqrt{\sum_{l=1}^{k} w_l}\}.$$

A practical way to compute the DTW distance between two time series is to build so-called cumulative distance matrix $\Gamma$. To compute the matrix we use dynamic programming with the following recurrence:

$$\gamma(i, j) = d(x_i, y_j) + \min\{\gamma(i-1, j), \gamma(i, j-1), \gamma(i-1, j-1)\},$$

and start conditions:

$$\gamma(0,0) = 0, \gamma(i,0) = \infty, \gamma(0, j) = \infty, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$$

The Euclidean distance between two sequences can be seen as a special case of DTW, where the *l*-th element of *W* is constrained such that $w_l = (i, j), i = j = l$. Note that it is only defined in the special case where the two sequences have the same length. The time and space complexity of DTW is $O(mn)$.

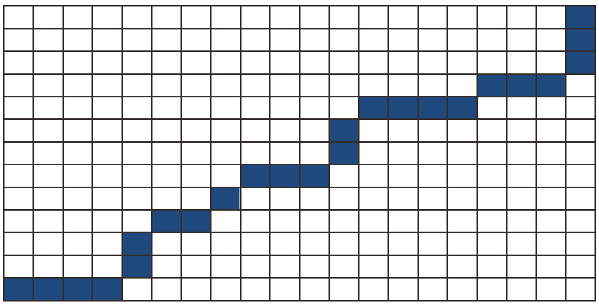

Fig. 1: Constructing a warping matrix, and searching for the optimal warping path

For multi-dimensional time series,

$$X = \begin{cases} x_{11}, x_{12}, \cdots x_{1i}, \cdots, x_{1m} \\ x_{21}, x_{22}, \cdots, x_{2i}, \cdots, x_{2m} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{q1}, x_{q2}, \cdots, x_{qi}, \cdots, x_{qm} \end{cases},$$

and

$$Y = \begin{cases} y_{11}, y_{12}, \cdots y_{1j}, \cdots, y_{1n} \\ y_{21}, y_{22}, \cdots, y_{2j}, \cdots, y_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_{q1}, y_{q2}, \cdots, y_{qj}, \cdots, y_{qn} \end{cases},$$

we can define the DTW distance between $X$ and $Y$ in the same way as in the one-dimensional time series. Only we need to modify the local cost function $d$ by

$$d(X_i, Y_j) = \sum_{l=1}^{q} (x_{li} - y_{lj})^2,$$

where

$$X_i = x_{1i}, x_{2i}, \cdots, x_{qi},$$
$$Y_j = y_{1j}, y_{2j}, \cdots, y_{qj}.$$

In order to speed up the DTW distance calculation, we use global constraints and local constraints on time warping[6]. The subset of the matrix that the warping path is allowed to visit is called the warping window. Figure 2 illustrates two of the most frequently used global constraints, the Sakoe-Chiba band (Sakoe and Chiba 1978) and the Itakura parallelogram (Itakura 1975).

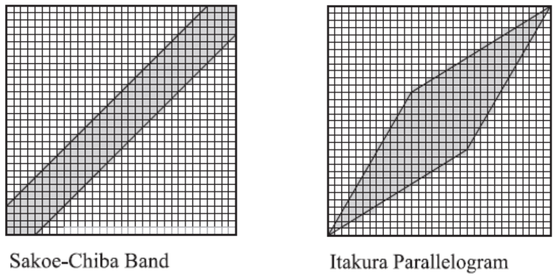


Fig. 2: Global constraints limit the scope of the warping path, restricting them to the gray areas. The two most common constraints in the literature are the Sakoe-Chiba band and the Itakura parallelogram optimal warping path

For global constraints, the adjustment window condition has the following form:

$$|i_k - j_k| \le r,$$

where $r$ is an appropriate positive integer called window length. This condition corresponds to the fact that time-axis fluctuation in usual cases never causes a too excessive timing difference. In the Sakoe-Chiba band, $r$ is a constant without correlation to $i, j$. In the Itakura parallelogram, $r$ has relationship with $i, j$, that is, $r$ is a function of $i, j$.

