ADC Resolution for Simultaneous Reception of Two Signals with High Dynamic Range

Baptiste Laporte-Fauret[1,2], Guillaume Ferré[1], Dominique Dallet[1], Bryce Minger[2], Loïc Fuché[2]
Univ. Bordeaux, Bordeaux INP, CNRS, IMS, UMR 5218, Talence (FRANCE)[1]
Thales, Cholet (FRANCE)[2]
Email: forename.name@{ims-bordeaux.fr[1], thalesgroup.com[2]}

***Abstract*—This paper discusses a new limit for the dynamic range of an Analog to Digital Converter (ADC) in presence of two simultaneous signals to process. An encountered method to determine the minimum required ADC resolution is analysed and compared to the well known formula established for a single received signal. A new approach is proposed to evaluate the resolution when the ADC is fed with a weak and a strong signal. Compared to previous works, we show the required resolution is enhanced by the presence of two signals through a spectral representation and mathematical developments.**

***Index Terms*—ADC, dynamic range, resolution, quantization noise, digitization, signal carrying**

I. Introduction

Radio receivers are now present in many sectors of activity, as well civilian than military. These mixed analog-digital architectures operate over large frequency ranges from a few kHz to several GHz. However, off-the-shelf components make it difficult to achieve the ADC main goal which is to process all the received signals. Indeed, if a weak and strong signal are simultaneously received, the digitization process can lead to the loss of the first one. This phenomenon is the consequence of the ADC bounded dynamic range. This situation is particularly true with the emergence of Internet of Thing (IoT) which leads to the processing of multiple signals with a wide range of power.

Dynamic range is a key notion to evaluate ADC performance. It is defined as the range between the noise floor and its maximum output level. This issue has been especially studied in [1] and [2] in the context of compressive sensing. The authors aim to reduce the required number of measurements in a digital acquisition by exploiting signal sparsity and underline the importance to keep an unchanged dynamic range in acquisition receivers. This problem is also addressed in the field of digital communications for instance in [3] but for a phase-domain ADC unlike the widely used In-phase Quadrature ADC.

In this paper, we aim to adjust the ADC resolution placed just after the analog front end of digital receivers. Therefore, we propose an analysis on how to evaluate this resolution when an ideally linear ADC is fed by:

1) a weak signal;
2) a weak and a strong signal

This paper is organized as follows. Section II recalls the required ADC resolution in presence of a single tone signal [4]. This method is also compared with an encountered technique [5] which evaluates the minimum resolution in presence of two signals but with overestimated results. Section III proposes a new limit for the dynamic range through simulations and mathematical developments while section IV presents conclusions and future prospects.

II. Resolution definition in the literature for instantaneous reception of two signals

A. Minimum resolution for a single tone signal reception

Before considering the reception of multiple signals, it is necessary to study the required ADC resolution for the digitization of a single analog signal. To do so, we analyse the quantization process which is the main cause of noise in an analog to digital system. This quantization noise represents the difference between the input signal and the ADC output and depends on the ADC resolution. It is therefore crucial to investigate these different parameters.

Let $x(t)$ be a sine wave and A its magnitude. The n bits ADC resolution is defined in (1) with the quantization step q for a FSR full scale voltage range.

$$q = \frac{FSR}{2^n} \tag{1}$$

As recalled in [4], we define the Root Mean Square (RMS) value for the signal $x_{RMS} = \frac{A}{\sqrt{2}}$ and for the quantization noise $\varepsilon_{RMS} = \frac{q}{\sqrt{12}}$. We take into consideration the quantization noise in order to obtain the Signal to Noise Ratio (SNR). Thus, the SNR value can be defined as expressed in (2) and (3).

$$(SNR)_{dB} = 20\log_{10}\left(\frac{x_{RMS}}{\varepsilon_{RMS}}\right) \tag{2}$$

$$(SNR)_{dB} = 6.02n + 1.76 + 20\log_{10}\left(\frac{2A}{FSR}\right) \tag{3}$$

The dynamic range for a single input signal can be expressed as the ADC range to resolution ratio as defined in (4). We can then deduce the required resolution for a given power ratio.

$$(DR)_{dB} = 20\log_{10}\left(\frac{FSR}{q}\right) = 6.02n \tag{4}$$

However, this definition does not apply to the simultaneous reception of two signals with different powers as explained in section II-B.

