Exercises Week 5

Econometrics

1. Suppose that the idiosyncratic errors in the erros-component model, $V_{i,t}$, are serially uncorrelated with constant variance, σ_V^2. Show that the correlation between adjacent differences, $\Delta V_{i,t}$ and $\Delta V_{i,t+1}$ is -0.5.

   Therefore, under the ideal FE assumptions, first differencing induces negative serial correlation of a known value.

2. **Exercise 7.24 in ETM**: Another estimator for panel data that was not discussed in the lecture is the **between-groups** estimator. It is given by running OLS in the equation

$$\bar{Y}_i = \bar{X}_i\beta + \mu_i + \bar{V}_i,$$

   where $\bar{Z}_i = (1/T)\sum_{t=1}^{T} Z_{i,t}$ for $Z = Y, X, V$. Let μ_i have variance σ_μ^2 and the $V_{i,t}$ have variance σ_V^2. Given these assumptions, show that the variance of the error terms in regression above is $\sigma_\mu^2 + \sigma_V^2/T$.

   Use this development to obtain another estimate for the variances needed to estimate random effects.

   Hint: $\bar{Z}_i = P_D Z$, where D is the dummy variables matrix defined in the lecture.

3. For this exercise you are going to use the *Rental* dataset from the *wooldridge* package. The data contains information on rental prices and other variables for college towns are for the years 1980 and 1990. The idea is to see whether a stronger presence of students affects rental rates. The unobserved effects model is

$$\log(rent_{it}) = \beta_0 + \delta_0 y90_t + \beta_1 \log(pop_{it}) + \beta_2 \log(avginc_{it}) + \beta_3 pctsu_{it} + a_i + u_{it},$$

   where $y90_t$ is a dummy for year 90, *pop* is city population, *avginc* is average income, and *pctstu* is student population as a percentage of city population (during the school year).

   (a) Estimate the equation by pooled OLS and report the results in standard form. What do you make of the estimate on the 1990 dummy variable? What do you get for $\hat{\beta}_3$?

   (b) Now, difference the equation and estimate by OLS. Compare your estimate of β_{pctstu} with that from part (a). Does the relative size of the student population appear to affect rental prices?

   (c) Estimate the model by fixed effects to verify that you get identical estimates and standard errors to those in part (c).