In addition to the global constraints, there has been active research on local constraints (Itakura 1975; Myers et al. 1980; Rabiner and Juang 1993; Sakoe and Chiba 1978; Tappert and Das 1978) for several decades. The basic idea is to limit the permissible warping paths, by providing local restrictions on the set of alternative steps considered. The effective intensity of the slope constraint can be measured by $P = s/t$. The larger the $P$ measure, the more rigidly the warping function slope is restricted. In practice, setting neither a too large nor a too small value for $P$ is desirable. The constraint might be designed based on domain knowledge, or from experience learned through trial and error.

Figures 3 illustrate the four original constraints suggested by Sakoe and Chiba (1978).

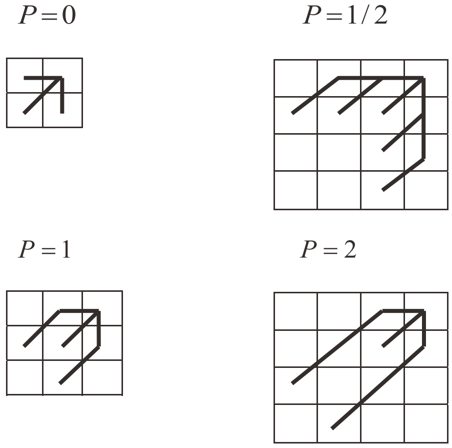


Fig. 3: Four local constraints on dynamic time warping, as suggested by Sakoe and Chiba

a) $\gamma(i,j)=\min\begin{cases} d(i,j)+\gamma(i,j-1) \\ d(i,j)+\gamma(i-1,j-1) \\ d(i,j)+\gamma(i-1,j) \end{cases}$

b)

$$\gamma(i,j)=\min\begin{cases} d(i,j)+d(i,j-1)+d(i,j-2)+\gamma(i-1,j-3) \\ d(i,j)+d(i,j-1)+\gamma(i-1,j-2) \\ d(i,j)+\gamma(i-1,j-1) \\ d(i,j)+d(i-1,j)+\gamma(i-2,j-1) \\ d(i,j)+d(i-1,j)+d(i-2,j)+\gamma(i-3,j-1) \end{cases}$$

c) $\gamma(i,j)=\min\begin{cases} d(i,j)+d(i,j-1)+\gamma(i-1,j-2) \\ d(i,j)+\gamma(i-1,j-1) \\ d(i,j)+d(i-1,j)+\gamma(i-2,j-1) \end{cases}$

d)

$$\gamma(i,j)=\min\begin{cases} d(i,j)+d(i,j-1)+d(i-1,j-2)+\gamma(i-2,j-3) \\ d(i,j)+\gamma(i-1,j-1) \\ d(i,j)+d(i-1,j)+d(i-2,j-1)+\gamma(i-3,j-2) \end{cases}$$

Importantly, the local constraints can be reinterpreted as global constraints.

For specific track-to-track association problems, for the sake of simplicity, firstly, we associate any two different radars sampling data. Given one of the radars the sampling interval $T_1$, the other $T_2$. Radars 1 starts at time $t_0$, and radars 2 starts at time $t_0'$. Suppose we can get the tracks

$$X=\begin{cases} x_{11}, x_{12}, \cdots x_{1i}, \cdots, x_{1m} \\ x_{21}, x_{22}, \cdots, x_{2i}, \cdots, x_{2m} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{q1}, x_{q2}, \cdots, x_{qi}, \cdots, x_{qm} \end{cases},$$

and

$$Y=\begin{cases} y_{11}, y_{12}, \cdots y_{1j}, \cdots, y_{1n} \\ y_{21}, y_{22}, \cdots, y_{2j}, \cdots, y_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_{q1}, y_{q2}, \cdots, y_{qj}, \cdots, y_{qn} \end{cases}.$$

