Universidade de São Paulo
Instituto de Física de São Carlos
Mathematical-Computational
Modeling

Minimum of a Scalar Field

Éverton Luís Mendes da Silva (10728171)

1 Introduction

In this project, computational methods will be approached to find minimum points in a scalar field. For this, three types of optimization were used: 'Random Search', 'Gradient Descent' and 'Simulated Annealing'.

2 Scalar Fields

The two scalar fields used for the methods are represented in the equations below.

$$\Phi_1(x, y) = x^2 + y^2 \tag{1}$$

$$\Phi_2(x, y) = -e^{-((x-1)^2+(y1)^2)/0.3^2} - 5e^{-16*((x-2)^2(y-2)^2)} \tag{2}$$

3 Analysis of Φ_1

In order to better understand the field, a plot of its level curves was made.

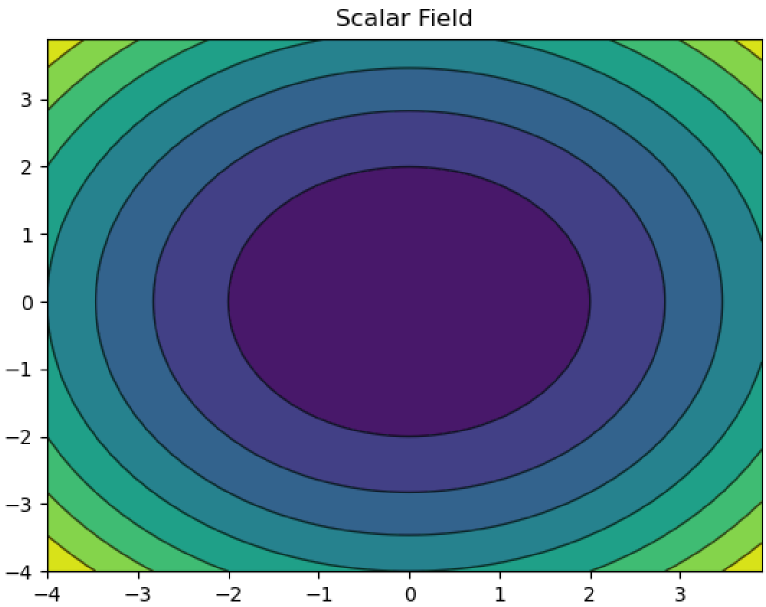

Figure 1: Level Curves

It is also possible to see the gradient of this equation in an interval.

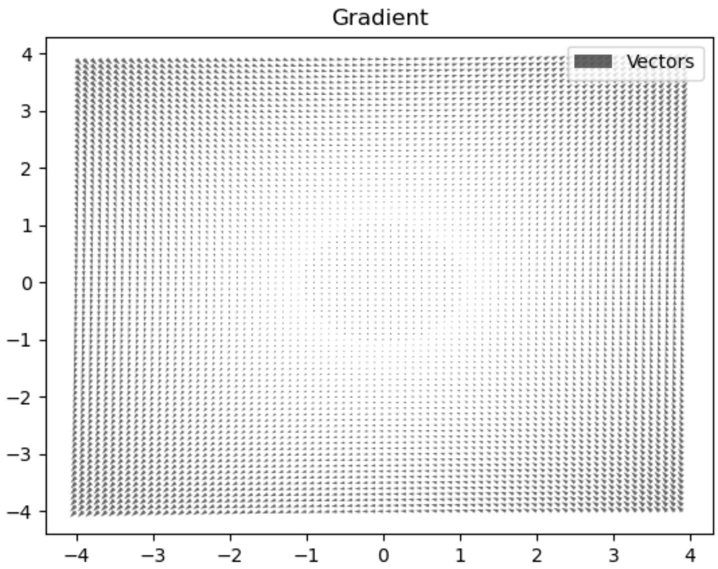

Figure 2

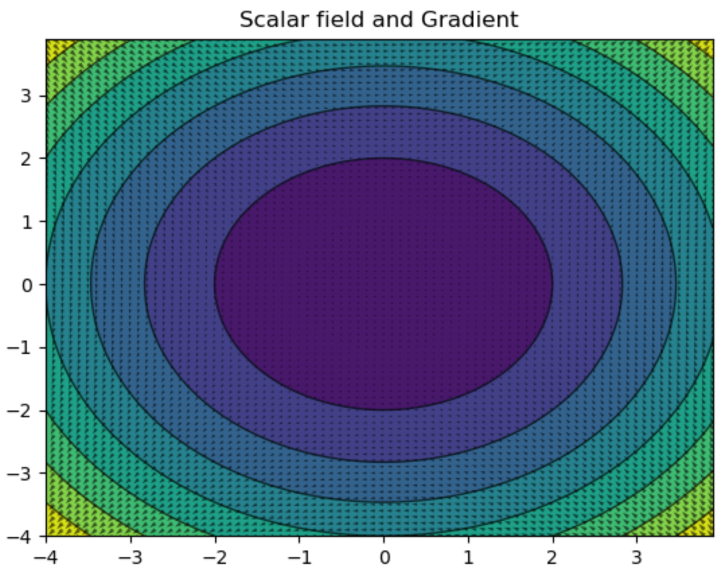

Figure 3

The next two images are the 'Random Search' and 'Gradient Descent' minimizations, respectively.

