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A SUBDIVISION ALGORITHM FOR COMPUTER

DISPLAY OF CURVED SURFACES

by

Edwin Earl Catmull

A dissertation submitted to the faculty of the
University of Utah in Partial Fulfillment of the requirements
for the degree of

Doctor of Philosophy

Department of Computer Science

University of Utah

December 1974

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ABSTRACT

This thesis presents a method for producing computer shaded pictures of curved surfaces. Three-dimensional curved patches are used, as contrasted with conventional methods using polygons. The method subdivides a patch into successively smaller subpatches until a subpatch is as small as a raster-element, at which time it can be displayed. In general this method could be very time consuming because of the great number of subdivisions that must take place; however, there is at least one very useful class of patches -- the bicubic patch -- that can be subdivided very quickly. Pictures produced with the method accurately portray the shading and silhouette of curved surfaces. In addition, photographs can be "mapped" onto patches thus providing a means for putting texture on computer-generated pictures.

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CHAPTER ONE

INTRODUCTION

A method for creating shaded pictures of curved surfaces is presented in this thesis. A motivation for the method is that we wish to produce high quality computer-generated images of surfaces and curved solid objects on a raster-scan output device. We would not only like the images to accurately represent the surfaces we choose but in addition we would like control over shading and texture. There has already been significant research directed toward these ends, especially on the hidden-surface [1,2] and shading [3,4] aspects of the problem. All such methods must must address the questions of how to model objects and then how to render them.

Polygons, and sometimes quadric patches, are used to model objects in current shaded-picture methods . There are some difficulties with using these simple pieces to model or approximate free-form curved surfaces. Approximation with polygons gives a faceted effect and a silhouette made up of straight-line segments. Quadric patches [5,6], while smooth in appearance, are not suitable for modelling arbitrary forms, since they don't provide enough degrees of freedom to satisfy slope continuity between patches.

There are two significant methods used for reducing or eliminating the undesirable visual effects that occur when polygons are used to approximate curved surfaces. The first method for getting rid of the faceted effect is that of Henri Gouraud [3]. With

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this method a scalar light intensity value is associated with each vertex of a polygon. Gouraud does linear interpolation of the intensity value between vertices and then subsequently across scan-lines. If adjoining polygons have the same intensities at the common vertices then this method yields continuous shading across the surface; however, the first derivative of the shading is discontinuous. Gouraud's method has been implemented by different groups making shaded-pictures. It is a simple and successful method but has a few shortcomings: the discontinuity of the derivative is noticable (the "Mach band effect"), it is difficult to do highlights, the shading is affected by the orientation of the polygon in the picture, and the silhouette is still made up of straight-line segments.

The second method developed to improve the appearance of the polygon approximation is that of Phong [4]. Since current methods of generating intensities for polygon surfaces include calculating a surface normal at the vertices, Phong decided to interpolate the entire surface normal vector between vertices and edges instead of the scalar intensity values that Gouraud used. This yields a normal at every display point which can be used to calculate the intensity. Although this normal may not be the mathematically correct one, it is close enough to use for intensity and highlight calculations. As Phong has noted, although there is still a discontinuity in the first derivative of the shading, the discontinuity is smaller than for Gouraud's method and hence less noticeable. Phong's method has been used to make some visually attractive photographs, but the problem of straight-line segments at the silhouette still remains.

Curved surface segments or "patches" can be used instead of polygons to model free-form curved surfaces. If such patches can be joined together with slope

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continuity across the boundaries then a picture of a surface can be made to appear "smooth" both in shading and at the silhouette. For patches to be useful in modelling a curved surface, techniques must be found for describing and manipulating the patches and for connecting them together with slope continuity across boundaries. One such patch is the bicubic patch, which is widely used (see Appendix A). Most of the ideas in this thesis will be applied to the bicubic patch, but this is not intended to imply a limitation on generality.

Generating pictures of curved patches requires techniques for

1) establishing a correspondence between points on the surface and the elements of the display raster,
2) removing hidden or, more generally, the "not seen" parts of patches, and
3) calculating light intensities to be displayed on the raster.