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B. Minimum resolution for a correct digitization of two simultaneous analog signals

We aim to obtain a new definition of the minimum ADC resolution when an input signal with two components is considered: one of low power and the other of high power. This problematic is not widely addressed in the literature with the exception of [5] which defines the dynamic range as expressed in (5) when a weak and a strong signal are considered. All expressions in this section are taken from [5] and only adapted to match the notations of this paper.

$$DR_{ws} = 2^{n_{ws}} q = \frac{(x_w + x_s)\sqrt{2}}{K} \quad (5)$$

With n_{ws} the resolution when a weak and a strong signal are considered; x_s and x_w resp. the RMS value of the strong and weak signal and K the scaling factor. This scaling factor is introduced in order to consider the situation where a signal is not ranging the full scale. For instance, K = 0.8 if the signal occupancy is 80% of the dynamic range.

By combining (5) and the quantization noise defined in section II-A, the weak signal to quantization noise ratio (SNR_w) can be obtained as expressed in (6).

$$SNR_w = \frac{x_w}{\varepsilon_{RMS}} = K \frac{x_w}{x_w + x_s} 2^{n_{ws}+1} \sqrt{\frac{3}{2}} \quad (6)$$

Finally, [5] considers the SNR value when a single weak signal is received in order to find the necessary ADC resolution to digitize simultaneously a weak and strong signal as in (7).

$$n_{ws} = n_w + \log_2\left(1 + \frac{x_s}{x_w}\right) - \log_2(K) \quad (7)$$

With $n_w = 5$ the resolution for a single weak signal in [5].

For instance, we notice 21 bits are required to achieve a 100 dB dynamic range depending on the value of K. This resolution is something that is not going to be possible with current hardware or in a near future for Nyquist ADC. It is rather interesting to develop a new evaluation of the minimum resolution for a correct and simultaneous reception of two signals. However, it is also troubling the resolution expressed in (7) is deteriorated compared to the one in [4] for a single weak signal reception. In fact, some primary results (developed in the next section) shows the opposite with an enhance dynamic range in that particular case. Thus, it would be pertinent to adapt the development of [5] with a new definition of the SNR_w by taking into consideration new parameters like the quantization noise or the number of points of the signal.

Consequently, we demonstrate, in the next part, these observed minimum ADC resolutions for the correct reception of two signals are overvalued.

III. Proposed Resolution Calculation for Instantaneous Reception of Two Signals

A. Observations

Our goal is to discuss a new resolution requirement for a given power ratio which is defined as the ratio between the full scale and the weak signal. Therefore, the dynamic range can be seen as the maximum power ratio achievable without the loss of the weak signal. The previously detailed method in section II-B is overestimated in presence of a weak and strong signal just like the method recalled in II-A. We generate a sum of two sine waves under the MATLAB software: one with magnitude 0.9 close to the normalized full scale and the other with an 80 dB power ratio. We observe in Fig. 1 that the weak signal is still visible even with a power ratio greater than the theoretical 72 dB dynamic range for a 12 bits resolution established for a single tone. In fact, we start losing it with a power ratio around 100 dB when the weak signal is mingled with the quantization noise. It does not necessary mean the signal is completely lost but we want to detect it by a basic thresholding method based on the Fast Fourier Transform (FFT). Therefore, it is relevant to compare this limit when a single weak signal fed the ADC.