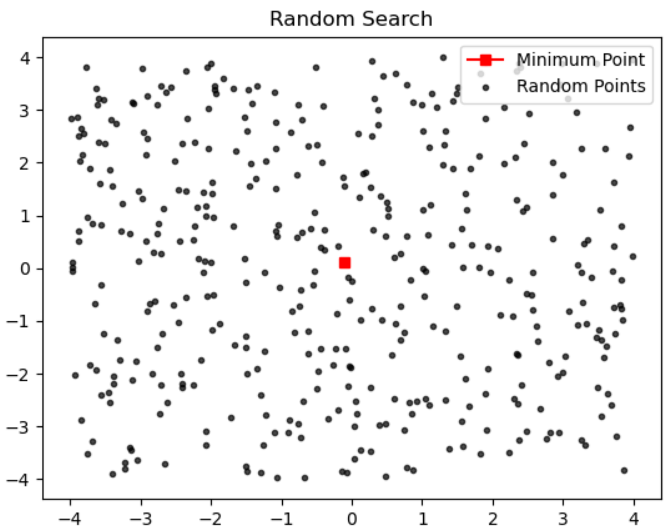

Figure 4: Average=0.2510, Deviation= 1.3898

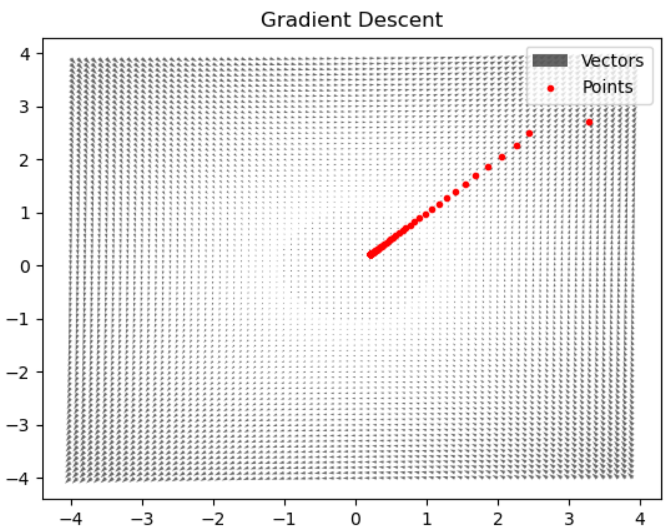

Figure 5:Average=0.3193, Deviation=1.3898

The next step is to analyze the minimum curves between the two methods, considering several starting points.

$$\text{Points} \rightarrow \begin{cases} A \rightarrow (x, y) = (2, 2) \\ B \rightarrow (x, y) = (-2, 1) \\ C \rightarrow (x, y) = (3, -3.2) \\ D \rightarrow (x, y) = (1, -2) \\ E \rightarrow (x, y) = (0, 3) \end{cases}$$

