Universidade de São Paulo
Instituto de Física de São Carlos
Mathematical-Computational Modeling

Direction Field

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1 Introduction

In order to better understand various physical phenomena in nature, the study of differential equations is essential for this. However, some become very complicated and need computational methods to visually analyze their properties.

2 Methods

The directional fields for the Malthusian, logistic and Lotka-Volterra model will be seen. In addition, solutions will be made using the Euler method. Below are the 3 types of models.

- Malthusian

$$\frac{dx}{dt} = rx \tag{1}$$

- Logistic

$$\frac{dx}{dt} = ax(1 - \frac{x}{N}) \tag{2}$$

- Lotka-Voltera

$$\begin{cases} \dot{x} = \alpha x - \beta xy \\ \dot{y} = -\gamma y + \delta xy \end{cases}$$

For simplicity, all constants have been set to 1 (r=N=$\alpha = \beta = \gamma = \delta = 1$).

3 Direction Fields

In each subsection, one of the equations mentioned above will be addressed.

3.1 Equation 1

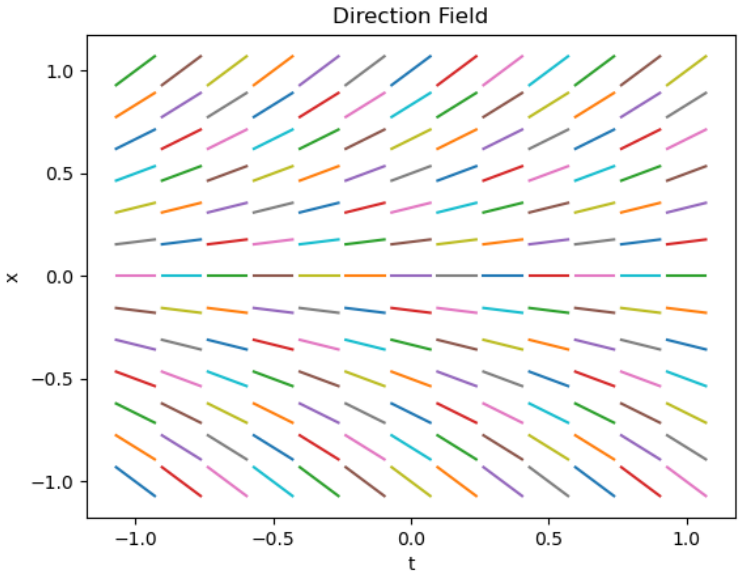

Figure 1

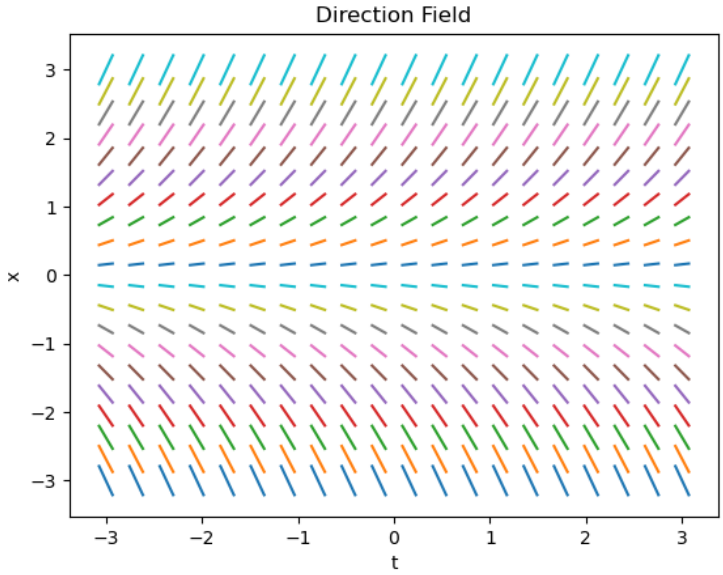

Figure 2

3.2 Equation 2

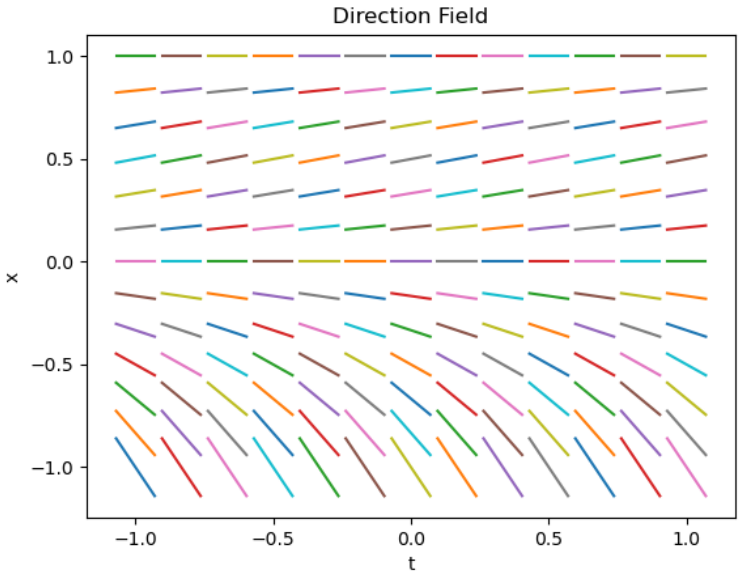

Figure 3

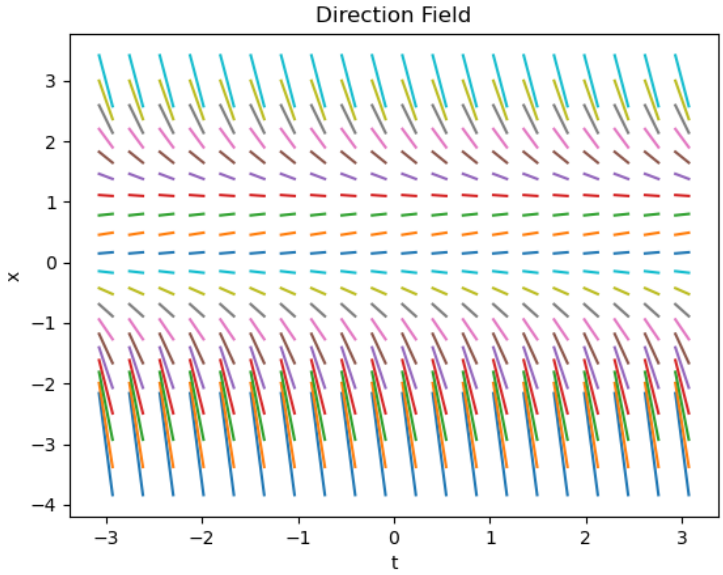

Figure 4

3.3 System of ODEs

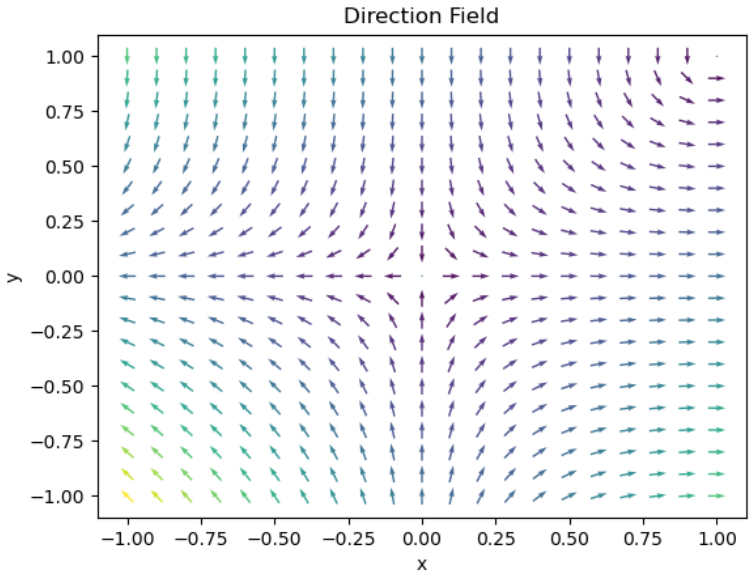

Figure 5

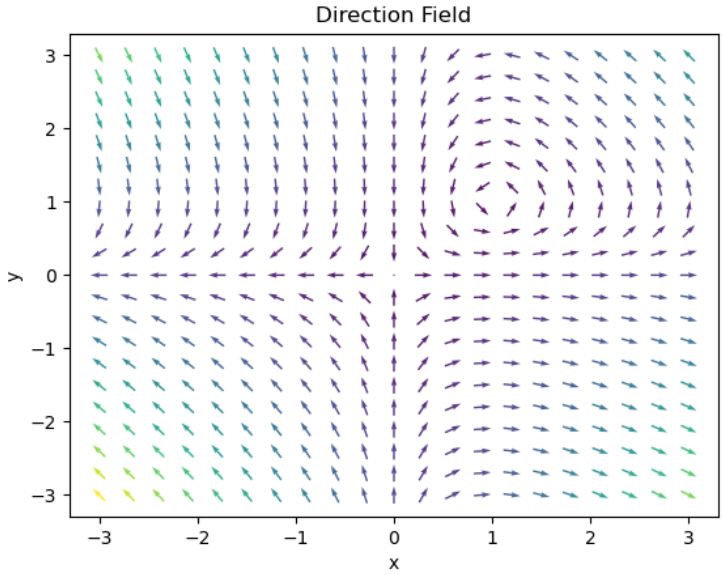

Figure 6

4 Euler Method

For this part of the study, a python code was implemented that leaves the choice of an initial condition free for the user. For that, the directional field visualization was made available.