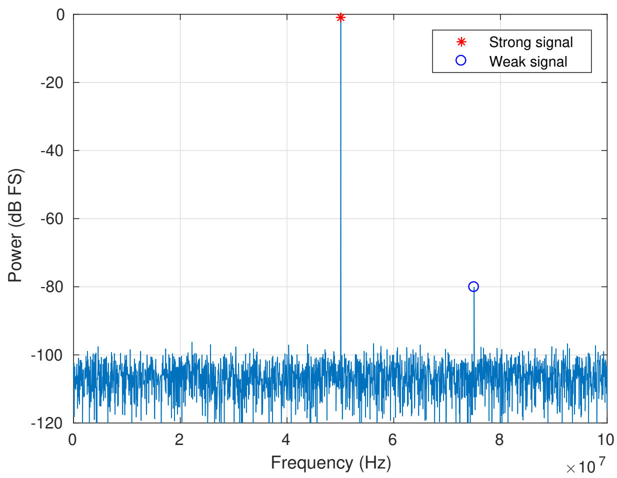

Fig. 1. Spectral representation with N = 4096 FFT points of a weak and strong signal with 80 dB dynamic range and 12 bits resolution

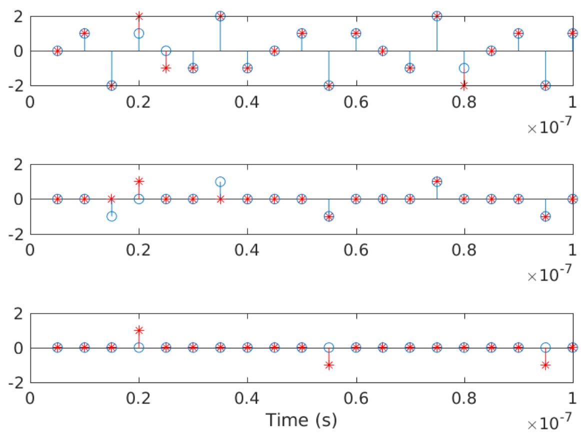

Fig. 2. Time domain representation of a single weak signal (o) and the strong signal subtracted after the ADC ($*$) with 60 (upside figure), 72 (middle figure) and 73 (downside figure) dB power ratio and 12 bits resolution

For a 12 bits ADC, Fig. 2 shows a signal with 72 dB power ratio alone (o) cannot be seen while it can be received when carried by a strong signal ($*$). In fact, for a 72 dB power ratio, the single tone signal is altered but can be retrieved with low SNR detection techniques as detailed [6] and not addressed here. Nevertheless, those techniques are useless if the signal is completely lost between the quantizations levels as it occurs for a single signal reception with a 73 dB power ratio.

As a result, it is possible to exceed the theoretical dynamic range in presence of two signals as detailed in the next section.

B. *Quantum limitations and signal carrying*

In a digitization process, the ADC resolution is of the utmost importance since it impacts on the quantization step. Indeed, an intra-quantum sine wave cannot be observed after a digitization process as we can see for the weak signal in Fig. 3. Nonetheless, this weak signal can be carried by a stronger one in order to cross the quantum level. In other words, the reception of multiple signals can, under certain conditions, enhance the dynamic range for a given resolution. The signal cannot always be superior to the quantization step. What's important is that it remains occasionally higher than q in order to be received and processed. Thus, we define a set of points $\mathcal{M}$ which respects the condition expressed in (8).

$$x_s(kT_e) + x_w(kT_e) > \frac{q}{2} \tag{8}$$

With T_e the ADC sampling frequency and $k \in \mathcal{M}$. Indeed, the signal composed of a strong ($x_s(kT_e)$) and weak ($x_w(kT_e)$) tone must be greater than a given number of quantum. We define $x_s(kT_e) = A_s \sin(2\pi f_s kT_e + \phi_s)$ and $x_w(kT_e) = A_w \sin(2\pi f_w kT_e + \phi_w)$ with A_s and A_w resp. the magnitude of x_s and x_w; f_s and f_w their carrier frequencies and ϕ_s and ϕ_w their phases.

We can then notice it is possible to find a stronger signal to carry the weak signal by crossing the quantum. Therefore, the minimum signal power that can be detected only depends on the quantization noise floor.

