Exercises Week 7

Econometrics

1. **SUR with same regressors**: Show that SUR estimation is numerically identical to OLS estimation when all equations share the same regressors, $X_1 = X_2 = \cdots = X_g$.

   You may use these properties of the Kronecker product:

   $$(A \otimes B)^T = A^T \otimes B^T,$$
   $$(A \otimes B)(C \otimes D) = (AC) \otimes (BD),$$
   $$(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}.$$

2. **SUR's efficiency**: Generate a sample of size 100 from the model

   $$y_{1,t} = \beta_1 x_{1,t} + u_{1,t},$$
   $$y_{2,t} = \beta_2 x_{2,t} + u_{2,t},$$

   with $\beta_1 = 2$, and $\beta_2 = 0.5$. For simplicity, assume that $x_{1,t}$ and $x_{2,t}$ are $NID(2,2)$ independent from each other, and that the $[u_{1,t}, u_{2,t}]^T \sim MVNID([0,0]^T, [1, \rho; \rho, 1])$; that is, the errors are sample from a multivariate normal with correlation ρ, a value of your choosing between 0 and 1.

   (a) Estimate both equations one by one using OLS. Report the value of $\hat{\beta}_1^{OLS}$, and $\hat{\beta}_2^{OLS}$, and their standard deviation.

   (b) Now estimate the parameters using SUR. Report the value of the estimates and their standard deviation.

   (c) Repeat at least 1000 times and plot the empirical distribution function (EDF) for the estimates of $\hat{\beta}_1^{OLS}$ and $\hat{\beta}_1^{SUR}$. Compare the densities and explain the results. Do the same for $\hat{\beta}_2^{OLS}$ and $\hat{\beta}_2^{SUR}$.

   (d) What happens to the distributions if you change ρ? Explain.

3. **SUR in CAPM**: The *Capm* data set in the *Ecdat* package contains data on stock prices for 3 industries and the market portfolio.

   (a) Regress the excess returns on your three stocks upon the excess return on the market portfolio (proxy), noting that this corresponds to the CAPM. Use three OLS for these regressions and include a constant. Report the estimates and their standard deviation.

   (b) Estimate the three equations at once using a SUR. Report the estimates and their standard deviation. Compare them against the previously obtained.

   (c) Test the CAPM by testing that the estimates for the constant terms are all zero using the SUR.

4. **Investment equation**: Grunfeld (1958) considered the following investment equation:

   $$I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}$$

   where I_{it} denotes real gross investment for firm i in year t, F_{it} is the real value of the firm (shares outstanding) and C_{it} is the real value of the capital stock. The data set consists of 10 large U.S. manufacturing firms over 20 years, 1935–1954. The data is contained in $Grunfeld$ in the *Ecdat* package.

   Consider the first three firms: G.M., U.S. Steel and General Electric.

(a) Run OLS of I on a constant, F and C for each of the 3 firms separately. Print the covariance matrix of the estimates.

(b) Run SUR for the first two firms. Compare with OLS.

(c) Run SUR for the three assigned firms. Compare these results with those above.

(d) Test for the diagonality of Σ across these three equations.

(e) Test for the equality of all coefficients across the 3 firms.