Exercises Week 5

Econometrics

1. **Exercise 8.2 in ETM**: Consider the simple IV estimator computed first with an $n \times K$ matrix W of instrumental variables, and then with another $n \times K$ matrix WJ, where J is a $K \times K$ nonsingular matrix. Show that the two estimators coincide. Why does this fact show that $\hat{\beta}_{IV}$ depends on W only through the orthogonal projection matrix P_W?

2. **Exercise 8.3 in ETM**: Show that, if the matrix of instrumental variables W is $n \times K$, with the same dimensions as the matrix X of explanatory variables, then the generalized IV estimator is identical to the simple IV estimator.

3. **Exercise 8.1 in ETM**: Consider a very simple consumption function, of the form $c_i = \beta_1 + \beta_2 y_i^* + u_i^*$, with $u_i^* \sim IID(0, \sigma^2)$, where c_i is the logarithm of consumption by household i, and y_i^* is the permanent income of household i, which is not observed. Instead, we observe current income y_i, which is equal to $y_i^* + v_i$, where $v_i \sim IID(0, \omega^2)$ is assumed to be uncorrelated with y_i^* and u_i. Therefore, we run the regression
$$c_i = \beta_1 + \beta_2 y_i + u_i.$$

   Under the plausible assumption that the true value β_{20} is positive, show that y_i is negatively correlated with u_i. Using this result, evaluate the *plim* of the OLS estimator $\hat{\beta}_2$, and show that this *plim* is less than β_{20}.

4. Generate a sample of size 50 from the model
$$y_t = \beta_1 + \beta_2 x_t + u_t,$$
   with $\beta_1 = 1$ and $\beta_2 = 1$. For simplicity, assume that x_t are $NID(2, 2)$ and that the u_t are $NID(0, 1)$. Now generate a noisy version of the regressor, $x_t^* = x_t + u_{x,t}$, with $u_{x,t} \sim N(0, 1)$, and an instrument $w_t = \rho x_t + u_{w,t}$ with $u_{w,t} \sim N(0, 1)$ and ρ a value of your choosing between 0 and 1. Estimate the equation
$$y_t = \beta_1 + \beta_2 x_t^* + u_t,$$
   using OLS and using IV with w_t as an instrument. Also estimate the true OLS equation given by
$$y_t = \beta_1 + \beta_2 x_t + u_t.$$

   Store all the different estimates for β_2.

   Repeat at least 100 times and plot the empirical distribution function (EDF) for the three different estimates of β_2, using OLS and IV in the equation with measurement errors and the OLS estimates in the correct specification. Compare the densities and explain the results.

   What happens to the distribution if you change ρ? Explain.

   Hint: Use the *density()* function from the *stats* package to compute the EDF.

5. **Exercises 8.25-27 in ETM**: The dataset *Money* from the *Ecdat* package are described in Exercise 4 on the Exercise Set for Week 4.

   (a) Using these data, estimate the model
$$m_t = \beta_1 + \beta_2 r_t + \beta_3 y_t + \beta_4 m_{t-1} + \beta_5 m_{t-2} + u_t \tag{1}$$

by OLS for the period 1968:1 to 1998:4. Then perform a Hausman test for the hypothesis that the interest rate, r_t, can be treated as exogenous, using r_{t-1} and r_{t-2} as additional instruments.

(b) Estimate Equation (1) by generalized instrumental variables, treating r_t as endogenous and using r_{t-1} and r_{t-2} as additional instruments. Are the estimates much different from the OLS ones? Are the reported standard errors the same? Explain why or why not.

(c) Perform a Sargan test of the overidentifying restrictions for the IV estimation you performed above. How do you interpret the results of this test?