Chapter two will deal with the first item: it will present a technique for establishing the correspondence between points on the surface and the raster elements, Chapters three and four will describe a specific method for quickly making the correspondence when bicubic patches are used. Chapter five will deal with item two: it will discuss the "hidden-surface" problem for patches. Item three -- calculating light intensities -- will be discussed in chapters six and seven.

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CHAPTER TWO

A GENERAL ALGORITHM FOR DISPLAYING CURVED PATCHES

An algorithm for establishing a correspondence between points on a patch and raster elements is described in this chapter. It applies to patches and surface sections in general, hence the algorithm presented will not be specific at the outset. Later on, when a specific kind of patch is used, more detail will be given. Before presenting that algorithm, however, some terms must be defined.

DEFINITIONS

A "raster-scan device" or "raster-display" is the device that we will consider for final output of an image. The rectangular array of "dots" that is produced on a raster-display is called the "raster." Each dot will usually be called a "raster-element." The raster element covers a very small area of the raster; however, it should not be thought of as a point. A row of raster-elements is a "scan-line." Scan-lines are usually produced in sequential order, termed "scan-line-order." Each raster-element has a brightness that is determined by the intensity value for that raster-element. The process of taking the intensity values and putting the dots on the raster with the corresponding intensities is called "displaying."

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A "frame-buffer" is a memory large enough to store all of the intensity values prior to displaying. An intensity value in the frame-buffer can be addressed in a way that corresponds to the position where the value will be displayed on the raster. Locations in the frame-buffer will also be called "raster-elements" since there is a strong one-to-one correspondence between those locations and the geometric locations of the raster-elements and because the distinction between the two is not important here. For our purposes, the frame-buffer is made with random-access memory so that values can be written into it in any order, as opposed to scan-line order only. The size of the frame-buffer is determined by the resolution of the raster-display and the number of "bits" used to store intensity values. For example, if the raster has 512 scan-lines and 512 raster-elements per line and each element has 8 bits for the intensity value, then the frame-buffer requires a storage capacity of 512x512x8 bits. For the most part we will ignore the raster-display and address ourselves to the issue of putting the right intensity values in the raster-elements of the frame-buffer.

The terms relating the original description of an object to its image will now be defined. "Object-space" is the three-dimensional space in which objects will ordinarily be described. In order to generate realistic pictures of objects we make a perspective transformation [1,7,8] of the object from object-space to "image-space." Image-space is also three-dimensional but the objects have undergone a perspective distortion so that an orthogonal projection of the object onto the x-y plane would result in the expected perspective image. We want the image-space to be three-dimensional in order to preserve depth information which will later be used to solve the

