

Exercises Week 11

Econometrics

1. You are going to show that the GMM estimator is equivalent to 3SLS. That is, show that

$$(P_W X_{\bullet})^T (\Sigma^{-1} \otimes I) = X_{\bullet}^T (\Sigma^{-1} \otimes P_W),$$

where $P_W X_{\bullet}$ is a block-diagonal matrix with blocks given by $P_W X_i$.

Hint: Write $P_W X_{\bullet}$ as a Kroenecker product.

2. **Exercises 12.29-30 in ETM:** Consider the demand-supply model

$$q_t = \beta_{1,1} + \beta_{2,1}x_{t2} + \beta_{3,1}x_{t3} + \gamma_{21}p_t + u_{t1},$$

$$q_t = \beta_{1,2} + \beta_{4,2}x_{t4} + \beta_{5,2}x_{t5} + \gamma_{22}p_t + u_{t2},$$

where q_t is the log of quantity, p_t is the log of price, x_{t2} is the log of income, x_{t3} is a dummy variable that accounts for regular demand shifts, and x_{t4} and x_{t5} are the prices of inputs. Thus the first equation is a demand function and the second equation is a supply function.

- (a) For this model, precisely what is the vector β_{\bullet} defined in the lecture?
- (b) How many overidentifying restrictions are there for each equation?
- (c) The file *demand_supply.csv* contains 120 observations generated by the model. Estimate this model by 2SLS.
- (d) Test the overidentifying restrictions for each equation.
- (e) Now estimate the model using 3SLS.

3. The dataset *smoke* in the *wooldridge* package contains information on smoking habits and other variables for a random sample of single adults from the United States. It contains 807 observations in 10 variables:

-educ: years of schooling

-cigpric: state cig. price, cents/pack

-white: 1 if white

-age: in years

-income: annual income

-cigs: cigs. smoked per day

-restaurn: 1 if restaurant smk. restrictions

-lincome: log(income)

-agesq: age squared

-lcigpric: log(cigprice)

We use the data to estimate a demand function for daily cigarette consumption.

- (a) A model to estimate the effects of smoking on annual income (perhaps through lost work days due to illness, or productivity effects) is

$$\log(\text{income}) = \beta_0 + \beta_1 \text{cigs} + \beta_2 \text{educ} + \beta_3 \text{age} + \beta_4 \text{age}^2 + u_1.$$

How do you interpret β_1 ?

- (b) To reflect the fact that cigarette consumption might be jointly determined with income, a demand for cigarettes equation is

$$\text{cigs} = \gamma_0 + \gamma_1 \log(\text{income}) + \gamma_2 \text{educ} + \gamma_3 \text{age} + \gamma_4 \text{age}^2 + \gamma_5 \log(\text{cigpric}) + \gamma_6 \text{restaurn} + u_2.$$

Assuming these are exogenous to the individual, what signs would you expect for γ_5 and γ_6 ?

- (c) Estimate the income equation by OLS and discuss the estimate of β_1 .
- (d) Estimate the reduced form for cigs . (Recall that this entails regressing cigs on all exogenous variables.) Are $\log(\text{cigpric})$ and restaurn significant in the reduced form?
- (e) Now, estimate the income equation by 2SLS. Discuss how the estimate of β_1 compares with the OLS estimate.
- (f) Do you think that cigarette prices and restaurant smoking restrictions are exogenous in the income equation?