$X$ is the observation result of radar 1, and $Y$ is the observation result of radar 2. If the observation of two tracks starts at different time, when constructing a warping matrix, and searching for the optimal warping path, for example fig.1, there will be one-to-many situations at the beginning and at the end. Such a case can be used to measure homology of the tracks. For example, let the box number with shadow at the bottom be $l_1$, the box number at the right end be $l_2$, if satisfying $l_1 \cdot T_1 = l_2 \cdot T_2$, the two tracks has the possibility of homology. If not satisfying the corresponding conditions

$$(l_1-1)\cdot T_1 = l_2 \cdot T_2$$

or

$$(l_1-1)\cdot T_1 = (l_2-1)\cdot T_2$$

or (1)

$$l_1 \cdot T_1 = (l_2-1)\cdot T_2$$

or

$$l_1 \cdot T_1 = l_2 \cdot T_2$$

we can determine the two tracks are not homologous.

For our problems, because the track is a special kind of time series, it has some properties. Discussing the track-to-track association problems, we must consider the time constraints based on DTW. For two tracks $X, Y$, if we choose $X_i$ and $Y_j$ to make a pair in the final warping path, that is, there exists $1 \le l \le k$ such that $w_l = (i,j)$ for $|i-j|$ large enough, and this will lose the practical meaning, because $X_i$ and $Y_j$ are treated as two points in the tracks, far apart each other in time, no matter how close in space, there will be no meaning.

For our convenience, we suppose $T_1 \le T_2$. For $T_1 \ge T_2$, we can obtain the same result. Firstly, we see the search path figure based on DTW as a figure of a point moving. Let

$$r=\left\lceil \frac{T_2}{T_1} \right\rceil \text{(if } T_1 \ge T_2\text{, then let } r=\left\lceil \frac{T_1}{T_2} \right\rceil\text{)},$$

here, the specific $r$ as a constraint number playing an important role, when constructing a warping matrix, and searching for the optimal warping path, for example Fig.4, a point in Fig.4 moves up $r$ steps as much as possible, and then it must moves right or moves along the diagonal direction. That is, as much as $r$ points in $X$ corresponds a point in $Y$. Because $X$ has more points than $Y$, we bend and compress $X$ and don't deal with $Y$ to obtain the searching area constraints.

