

# Exercises Week 1

## Econometrics

1. **Ex. 2.6 in ETM:** Prove that, if the  $k$  columns of  $X$  are linearly independent, each vector  $z$  in  $\mathcal{S}(X)$  can be expressed as  $Xb$  for one and only one  $k$ -vector  $b$ . Hint: Suppose that there are two different vectors,  $b_1$  and  $b_2$ , such that  $z = Xb_i$ ,  $i = 1, 2$ , and show that this implies that the columns of  $X$  are linearly dependent.
2. **Ex. 2.9 in ETM:** Prove algebraically that  $P_X M_X = O$ . Use only the definition that  $P_X + M_X = I$ , and the idempotency of  $P_X$ .
3. **Ex. 3.1 in ETM:** Generate a sample of size 25 from the model

$$y_t = \beta_1 + \beta_2 y_{t-1} + u_t,$$

with  $\beta_1 = 1$  and  $\beta_2 = 0.8$ . For simplicity, assume that  $y_0 = 0$  and that the  $u_t$  are  $NID(0, 1)$ . Use this sample to compute the OLS estimates  $\hat{\beta}_1$  and  $\hat{\beta}_2$ . Repeat at least 100 times, and find the averages of the  $\hat{\beta}_1$  and  $\hat{\beta}_2$ . Use these averages to estimate the bias of the OLS estimators of  $\beta_1$  and  $\beta_2$ .

Repeat this exercise for sample sizes of 50, 100, and 200. What happens to the bias of  $\hat{\beta}_1$  and  $\hat{\beta}_2$  as the sample size is increased?

4. **Section 2.7 in AGME:** Follow the example on the Capital Asset Pricing Model (CAPM) from AGME (PDF attached titled *CAPM\_example*) and replicate the tables. The data is available on the *Ecdat* library under *Capm*.
5. **Exercise 2.3 in AGME:** Work on the empirical regression exercise from Asset Pricing in AGME (PDF attached titled *FactorModel\_AGME*). The data is available on the *assets2.dat* file.

On **g**, "partial slopes" refer to the individual betas.