Exercises Week 1

Econometrics

1. **Ex. 2.9 in ETM**: Prove algebraically that $P_X M_X = 0$. Use only the definition that $P_X + M_X = I$, and the idempotency of P_X.

2. **Ex. 2.10 in ETM**: Let X and W be two $n \times k$ matrices such that $S(X) \neq S(W)$. Show that the $n \times n$ matrix $P := X(W'X)^{-1}W'$ is idempotent but not symmetric.

This matrix is called the projection matrix on to $S(X)$ along $S(W)$, and it will be used in IV regression.

3. **Ex. 2.12 in ETM**: Show algebraically that, if P_X and M_X are complementary orthogonal projections, then M_X annihilates all vectors in $S(X)$, and P_X annihilates all vectors in $S^{\perp}(X)$.

3. **Normality of OLS estimator**: In this exercise, you are going to conduct a Monte Carlo simulation to show graphically that the OLS estimator follows a normal distribution under all the assumptions.

Hence, you will need to follow the steps below:

- Set the sample size $n = 100$.
- Then, for $R = 1000$ repetitions, do the following:
  i. Generate a regressor x from a normal distribution with mean 0 and variance 1.
  ii. Generate an error term u from a normal distribution with mean 0 and variance σ^2 of your choosing.
  iii. Generate a dependent variable y from the following model:

$$y = \beta_1 x + u,$$

  for $\beta_1 = 1$.

  – Why we dont need to generate an intercept (constant term)?

  iv. Estimate the model above and store the OLS estimator $\hat{\beta}_1$ in a vector of size R.
- Plot the histogram of $\hat{\beta}_1$.
- Compare the histogram with the normal distribution with mean β_1 and variance $\sigma^2(X'X)^{-1}$.
  – What is $plim(\frac{1}{n}X'X)^{-1}$ for this model?
- Increase the sample size and comment on the results.

- **Hint**: You can use the function `randn()` to simulate both the regressor x and the error term u.

1. **Section 2.7 in AGME**: Follow the example on the Capital Asset Pricing Model (CAPM) from AGME (PDF attached titled *CAPM___example*) and replicate the tables. The data is available on the* Ecdat* library under *Capm*.

**Note that there are small differences with the results in the book. It may be due to small errors at capturing the data.*

For now, do not pay much attention to the discussion on consistency and asymptotic approximations. We will talk about that in future lectures.