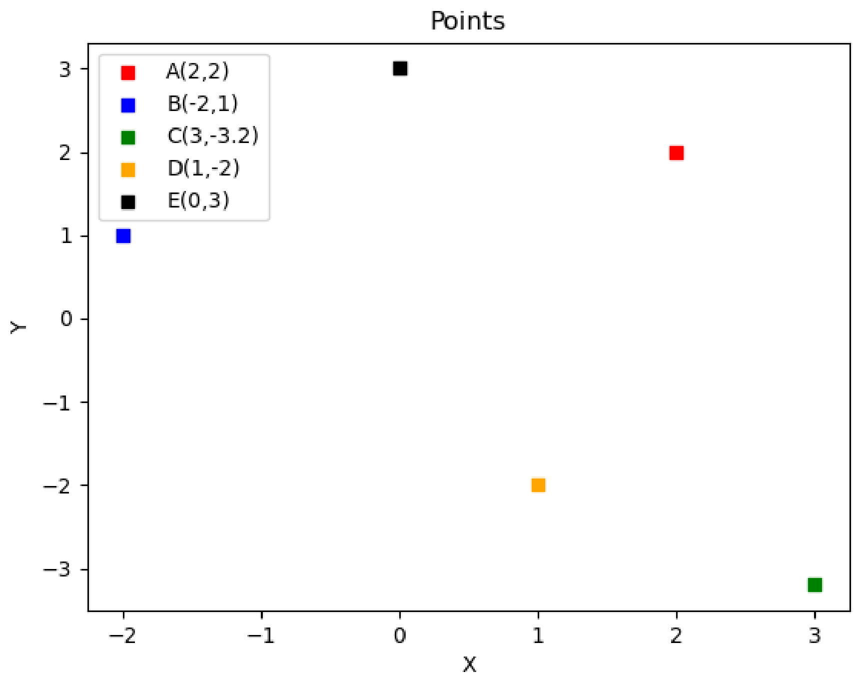

Figure 6

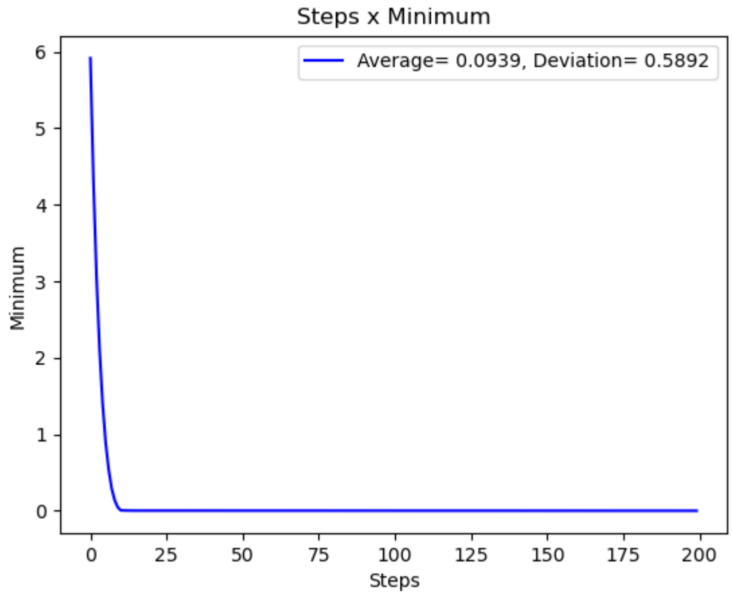

Figure 7: Gradient Descent of A

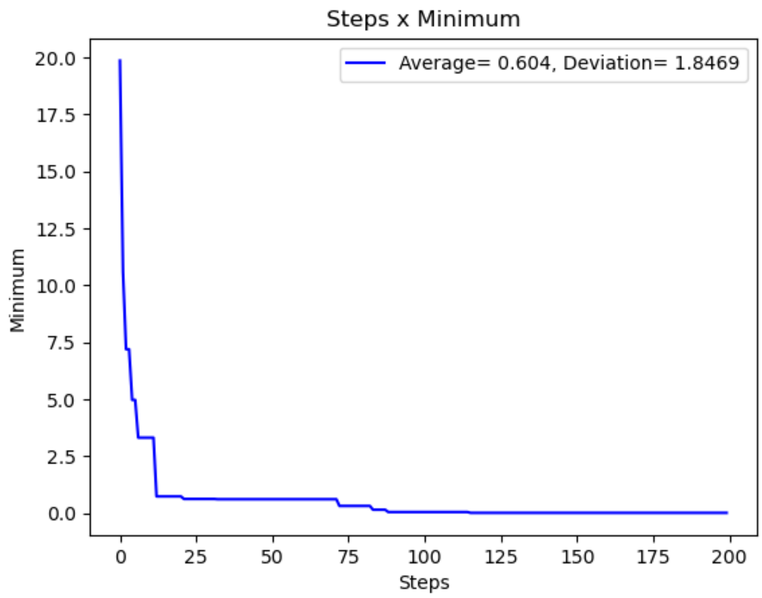

Figure 8: Random Search of A

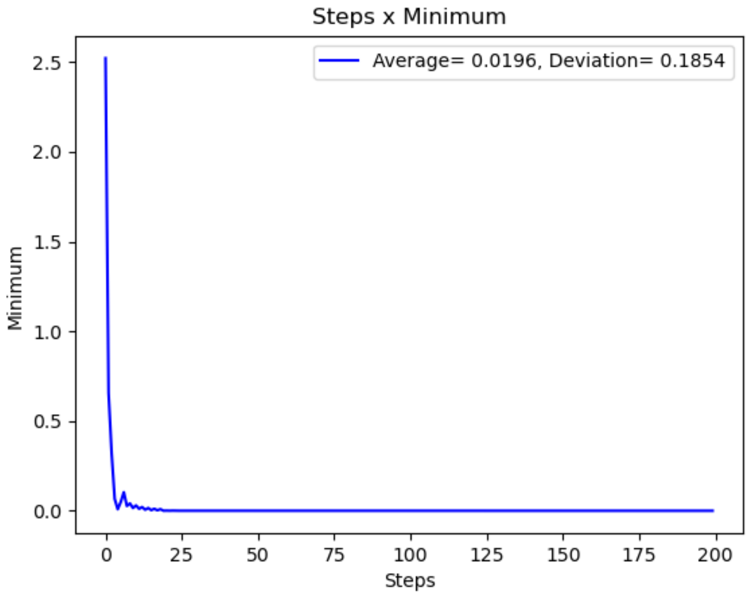

Figure 9: Gradient Descent of B

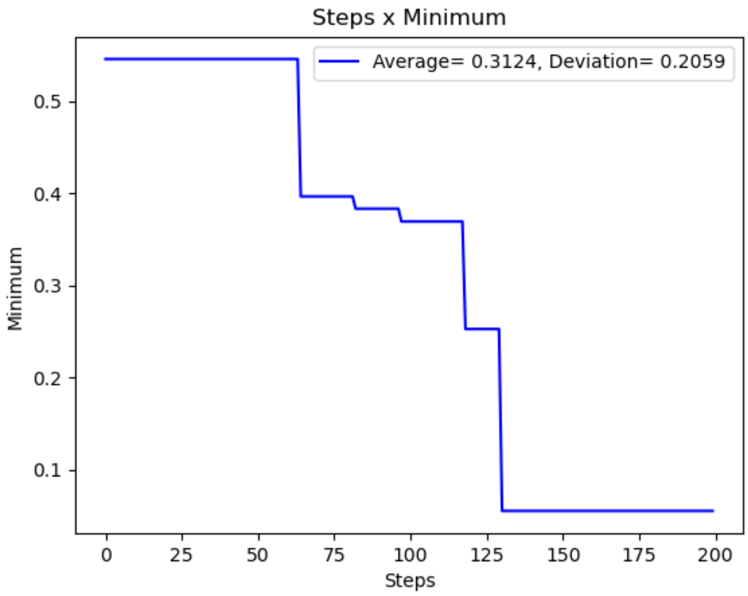

