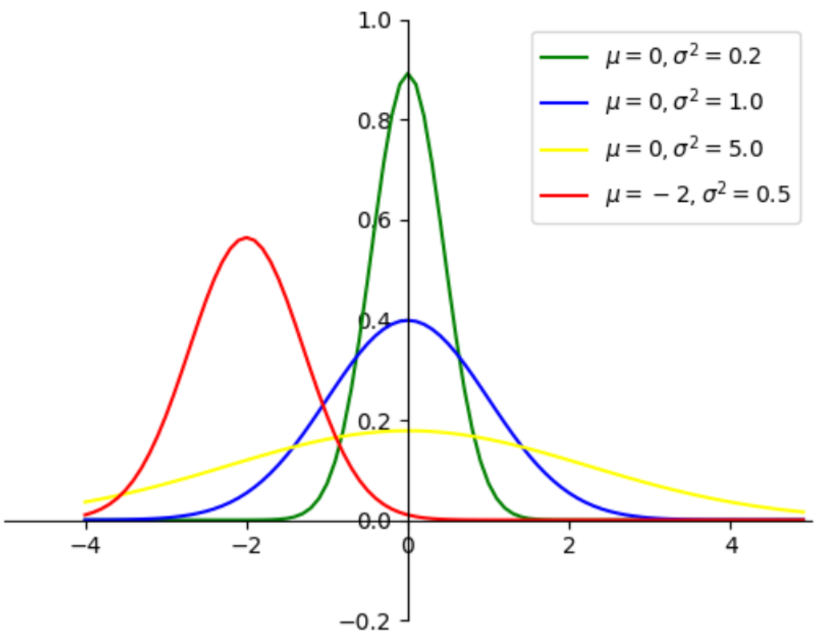
Assignment6

1. NumPy Warm-up



$$\int_R G(x)dx = 1.0000000000000002 \approx 1 \quad (\ \mu = 0, \sigma^2 = 0.2 \ )$$

$$\int_R G(x)dx = 0.9999949896680438 \approx 1 \quad (\ \mu = 0, \sigma^2 = 1.0 \ )$$

$$\int_R G(x)dx = 0.9999994266968562 \approx 1 \quad (\ \mu = 0, \sigma^2 = 5.0 \ )$$

$$\int_R G(x)dx = 0.9999999999984623 \approx 1 \quad (\ \mu = -2, \sigma^2 = 0.5 \ )$$

So the identity is verified.

2. Numerics and Linear Algebra

2.1 L2 norm by LU

For Vandermonde

when M is 8, N is 8, L2 norm is 6.1814×10^{-16}

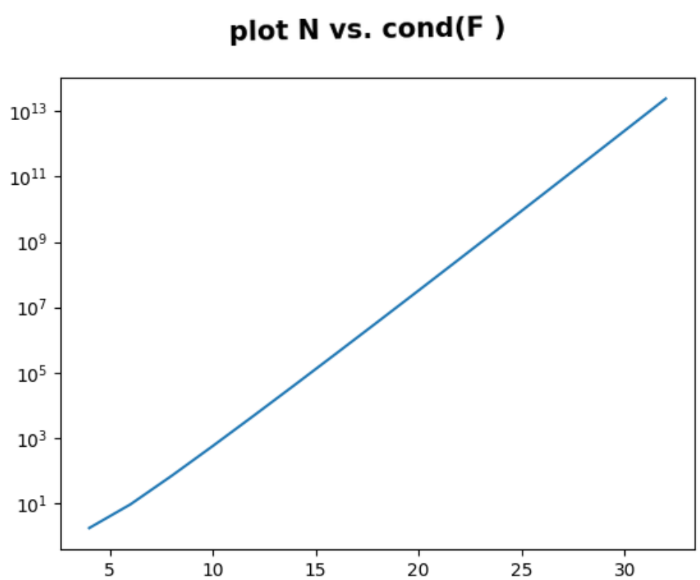
when M is 16, N is 16, L2 norm is 3.7583×10^{-15}

