Exercise 2.3 (Asset Pricing – Empirical)

In the recent finance literature it is suggested that asset prices are fairly well described by a so-called factor model, where excess returns are linearly explained from excess returns on a number of 'factor portfolios'. As in the CAPM, the intercept term should be zero, just like the coefficient for any other variable included in the model the value of which is known in advance (e.g. a January dummy). The data set ASSETS2 contains excess returns on four factor portfolios for January 1960 to December 2002:[26]

rmrf:	excess return on a value-weighted market proxy
smb:	return on a small-stock portfolio minus the return on a large-stock portfolio (Small minus Big)
hml:	return on a value-stock portfolio minus the return on a growth-stock portfolio (High minus Low)
umd:	return on a high prior return portfolio minus the return on a low prior return portfolio (Up minus Down)

All data are for the USA. Each of the last three variables denotes the difference in returns on two hypothetical portfolios of stocks. These portfolios are re-formed each month on the basis of the most recent available information on firm size, book-to-market value of equity and historical returns, respectively. The *hml* factor is based on the ratio of book value to market value of equity, and reflects the difference in returns between a portfolio of stocks with a high book-to-market ratio (value stocks) and a portfolio of stocks with a low book-to-market ratio (growth stocks). The factors are motivated by empirically found anomalies of the CAPM (for example, small firms appear to have higher returns than large ones, even after the CAPM risk correction).

In addition to the excess returns on these four factors, we have observations on the returns on ten different 'assets' which are ten portfolios of stocks, maintained by the Center for Research in Security Prices (CRSP). These portfolios are size-based, which means that portfolio 1 contains the 10% smallest firms listed at the New York Stock Exchange and portfolio 10 contains the 10% largest firms that are listed. Excess returns (in excess of the riskfree rate) on these portfolios are denoted by $r1$ to $r10$, respectively.

In answering the following questions use $r1$, $r10$ and the returns on two additional portfolios that you select.

- **a.** Regress the excess returns on your four portfolios upon the excess return on the market portfolio (proxy), noting that this corresponds to the CAPM. Include a constant in these regressions.
- **b.** Give an economic interpretation of the estimated β coefficients.
- **c.** Give an economic and a statistical interpretation of the R^2s.
- **d.** Test the hypothesis that $\beta_j = 1$ for each of the four portfolios. State the assumptions you need to make for the tests to be (asymptotically) valid.
- **e.** Test the validity of the CAPM by testing whether the constant terms in the four regressions are zero.

[26] All data for this exercise are taken from the website of Kenneth French; see http://mba.tuck/dartmouth.edu/pages/faculty/ken.french.

f. Test for a January effect in each of the four regressions.

g. Next, estimate the four-factor model

$$r_{jt} = \alpha_j + \beta_{j1} rmrf_t + \beta_{j2} smb_t + \beta_{j3} hml_t + \beta_{j4} umd_t + \varepsilon_{jt}$$

by OLS. Compare the estimation results with those obtained from the one-factor (CAPM) model. Pay attention to the estimated partial slope coefficients and the R^2s.

h. Perform F-tests for the hypothesis that the coefficients for the three new factors are jointly equal to zero.

i. Test the validity of the four-factor model by testing whether the constant terms in the four regressions are zero. Compare your conclusions with those obtained from the CAPM.

Exercise 2.4 (Regression – True or False?)

Carefully read the following statements. Are they true or false? Explain.

a. Under the Gauss–Markov conditions, OLS can be shown to be BLUE. The phrase 'linear' in this acronym refers to the fact that we are estimating a linear model.

b. In order to apply a t-test, the Gauss–Markov conditions are strictly required.

c. A regression of the OLS residual upon the regressors included in the model by construction yields an R^2 of zero.

d. The hypothesis that the OLS estimator is equal to zero can be tested by means of a t-test.

e. From asymptotic theory, we learn that – under appropriate conditions – the error terms in a regression model will be approximately normally distributed if the sample size is sufficiently large.

f. If the absolute t-value of a coefficient is smaller than 1.96, we accept the null hypothesis that the coefficient is zero, with 95% confidence.

g. Because OLS provides the *best* linear approximation of a variable y from a set of regressors, OLS also gives *best* linear unbiased estimators for the coefficients of these regressors.

h. If a variable in a model is significant at the 10% level, it is also significant at the 5% level.