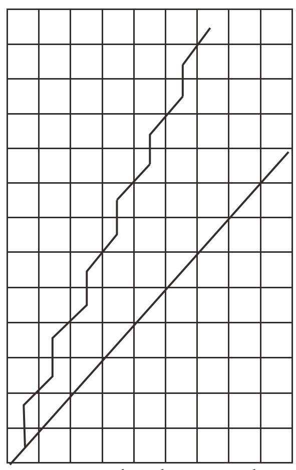

Fig. 4: Searching area constraints between the straight line and the broken line

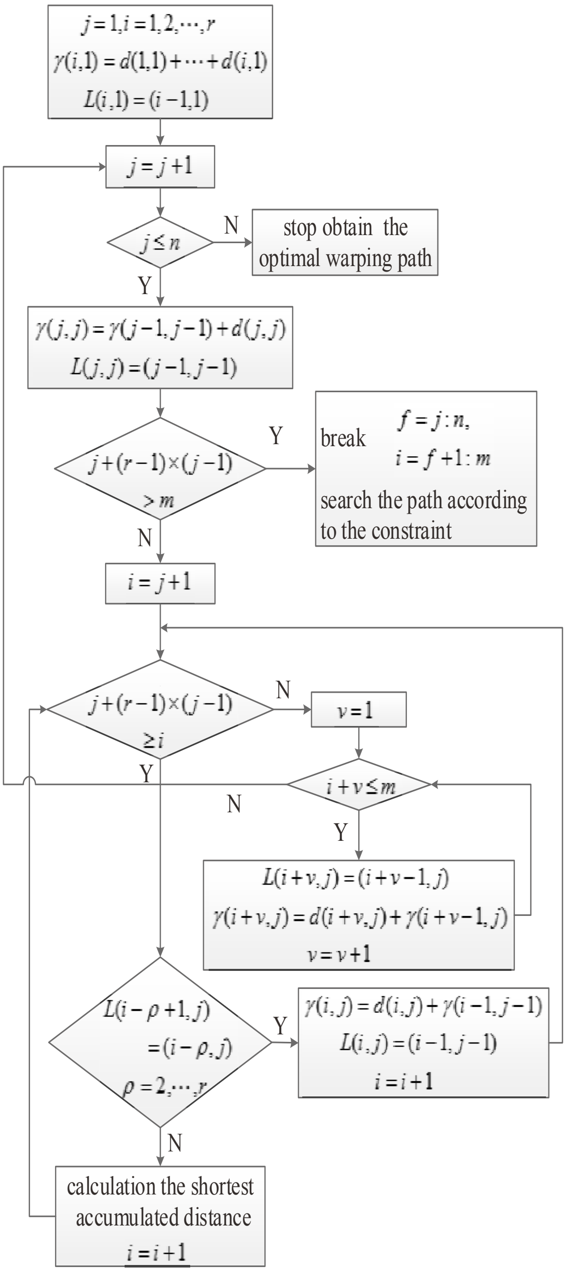


Fig.5: DTW algorithm for asynchronous track-to-track association with the same starting and ending time

Suppose the track $X$ has the length $m$, the track $Y$ has the length $n$. We consider two cases in the following. One is the radars start and end at the same time, and they only has different sampling rate. Let one of the radars the sampling interval be $T_1$, the other $T_2$. The flow chart is in figure 5. The other is that the radars start and end at different time, and sampling intervals are also $T_1$ and $T_2$, respectively. We can use the conditions for starting and ending to determine the homology initially as described before. The flow chart is in figure 6.

For asynchronous track-to-track association with the same starting and ending time, the time complexity of DTW algorithm has relation to $r$. It is about $O(\frac{\min\{m^2,n^2\}}{r})$. The space complexity of algorithm is a linear function of track numbers in monitoring area and the accumulative time for track.

## 3 Simulation

Suppose we have two radars of polar coordinates located in different place. Radars are away from each other for 100000 meters. They follow the tracks of 5 objects simultaneously. The distance and angle error of radar 1 are $\sigma_{r1}=100m$, $\sigma_{\theta 1}=0.001rad$, respectively. The sampling interval is $T_1=2s$. The distance and angle error of radar 2 are $\sigma_{r2}=80m$, $\sigma_{\theta 1}=0.003rad$, respectively. The sampling interval is $T_2=3s$.

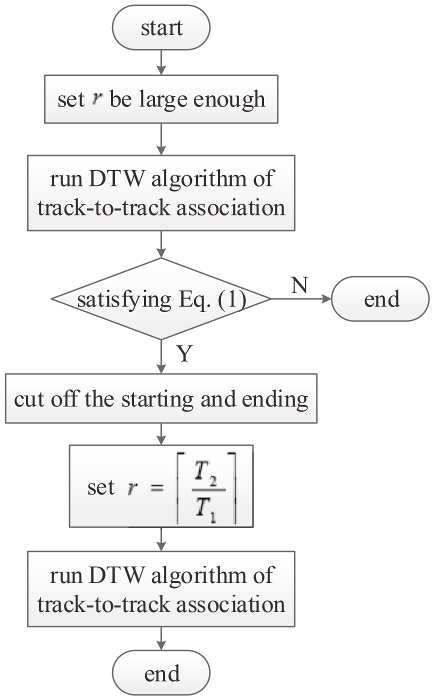


Fig.6: DTW algorithm for asynchronous track-to-track association with different starting and ending time

Case 1: Radar 1 and radar 2 start and end working at the same time. Based on data association and EKF (Extent Kalman Filter), we obtain the tracks. Moreover, here, we suppose all the trajectories can be tracked successfully. Figure 7 is the tracks of radar 1 and radar 2 after EKF. Figure 8 is the optimal warping path of matching partially. Table 1 is the comparison of the distance.