4.1 Equation 1

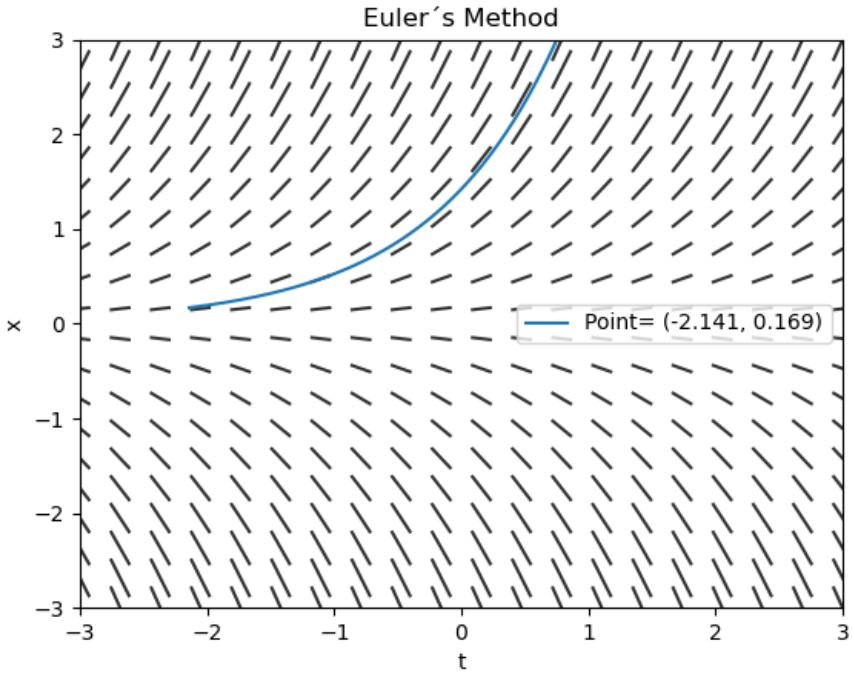

Figure 7

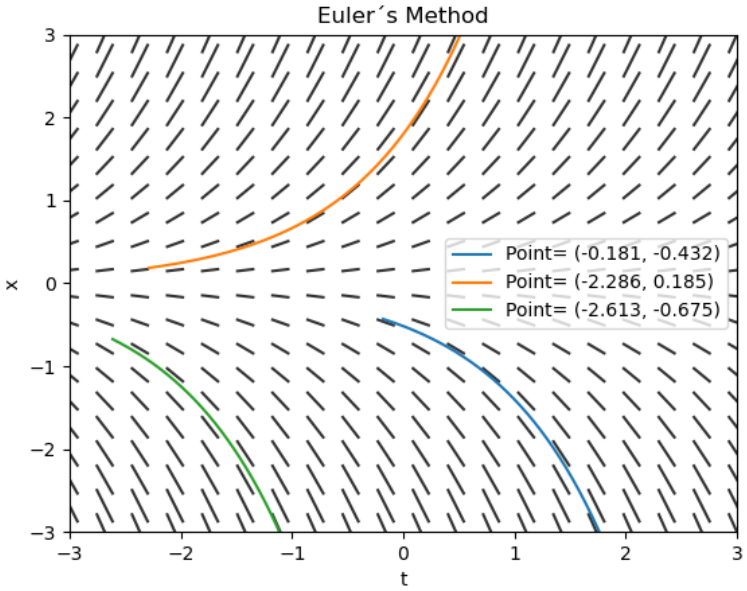

Figure 8

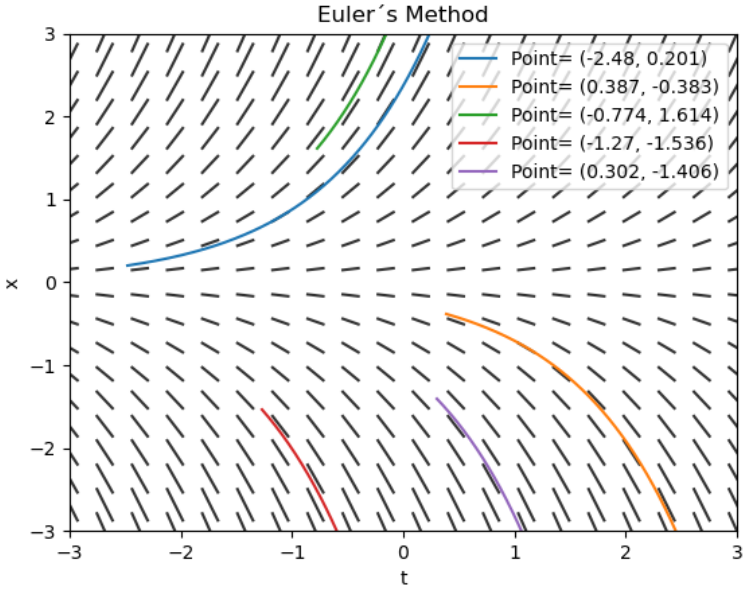

Figure 9

4.2 Equation 2