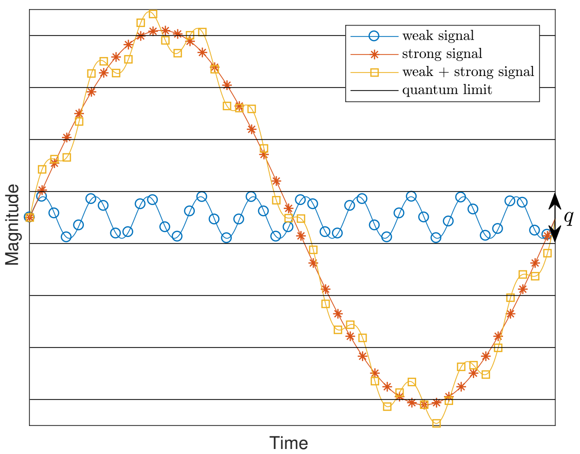

Fig. 3. Weak signal carried by a strong signal and crossing quantization levels (before digitization)

C. *Proposed new lower bound*

Since the correct detection of a carried weak signal no longer only depends on the quantization step, we study the quantization noise floor which is the new limiting factor.

The average noise floor, denoted $(B_m)_{dB}$, is defined as the quantization noise power spectral density. We define $X(k)$ in (9) as the Discrete Fourier Transform (DFT) at normalized frequency $\frac{k}{N}$ with $x(i)$ the input data samples from a sine wave and $i, k = 0, 1, 2, \cdots, N-1$.

$$X(k) = \sum_{i=0}^{N-1} x(i) e^{-j2\pi i \frac{k}{N}} \tag{9}$$

The acquisition is considered with coherent frequencies. The $(B_m)_{dB}$ level is set according to a reference value like the power of a sine wave with magnitude equal to the full scale. In order to determine a theoretical relation of B_m, we take as reference the input signal when only the quantization noise is considered. The quantization noise power can be obtained such as the signal power without the continuous ($k = 0$) and fundamental values ($k = l$) as expressed in (10).

$$B_q = \sum_{k=0}^{N-1} |X(k)|^2 - |X(0)|^2 - 2|X(l)|^2 \tag{10}$$

With $|X(0)|^2$ the power of the continuous value and $|X(l)|^2$ half the signal power. We also know the average noise power spectral density can be expressed in (11).

$$B_m = \frac{B_q}{N-3} \tag{11}$$

(11) can be normalised in (12) and (13) by $|X(l)|^2$, the positive part of the signal power to adapt it from the full scale.

$$10\log_{10}\frac{B_m}{|X(l)|^2} = 10\log_{10}\frac{B_q}{(N-3)|X(l)|^2} \tag{12}$$

$$(B_m)_{dB} = 3 - 10\log_{10}(N-3) - 10\log_{10}\left(\frac{2|X(l)|^2}{B_q}\right) \tag{13}$$

$10\log_{10}\left(\frac{2|X(l)|^2}{B_q}\right)$ represents the signal to quantization noise ratio in dB which is known to be equal to $6.02n + 1.76$ as previously explained in (3) for a full amplitude sine wave. We can then deduce the final expression of the average noise floor (14) which depends on the FFT algorithm number of points.

$$(B_m)_{dB} = -6.02n - 10\log_{10}(N-3) + 1.24 \tag{14}$$

However, in order to determine a value for the dynamic range for a given ADC resolution, the variance of the estimator needs to be determined. We consider a complex white noise process $w[n]$ with variance σ_w^2 as explained in [7] and [8]. $\hat{P}_w(e^{jw})$ represents the periodogram of $w[n]$. To compute the covariance, we study the correlation between samples of periodogram at two points ω_1 and ω_2. We note $\mathrm{Cov}[a, b] = \mathbb{E}[(a - \mathbb{E}(a))(b - \mathbb{E}(b))]$ and $\mathrm{Var}[a] = \mathbb{E}[(a - \mathbb{E}(a))^2]$. After some developments detailed in [8], we obtain the covariance of the periodogram in (15).

$$\mathrm{Cov}[\hat{P}_w(e^{jw_1}), \hat{P}_w(e^{jw_2})] = \sigma_w^4 \left(\frac{\sin(N(\omega_1 - \omega_2)/2)}{N \sin((\omega_1 - \omega_2)/2)} \right)^2 \tag{15}$$