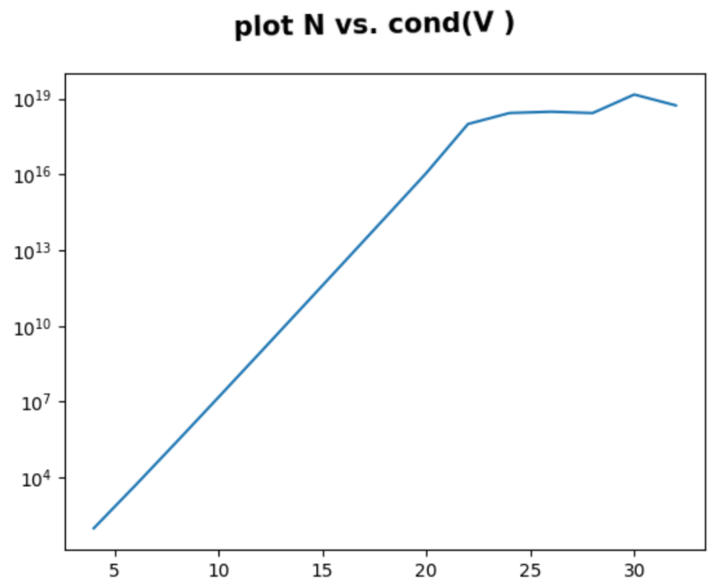
For Fourier

when M is 8, N is 8, L2 norm is 3.8459×10^{-16}

when M is 16 N is 16 L2 norm is 1.6279×10^{-15}

2.2 Cond graph





reasons of trend

As the order of the equation increases, the numerical instability of the solution increases due to the increasing pathological nature of the coefficient matrix, and therefore the cond also increases， but I can't explain why the Vandemonium matrix cond drops after 30.

2.3 table of Cond and positive definite

N	4	6	8	10	12	14	16	18
Posdef (V)	1	1	1	1	1	1	0	0
Posdef (F)	1	1	1	1	1	1	1	1
cond (V)	98.87	4924.37	2.67×10^{5}	1.52×10^{7}	8.83×10^{8}	5.22×10^{10}	3.12×10^{12}	1.88×10^{15}
cond (F)	1.73	9.17	68.55	526.76	4830.61	4.27×10^{4}	3.85×10^{5}	3.53×10^{6}

N	20	22	24	26	28	30	32
Posdef(V)	0	0	0	0	0	0	0
Posdef(F)	1	0	1	0	0	0	0
cond(V)	1.53×10^{16}	9.94×10^{17}	2.71×10^{18}	3.05×10^{18}	2.70×10^{18}	1.46×10^{19}	5.42×10^{18}
cond(F)	3.27×10^{7}	3.07×10^{8}	2.90×10^{9}	2.75×10^{10}	2.64×10^{11}	2.54×10^{11}	2.45×10^{11}

- largest value of N where AV is positive definite： 14
- the condition number of that V： 5.22×10^{10}
- largest value of N where AF is positive definite：24
- condition number of that F： 2.90×10^{9}

Connection:

The larger N is, the larger cond is, and the larger cond is, the more likely it is not positive definite.

2.4 L2 norm under Cholesky

For Vandermonde

when M is 8, N is 8 , L2 norm is 2.8601×10^{-11}

For Fourier

when M is 8, N is 8 , L2 norm is 1.0283×10^{-14}

Compared to LU

L2 norm made by Cholesky decomposition is bigger than LU's, howerver they are both too small.

3. Least Squares Problems and QR

3.1 QR decomposition

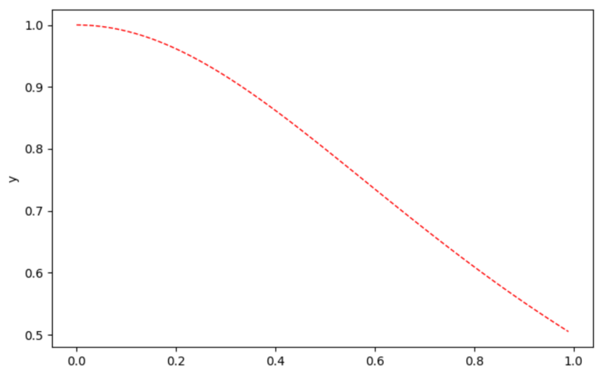
M is 16 ,N is 4：

$$c = [1.00166564 \quad -0.02698999 \quad -1.01909544 \quad 0.54698731]^T$$

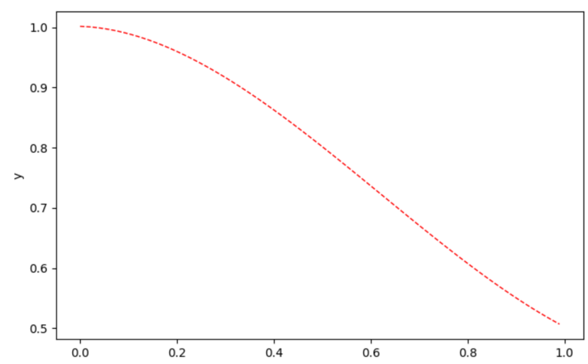
M is 16 ,N is 8：

$$c = [1.00000188 \quad -1.07143503 \times 10^{-3} \quad -9.75766185 \times 10^{-1} \quad -2.00107692 \times 10^{-1} \quad 1.80128191 \quad -1.67195234 \quad 6.25188414 \times 10^{-1} \quad -7.75730913 \times 10^{-2}]^T$$