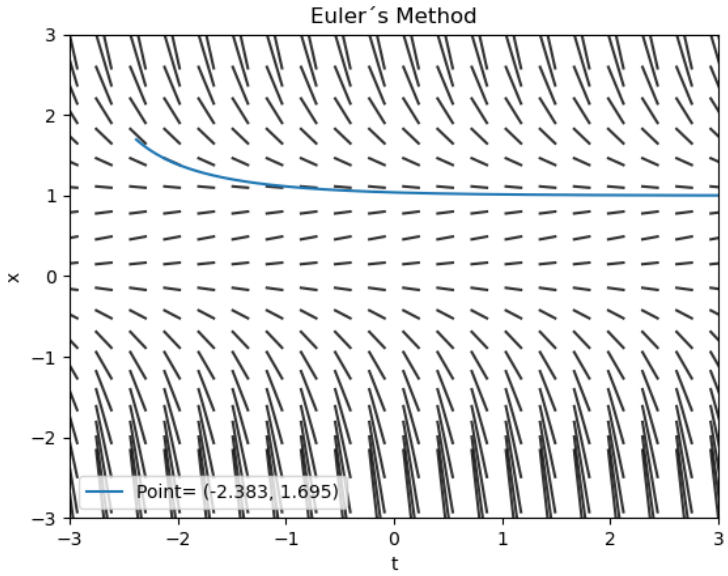

Figure 10

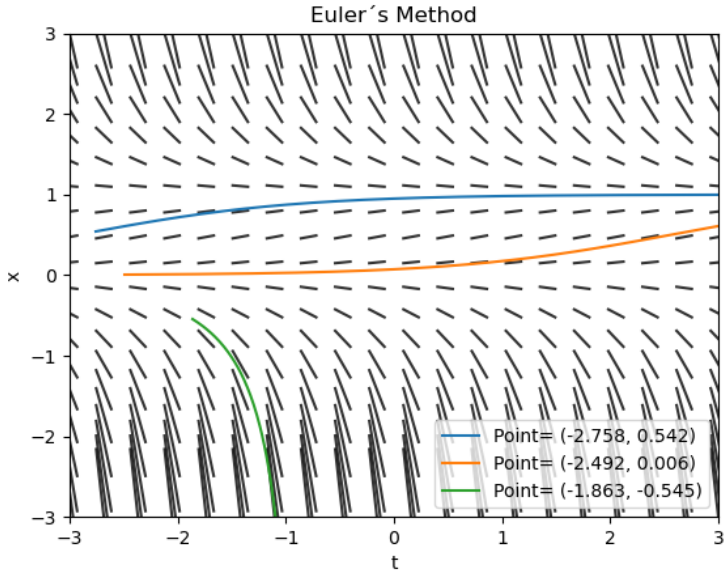

Figure 11

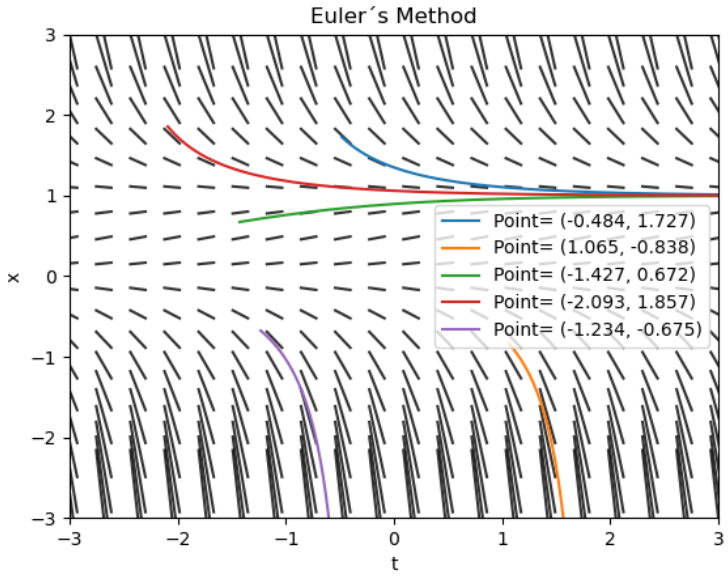

Figure 12

4.3 System of ODEs

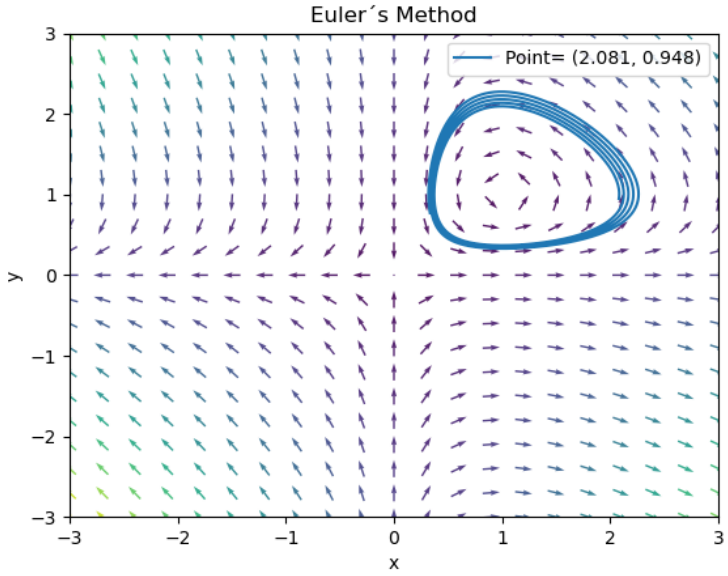