Finally, the variance of the periodogram is given by the limit of (15) when $\omega_1 \to \omega_2$ as expressed in (16). The variance is shown to be constant and independent on the number of data samples: the estimator is not consistent.

$$\mathrm{Var}[\hat{P}_w(e^{jw})] = \sigma_w^4 \tag{16}$$

For an ideal ADC, the quantization process can be modeled as the addition of a white noise with $\sigma_w^2 = \frac{q^2}{12N}$ (as explained in [9] or [10]). We also need to normalise it with the B_m expression since we work with the full scale. Thus, we can propose in (17) a new lower limit of the dynamic range DR_p to detect properly a weak and a strong signal.

$$(DR_p)_{dB} = (B_m)_{dB} - 10\log_{10}\left(\left(\frac{q^2}{12N}\right)^2 / B_m^2\right) \tag{17}$$

Fig. 4 compares the encountered method [5], the expression for a single tone signal recalled in [4] and the new proposed theoretical limit defined in (17). For a 12 bits ADC, we can notice a significant difference between the three dynamic range. While the theoretical dynamic range established for a single tone signal in [4] is 72 dB, it has been shown by mathematical developments that a 95 dB power ratio can be received. This value is not absolute. To refine it, we simulate 10^3 sine waves with random phases ϕ_s and ϕ_w and we consider a 95 dB dynamic range between the strong signal close to the full scale and the weak signal. The weak signal is detected by thresholding with the limit defined in (17) and $N = 2^n$ FFT points. The frequencies obtained are compared to the ones specified for the simulation in order to be certain that the correct signal is detected. With this process, only 57% of weak signals are detected. The weak signal is almost always detected if we consider a 90 dB power ratio. On the contrary, the weak signal starts to be completely lost for a 104 dB power ratio.

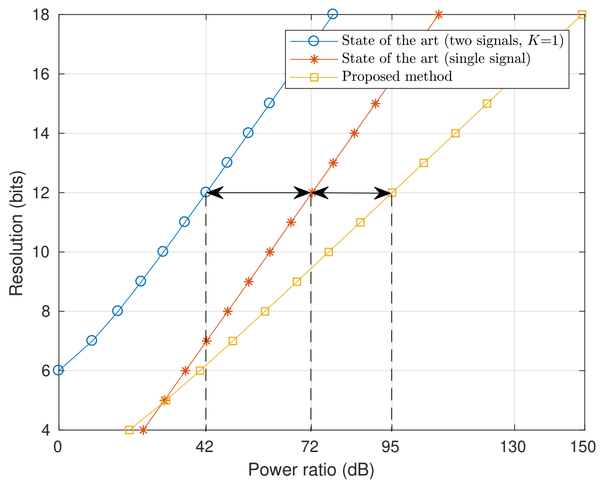

Fig. 4. Comparison between encountered method, single tone method and our new limit for $N = 2^n$ FFT points

These results can be compared with the method developed in [5] which, unlike [4], takes into consideration the reception of a strong and weak signal. Its required resolution prove to be overestimated for such a dynamic range since only 42 dB are obtained for a 12 bits ADC in the best case scenario.

IV. Conclusion

A new method to evaluate the necessary ADC resolution was introduced when two signals, one weak and one strong, are simultaneously received. The required resolution is enhanced compared to the previously defined metric with a single tone signal. The proposed limit is also more relevant than an encountered method in [5] where the results are overvalued. This work could be extended to the demodulation of digital communications signals, the consideration of transition noise in a non-ideal ADC and also the determination of the required number of points of the set to carry the weak signal.

Although the gain is noticeable compared to the one observed for a single tone, it is still not enough. With the IoT, it is now crucial to achieve around 130 dB dynamic range. This range requires ADC with more bits of resolution like 16 bits for a weak and a strong signal. However, current technologies don't allow better resolution than 12 or 14 bits in order to achieve a high dynamic range. Therefore, it is necessary to develop new methods in order to overtake the bounded ADC dynamic range such as companding techniques.

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