Figure 10: Random Search of B

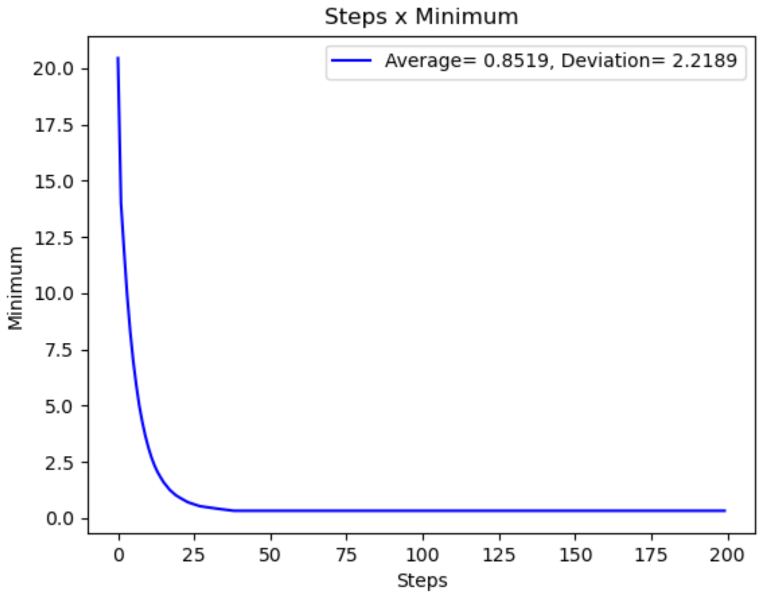

Figure 11: Gradient Descent of C

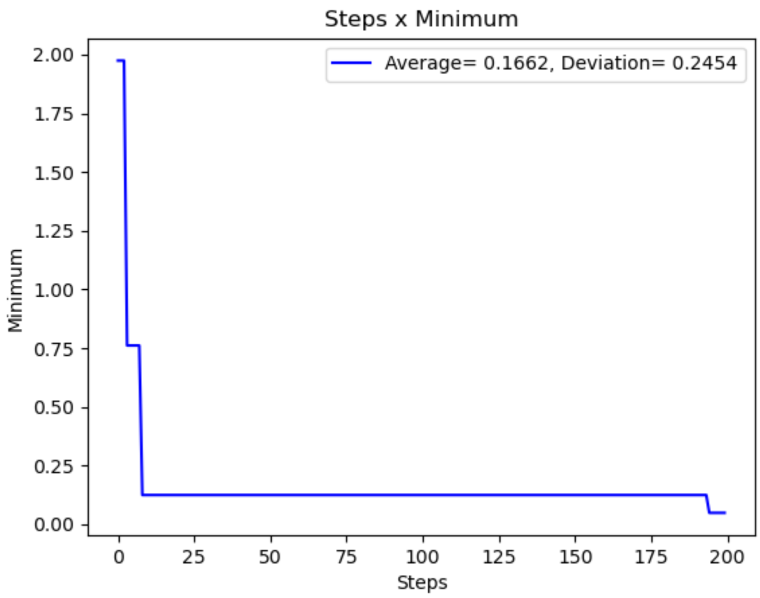

Figure 12: Random Search of C

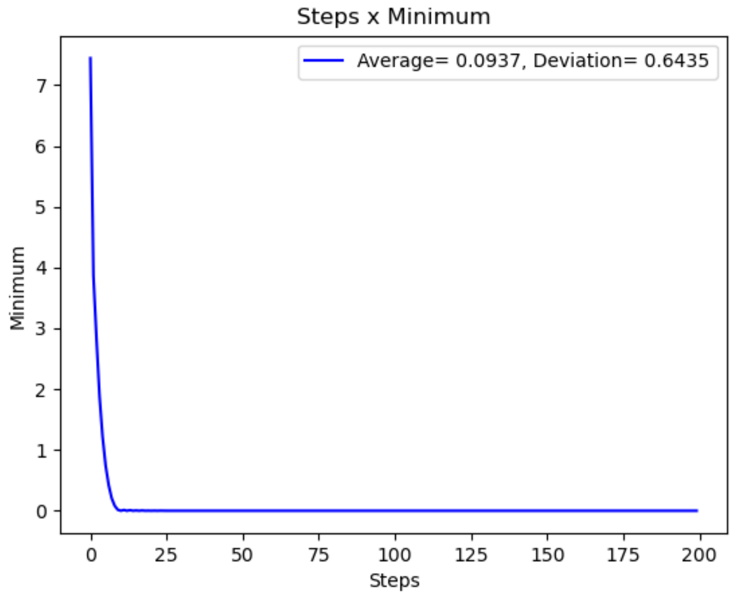

Figure 13: Gradient Descent of D

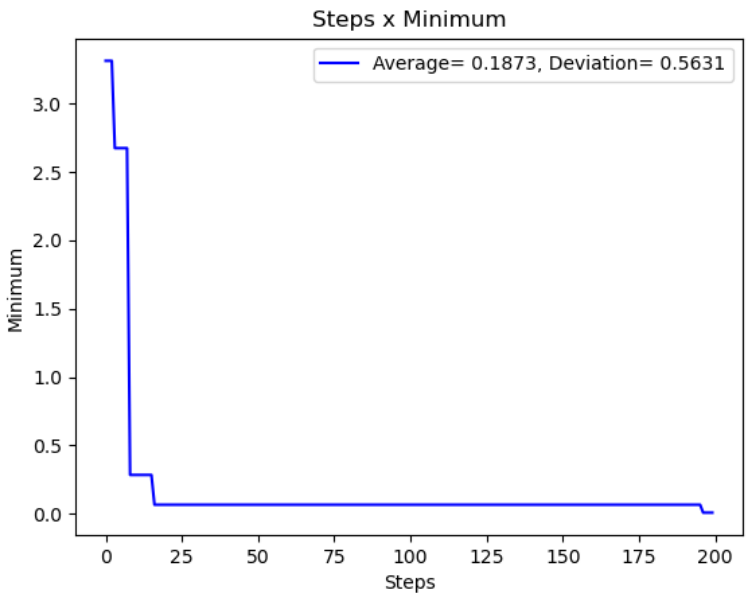