Figure 13

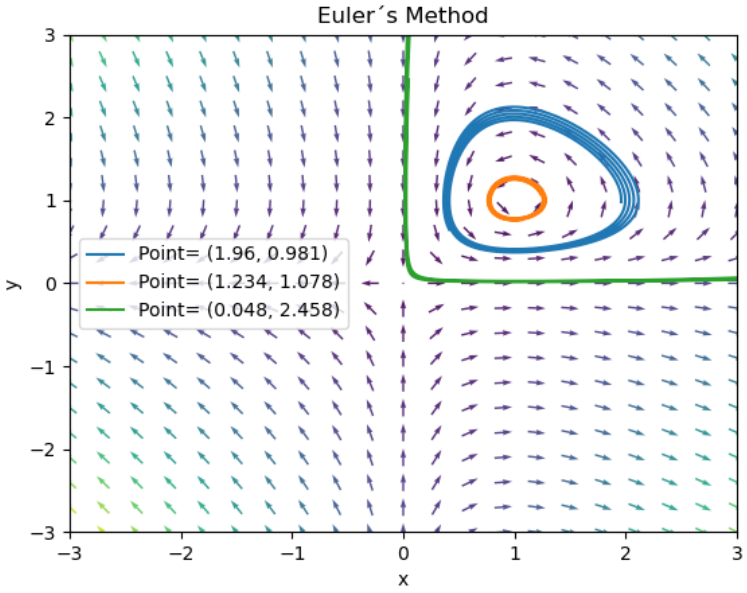

Figure 14

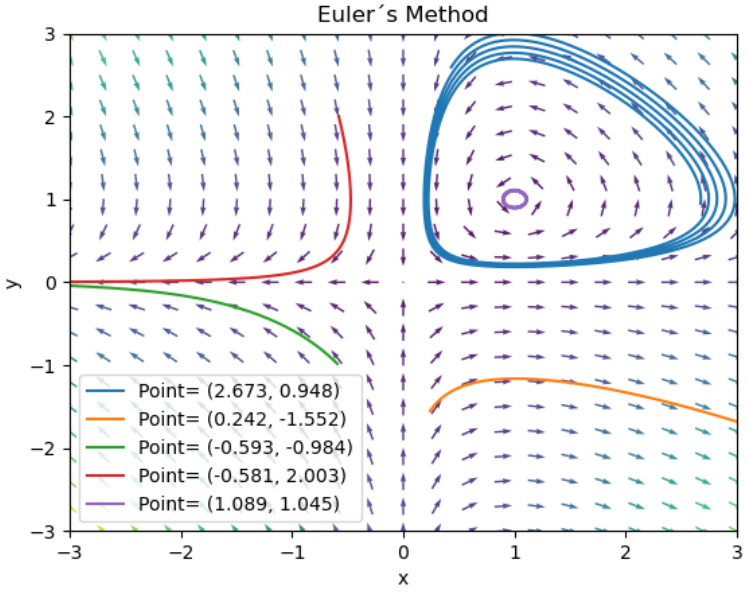

Figure 15

5 References

[1] Costa, Luciano da Fontoura. Didatic's Texts to learn about the methods abord (CDT's).

[2] da Silva, Éverton Luís Mendes. Python programs to do the calculations and graphs. https://github.com/evertonmendes/Mathematical-Computational-Modeling