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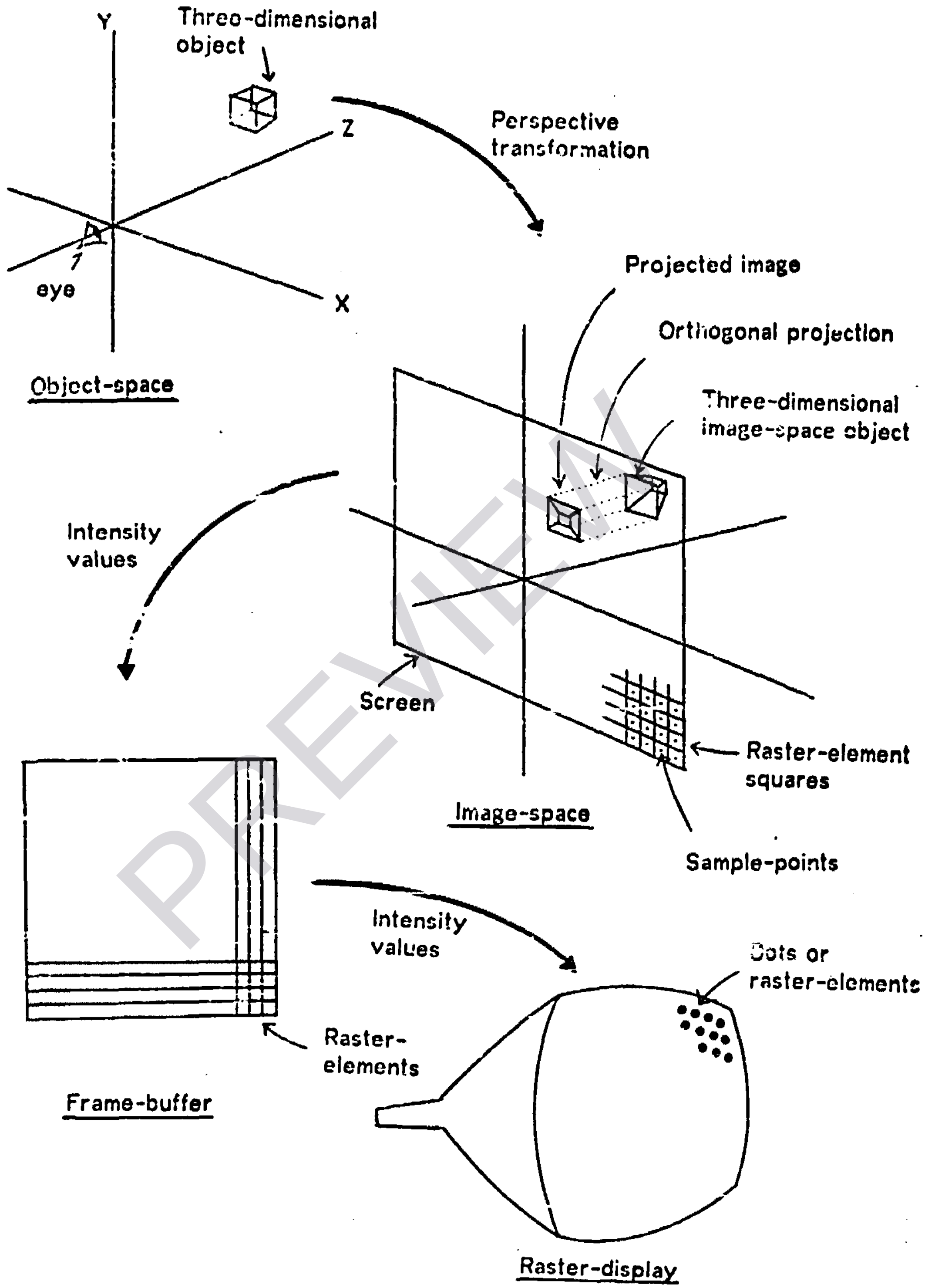

Figure 2-1

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hidden-surface problem. The orthogonal projection of the image-space object onto the x-y plane is called the "projected image." That part of the x-y plane which will be associated with the raster is called the "screen."

We must define the relationship between the image-space and the raster in order to transfer information from the projected image to the raster. Recall that the screen is the portion of the x-y plane of the image-space that corresponds to the raster. The area of the screen is divided into small squares called "raster-element squares." There is, of course, a one-to-one correspondence between raster-element squares and raster elements. The center of each raster-element square will be called a "sample-point." A diagram depicting the relationships of the above terms is shown in figure 2-1.

THE SUBDIVISION ALGORITHM

The algorithm for establishing the correspondence between a patch and the raster-elements will now be presented. The algorithm, hereafter called the "subdivision algorithm," works for either patches or segments of patches, called "subpatches." Figure 2-2 illustrates a portion of the screen where the dots represent the sample-points. (The outlines of the raster-element squares are not shown.) The curved lines represent the edges of a projected patch. Even though only the projection is shown, we assume that enough information about the patch is maintained so that the light intensity for any location on the patch can be calculated.

A statement of the algorithm is:

If the patch (subpatch) is small enough so that its projection covers only

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> one sample-point, then compute the intensity of the patch and write it into the corresponding element of the frame-buffer; otherwise, subdivide the patch into smaller subpatches; and repeat the process for each subpatch.

Figure 2-3 shows a patch subdivided into four subpatches where most of the subpatches still cover more than one sample-point. In figure 2-4 the subpatches that are too large are again subdivided. Subdivision continues until no subpatch covers more than one sample-point.

Readers familiar with other computer-generated shaded-picture efforts will recognize a similarity between the method presented here and Warnock's hidden surface algorithm [9]. Warnock solved the hidden surface problem for polygons by recursively subdividing the screen space into successively smaller sections until all questions about the ordering of polygons left in a section were easy to answer. Warnock's algorithm differs from the one presented here in that the former subdivides the screen, while the latter subdivides the surface being rendered.