Figure 14: Random Search of D

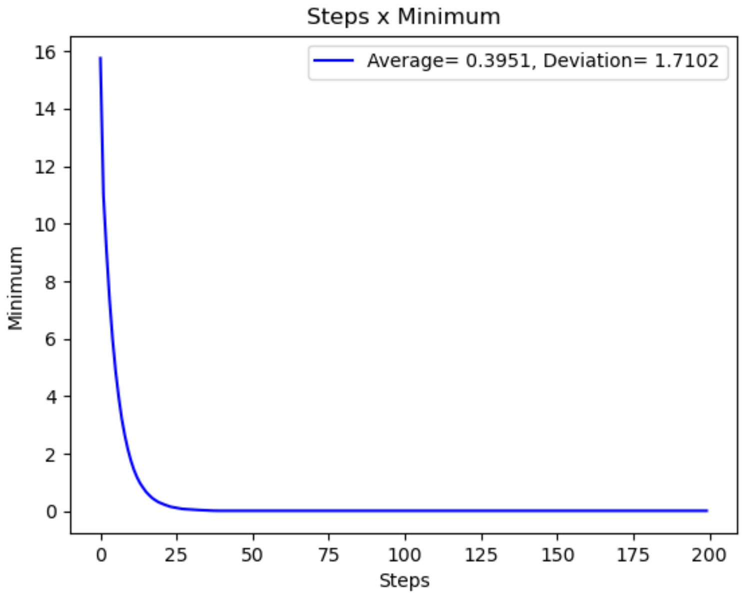

Figure 15: Gradient Descent of E

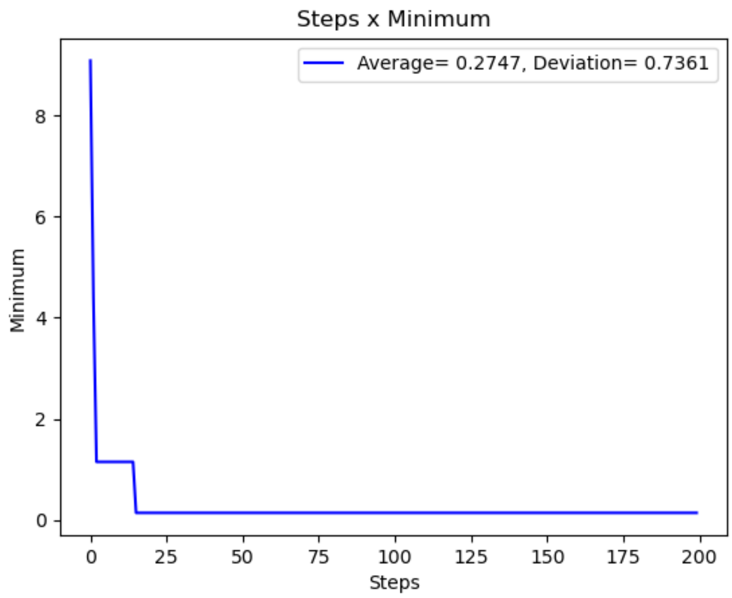

Figure 16: Random Search of E

4 Analysis of Φ_2

In order to better understand the field, a plot of its level curves was made.