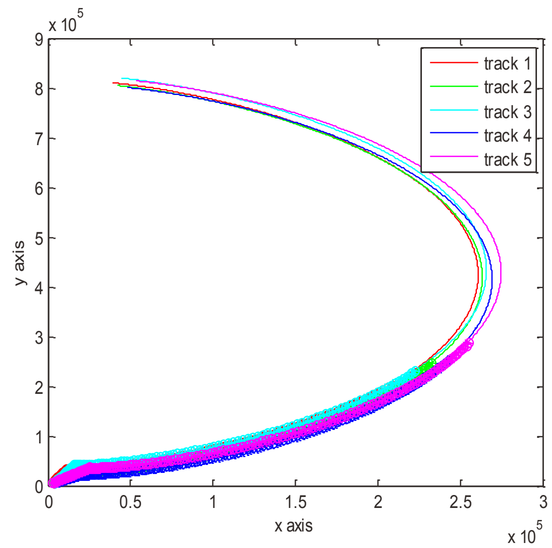


Fig .7: The tracks of radar 1 and radar 2 after EKF in case 1

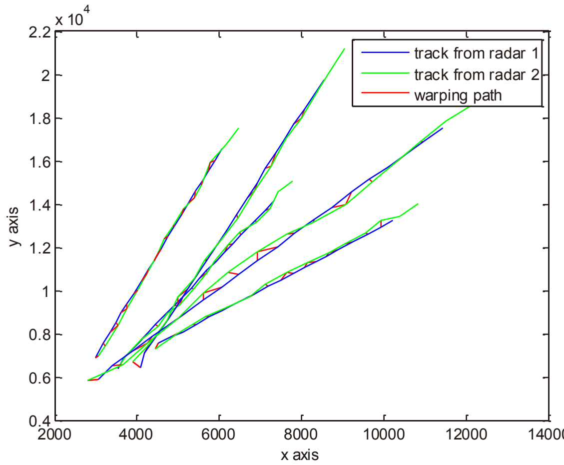


Fig. 8: The optimal warping path of matching partially in case 1

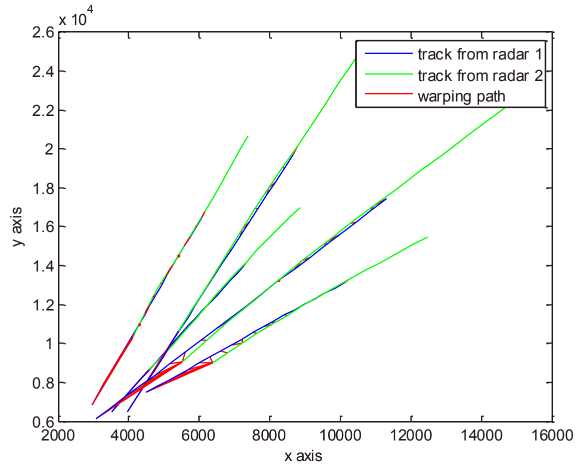


Fig. 10: The optimal warping path of matching partially in the condition of not cutting off the starting and ending in case 2

Table 1: the comparison of the distance in case 1

| distance | T1 of R1 | T2 of R1 | T3 of R1 | T4 of R1 | T5 of R1 |
|---|---|---|---|---|---|
| T1 of R2 | **1.2722 e+08** | 1.5870 e+11 | 5.2954 e+10 | 2.2598 e+11 | 5.6934 e+11 |
| T2 of R2 | 2.9084 e+10 | **1.6418 e+08** | 2.0422 e+10 | 1.0979 e+10 | 1.4156 e+11 |
| T3 of R2 | 1.1101 e+10 | 3.0185 e+10 | **1.5148 e+08** | 9.2417 e+10 | 2.9660 e+11 |
| T4 of R2 | 6.4349 e+10 | 7.6848 e+09 | 5.0744 e+10 | **1.6819 e+08** | 1.0916 e+11 |
| T5 of R2 | 1.7347 e+11 | 2.5222 e+10 | 6.5722 e+10 | 2.0530 e+10 | **2.1634 e+08** |

In table 1, the element located in i-th row and j-th column represents the distance between i-th track of radar 2 and j-th track of radar 1. The j-th column shows the comparison of the distance between five tracks from radar 2 and the j-th track from radar1. From the table, correct matching can be verified easily in the diagonals.