The patch subdivision algorithm as stated is very simple but some questions remain: How is the subdivision process terminated? What if a patch covers no sample-points? What if part of the patch intersects the edge of the screen or is behind the eye? How many times must a patch be subdivided? Finally, what kinds of problems does the discrete sampling introduce? Each of these issues will be discussed in turn.

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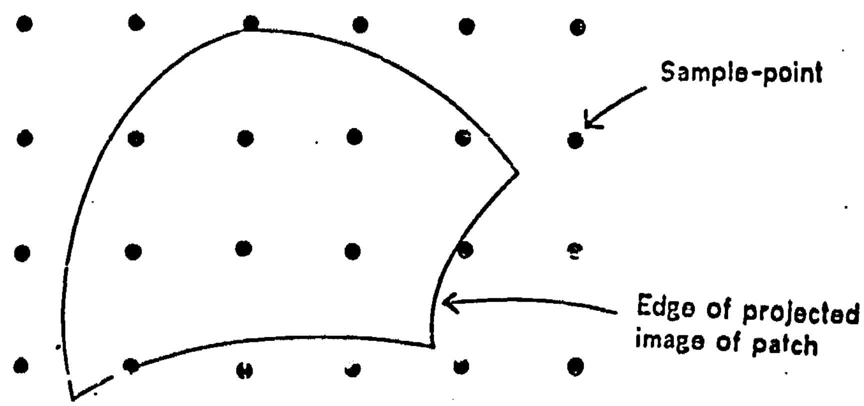

Figure 2-2

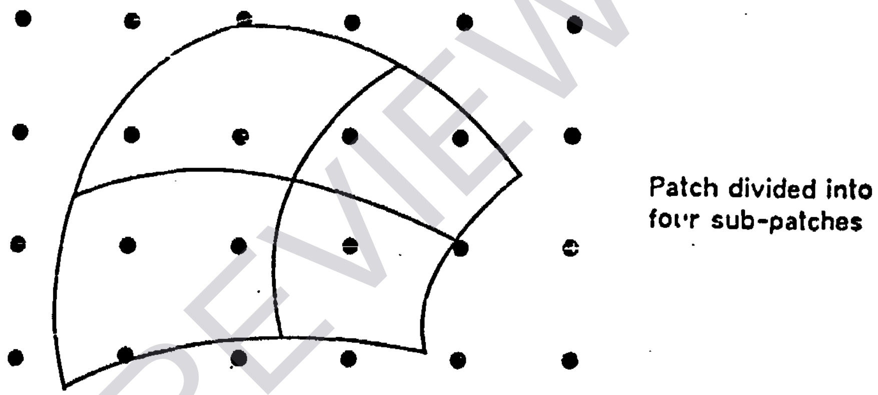

Figure 2-3

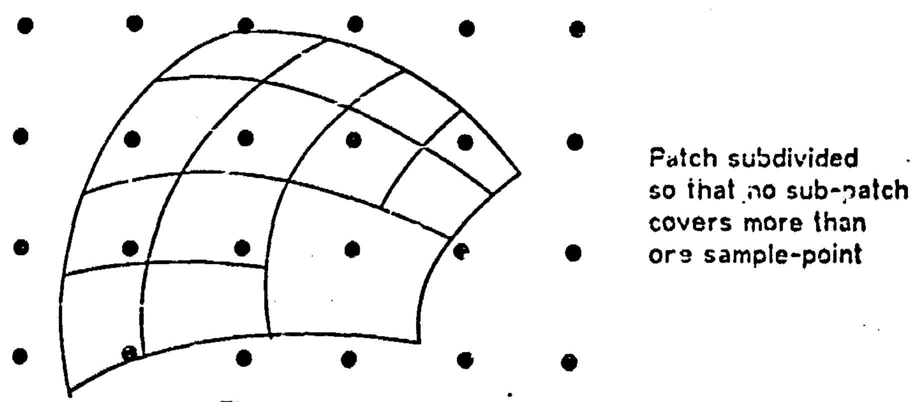

Figure 2-4

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