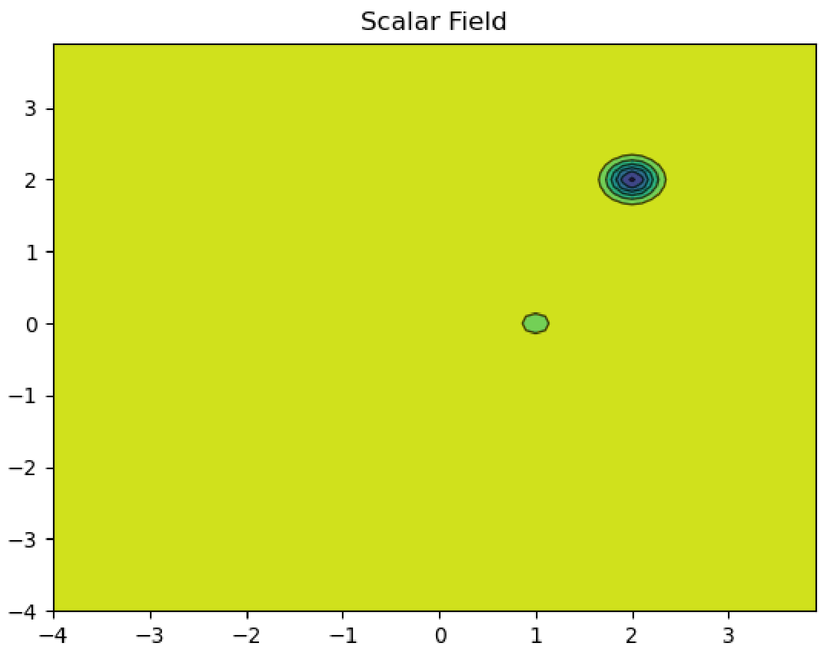

Figure 17: Level Curves

Following the same idea it is possible to transform the scalar field into a surface in R^3.

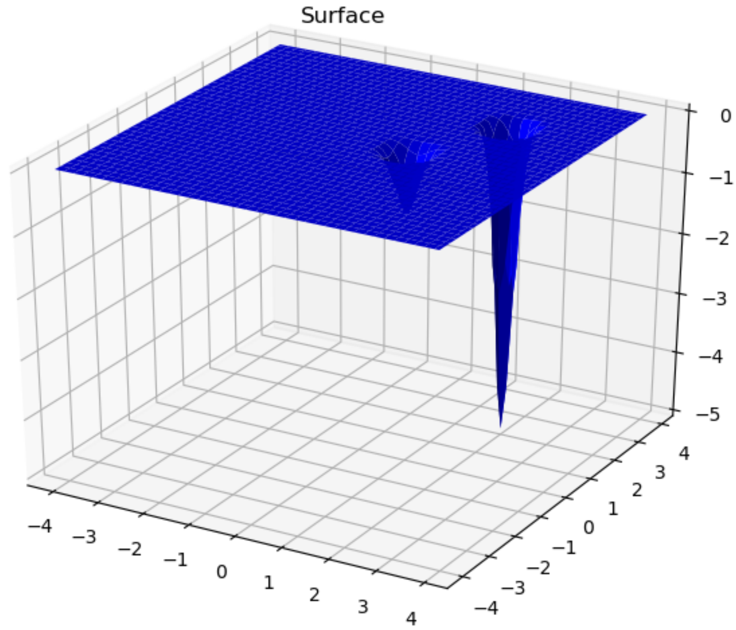

Figure 18

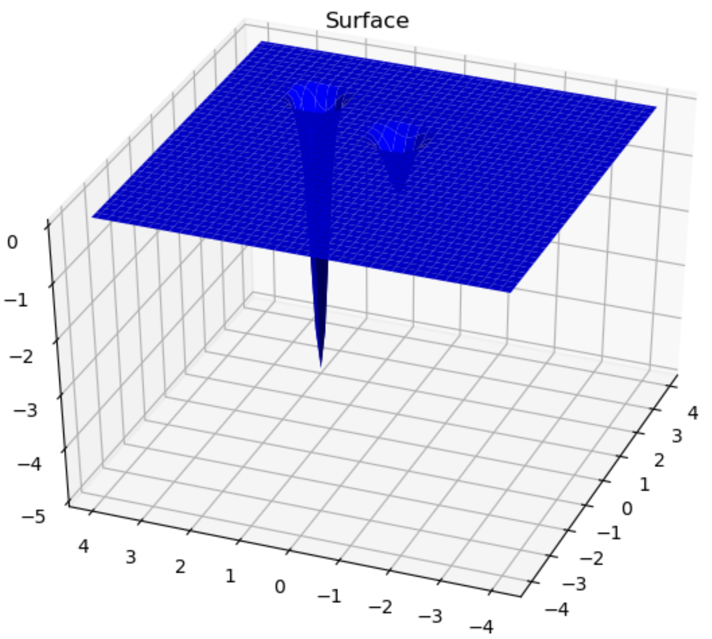

Figure 19

It is also possible to see the gradient of this equation in an interval.