Case 2: Radar 1 and radar 2 start and end working at different time. Figure 9 is the tracks of radar 1 and radar 2 after data association and EKF. Furthermore, like case 1, suppose all the trajectories can be tracked successfully here. Figure 10 is the optimal warping path of matching partially. Table 2 is the comparison of the distance.

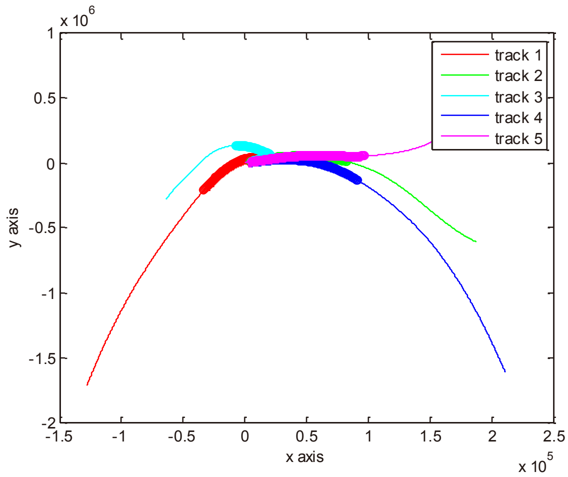


Fig .9: The tracks of radar 1 and radar 2 after EKF in case 2

Table 2: the comparison of the distance in the condition of not cutting off the starting and ending in case 2

| distance | T1 of R1 | T2 of R1 | T3 of R1 | T4 of R1 | T5 of R1 |
|---|---|---|---|---|---|
| T1 of R2 | **3.7280 e+09** | 1.1713 e+12 | 2.3266 e+12 | 6.2457 e+11 | 1.6082 e+12 |
| T2 of R2 | 1.2382 e+12 | **2.9140 e+08** | 7.0930 e+11 | 4.7020 e+11 | 2.9845 e+10 |
| T3 of R2 | 2.4165 e+12 | 6.2671 e+11 | **8.1379 e+07** | 2.1019 e+12 | 6.6038 e+11 |
| T4 of R2 | 5.6897 e+11 | 4.5521 e+11 | 2.1312 e+12 | **1.5904 e+09** | 6.6597 e+11 |
| T5 of R2 | 1.6741 e+12 | 2.4106 e+10 | 7.1272 e+11 | 7.0064 e+11 | **8.9395 e+07** |

Similar to table 1, the element in i-th row and j-th column represents the distance between i-th track of radar 2 and j-th track of radar 1. The j-th column shows the comparison of the distance between five tracks from radar 2 and the j-th track from radar1. From the table, we can see the elements representing the distance in the diagonals are much smaller. So, the right association can be verified easily because the distance can distinguish the homology clearly. After cutting off the starting and ending, the algorithm can implement the matching more accurately.

## 4 Conclusion

For distributed multi-target tracking system, by using dynamic time warping distance to measure the homology of any two tracks, this method can avoid the registering in the time domain, and can decrease the error to some extent. According the specific track-to-track association problems, we confine the search area in the dynamic time warping. This can improve the search speed as global constraints on DTW. The algorithm only makes a preliminary simulation, and these simple simulation results prove the effectiveness of the algorithm to a certain degree and also it needs further exploration and improvement. In the proposed algorithm, the satisfying condition Eq(1) is an approximate approach .What we should do in the future is to explore a more practical and theoretical expression according to the actual environment or applications. At present, the algorithm is only a kind of block procedure. Furthermore, in the future, we hope to improve it into a sequential procedure or adjust it by using sliding window technique, so that can be more suitable for engineering application. At the same time, the improvement or adjustment can save the calculation cost greatly and ensure the high speed in real-time.

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