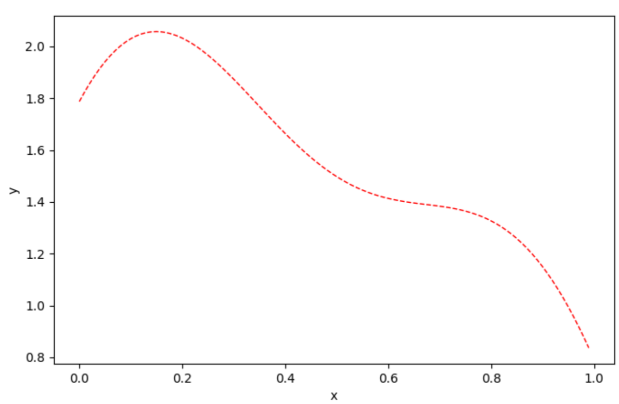
3.2 Plot the gV , gF



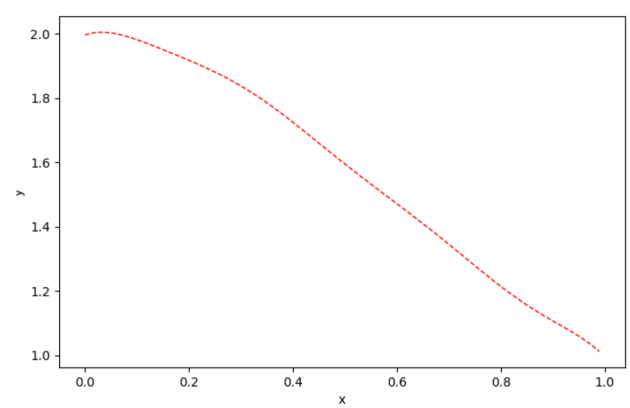
gV, M = 16, N = 4



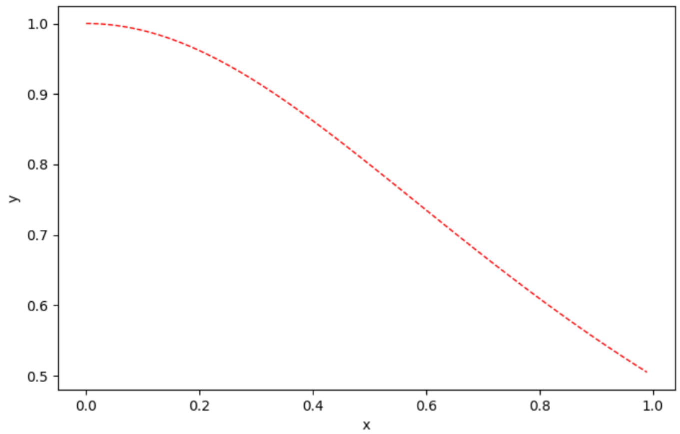
gV, M = 16, N = 8



gF, M = 16, N = 4



gF, M = 16, N = 8



function f(x)

From the above graph, we can see that Vandermonde fits f(x) better, while Fourier shows a larger volatility when N is 4.

By plotting the solutions obtained in 3.1, it is obtained that the fitted graph is highly similar to f(x) under the Vandermonde matrix, but the image obtained by QR decomposition is more accurate than LU decomposition under the Fourier matrix. ((The image of 3.1 will not be posted due to the limitation of space)

4. Image Alignment

4.1 Get new picture

I used least squares method, with LU, Cholesky, and QR decomposition respectively to get matrix A and B, and they are the same.

$$A = \begin{bmatrix} 8.83269474 \times 10^{-1} & 2.67849991 \times 10^{-1} \\ -2.65621407 \times 10^{-1} & 8.70814325 \times 10^{-1} \end{bmatrix}$$

$$B = \begin{bmatrix} 5.64529375 \\ 2.75936885 \times 10^{2} \end{bmatrix}$$

alignment by LU

alignment by QR

alignment by Cholesky

4.2 difference

Although the above decompositions give the same results in this problem, they can yield different results according to the results obtained in 3.2.

For example, the QR decomposition is more accurate and stable than the LU decomposition under certain conditions. The larger amount of data or the more unstable matrix may lead to larger deviations in the final image when aligning is performed.