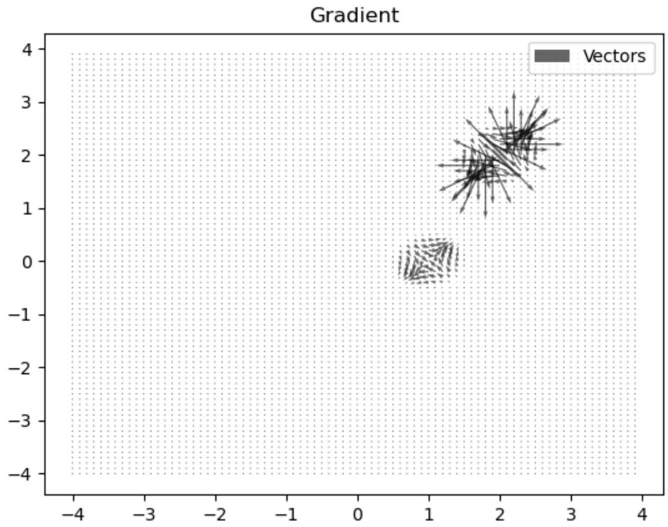

Figure 20

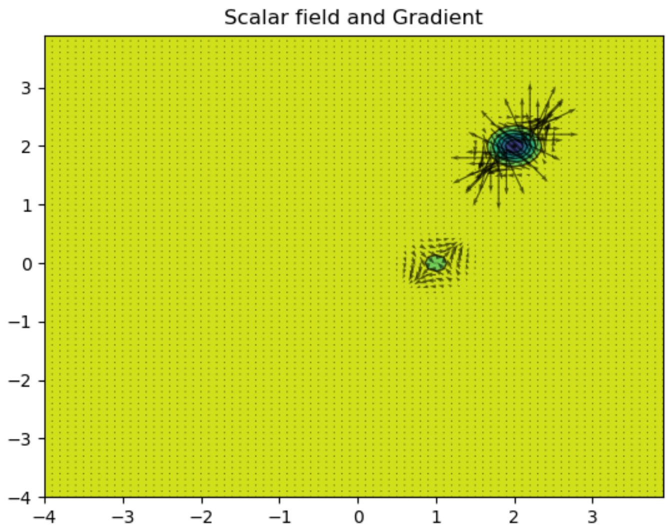

Figure 21

In addition, as we now have a more in-depth view of the scalar field, it is possible to find the minimum using the 'Simulated Annealing' method.

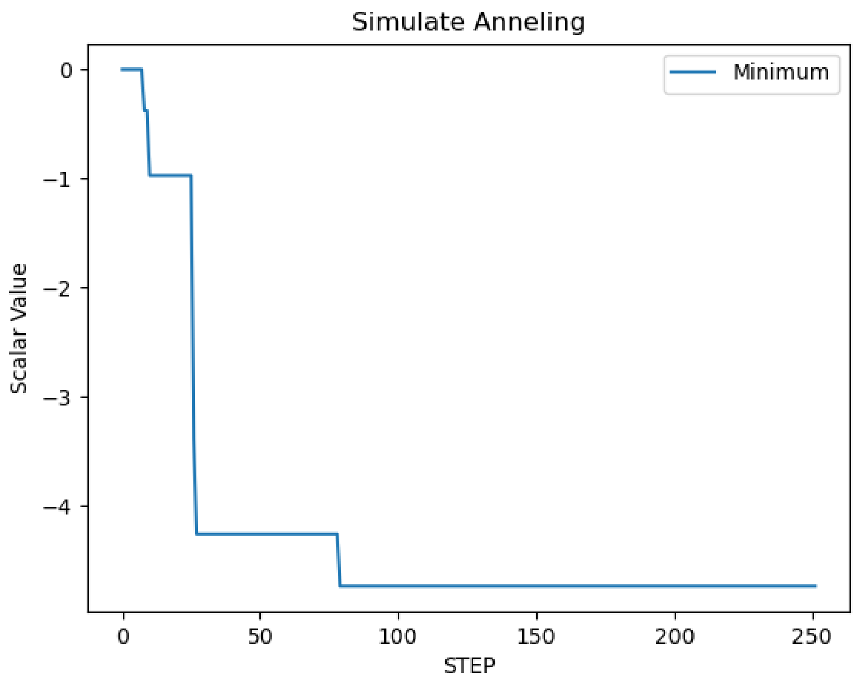

Figure 22: Temperature=50, cooling=0.9

In order to verify the efficiency, the minimum curves of this procedure were compared with the 'Random Search'.

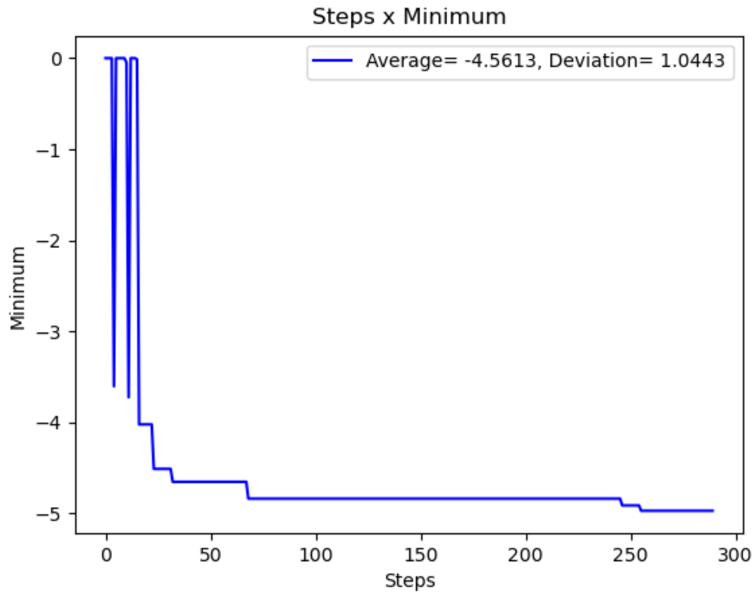

Figure 23: Simulated Annealing 1

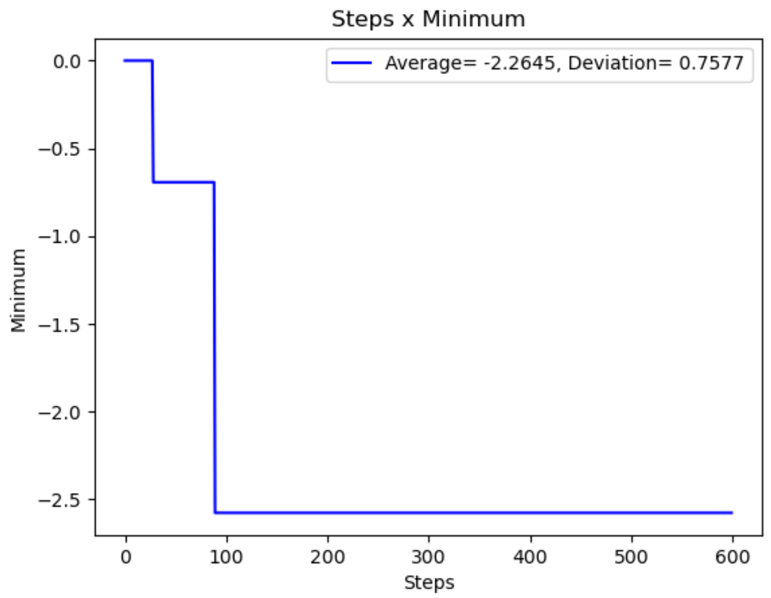

Figure 24: Random Search 1

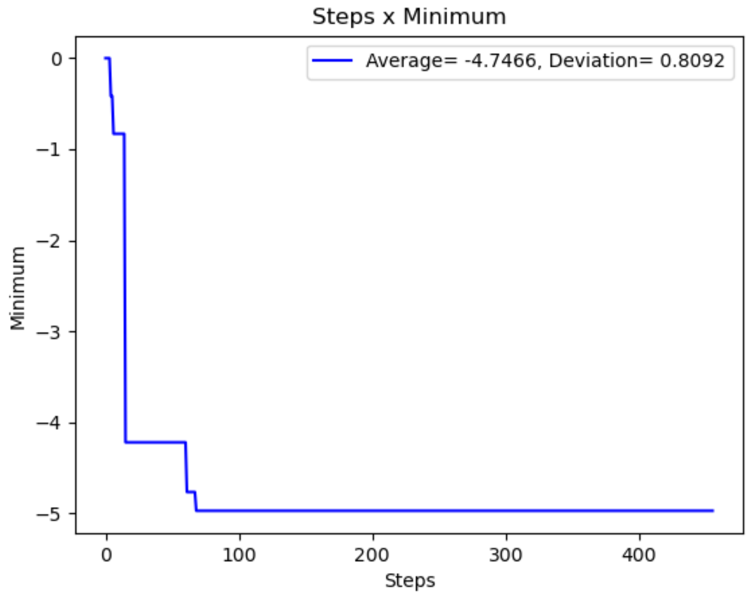

Figure 25:Simulated Annealing 2

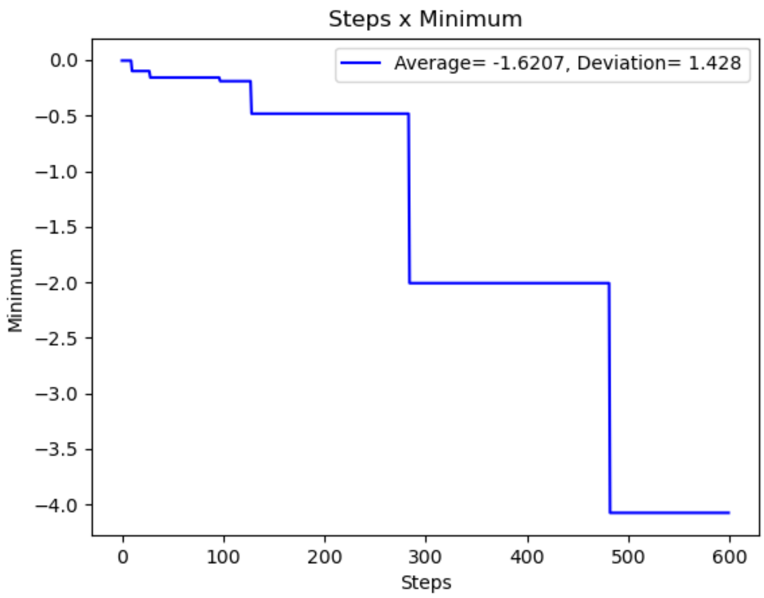

Figure 26: Random Search 2

5 References

[1] Costa, Luciano da Fontoura. Didatic's Texts to learn about the methods abord (CDT's).

[2] da Silva, Éverton Luís Mendes. Python programs to do the calculations and graphs. https://github.com/evertonmendes/Mathematical-Computational-Modeling